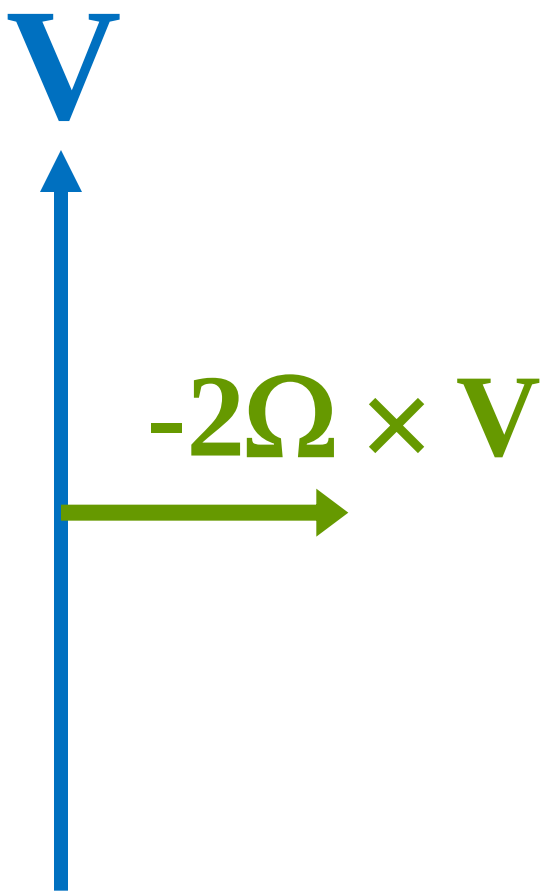
Coriolis II

The Coriolis Effect on
a rotating planet

What does the Coriolis term say?

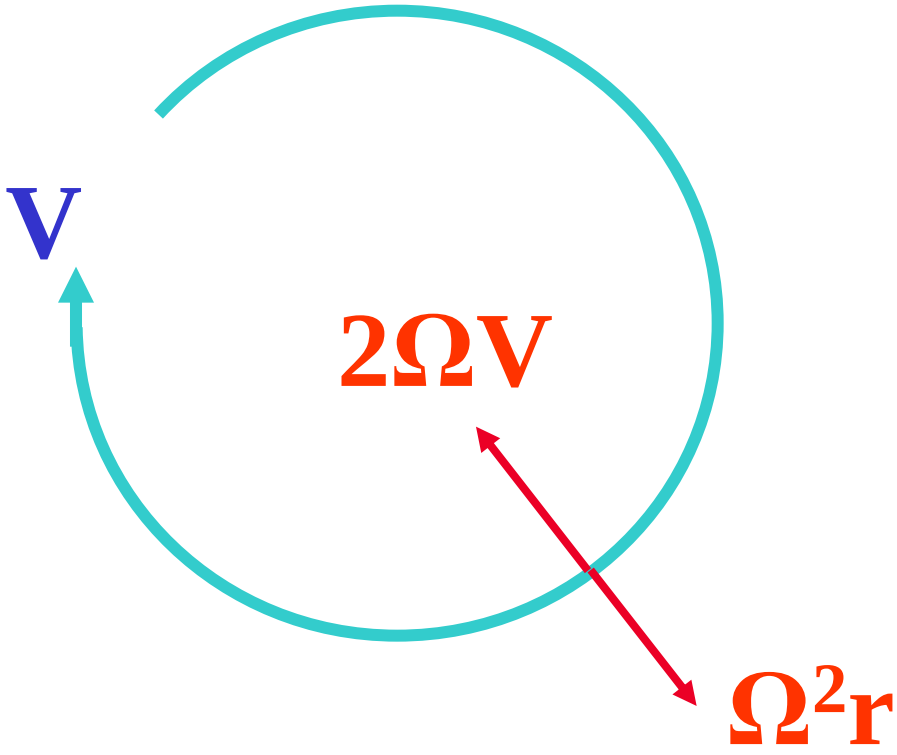
$$-2\Omega \times \mathbf{V}$$



. . . . and what does this mean?

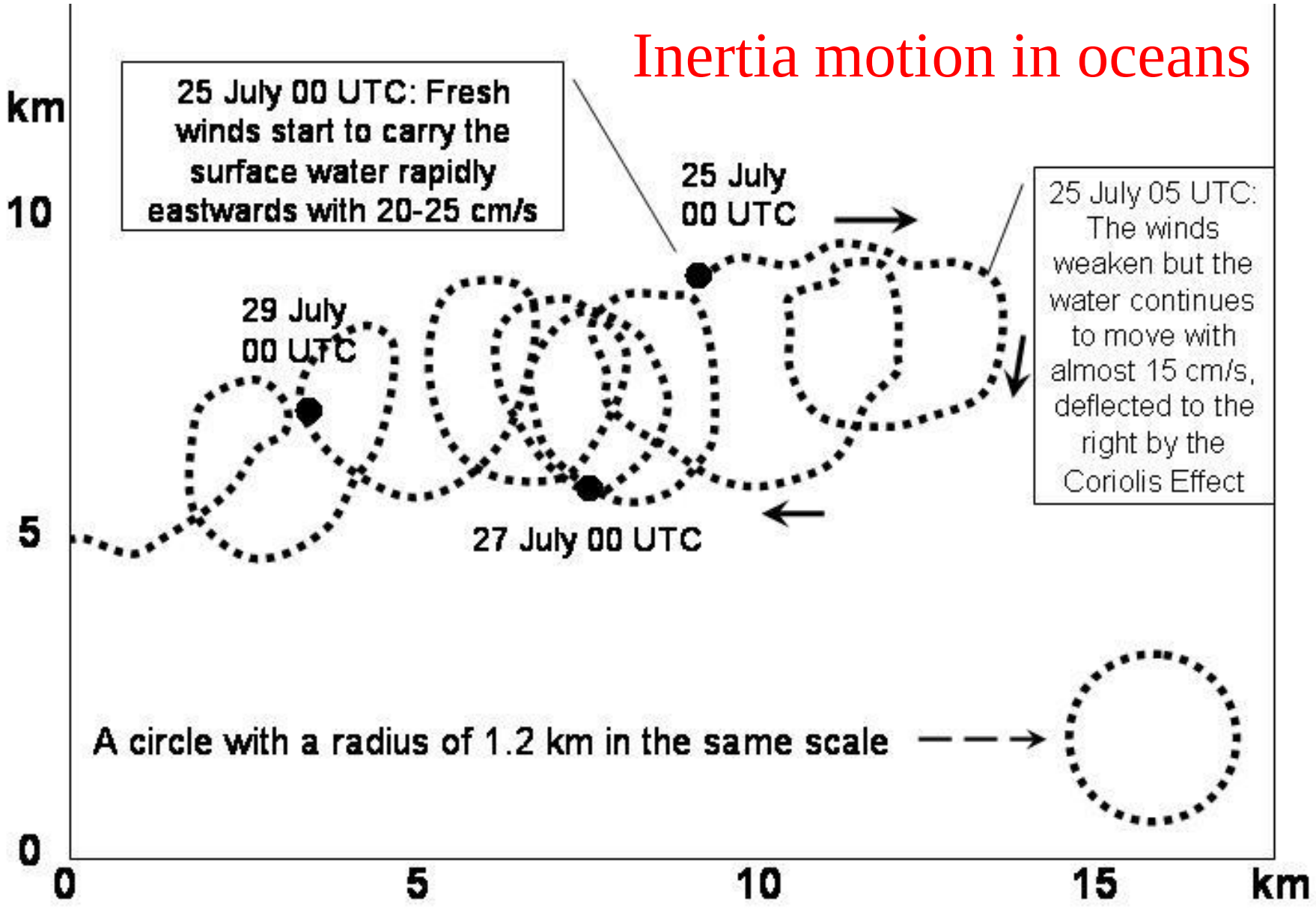
What does -2Ω×V mean???

It defines the "inertia circle" motion

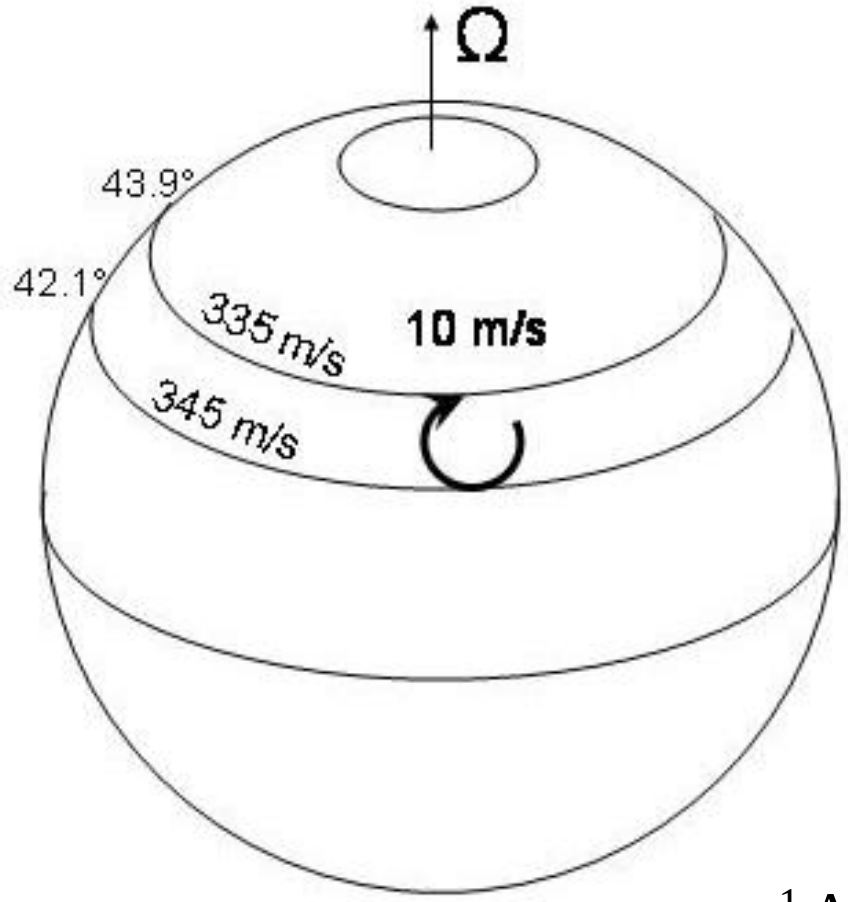


The radius of the inertia circle
r = 2V/Ω

Inertia motion in oceans



Frictionless motion along the earth's surface[1] are confined to small inertia circle trajectories

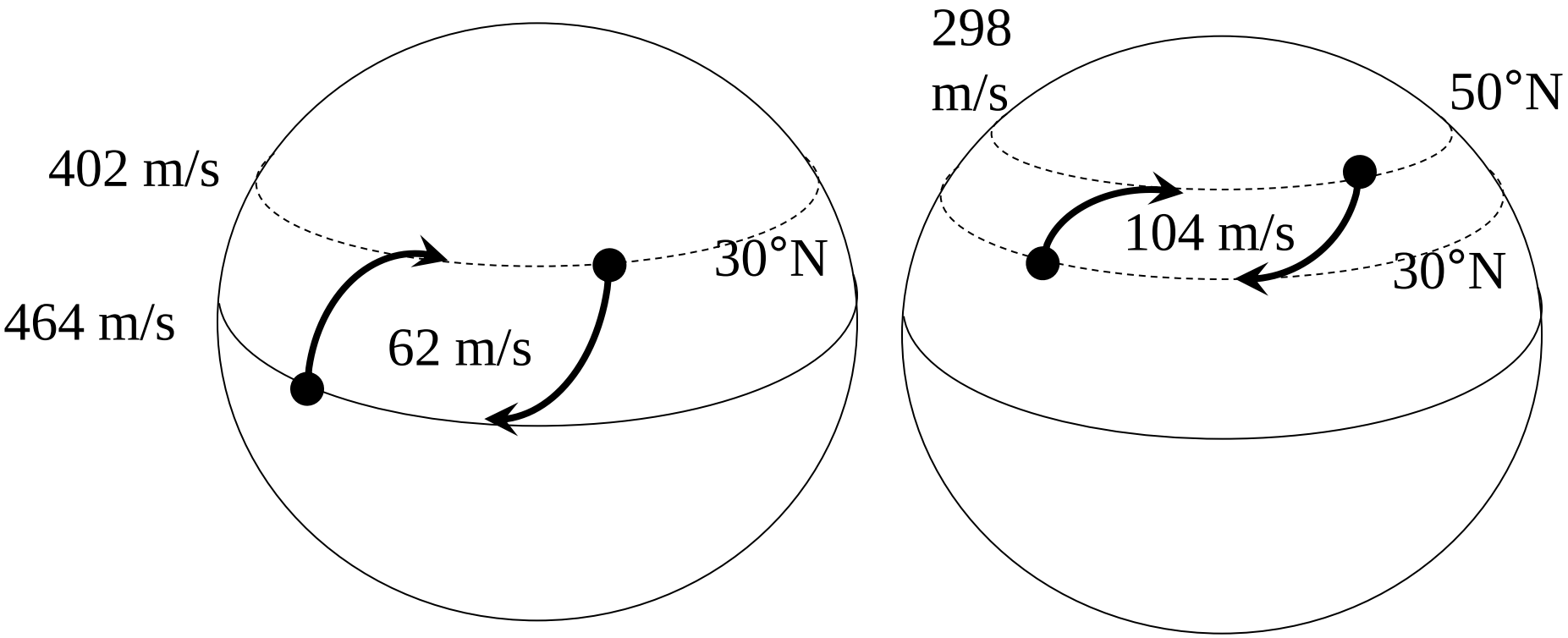


There is no increase in speed (kinetic energy)

[1] Away from the tropical latitudes

AND NOT LIKE THIS!

The popular, but erroneous “Hadley’s Principle” using conservation of absolute velocity

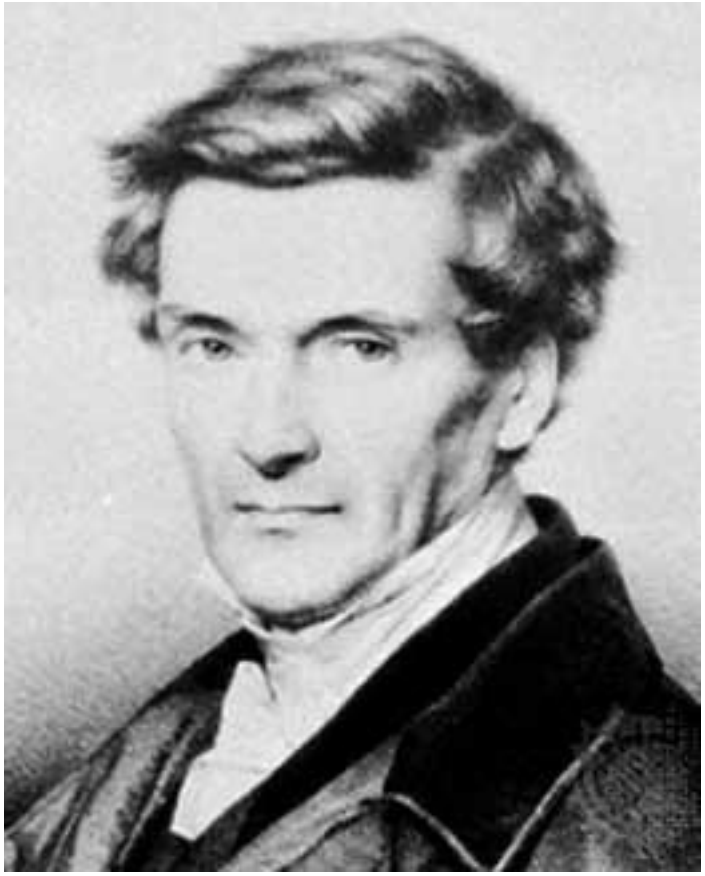


Common explanation: the excessive winds are retarded by friction

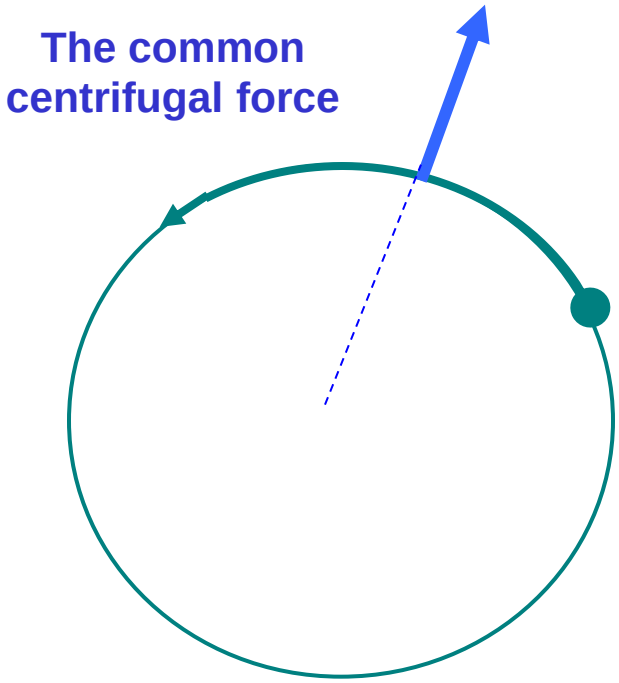
The popular breakthrough for George Hadley's "Principle", championed by a German meteorologist Heinrich W. Dove, came in the 1830's more or less exactly when Coriolis presented his paper

The period 1860-1900 saw several correct analyses of the deflection over the earth's surface based on Coriolis (1835) but from about 1905 the confusion stats to set in:

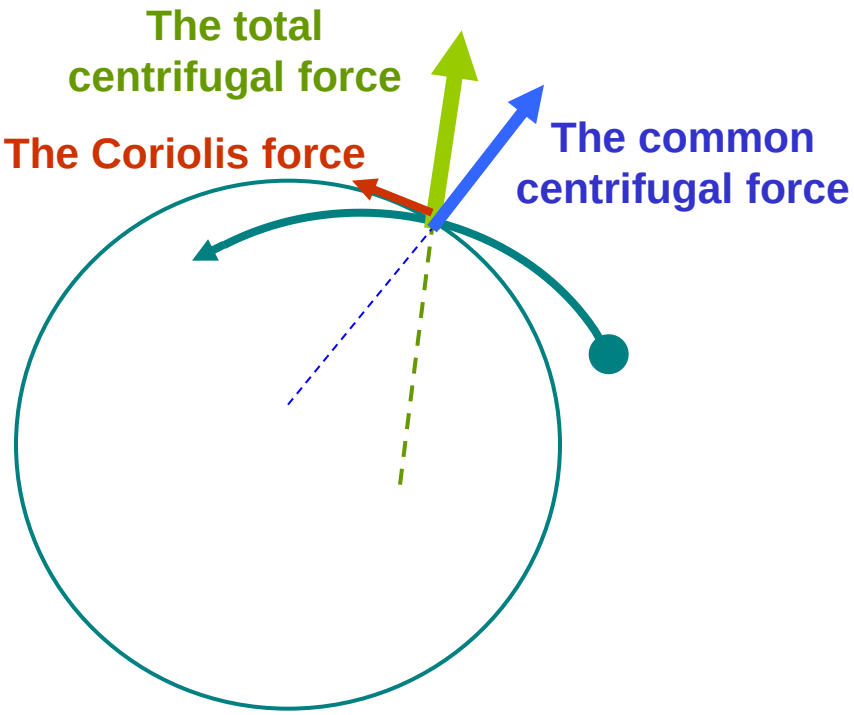
"Hadley's Principle" is so simple and easy to understand **- it cannot be wrong!**



So how did these Coriolian analyses look like?



A stationary object in the rotating system



An object moving (inwards) in the rotating system

The Coriolis force is the extra force that has to be added to the common centrifugal force for an relatively moving object

This is where we left Coriolis (1835) in the previous lecture

The Coriolis force (per unit mass)

The centrifugal acceleration

$$\left(\frac{d\mathbf{V}_r}{dt}\right)_r = \frac{d\mathbf{V}}{dt} - 2\boldsymbol{\Omega}\times\mathbf{V}_r - \boldsymbol{\Omega}\times(\boldsymbol{\Omega}\times\mathbf{r})$$

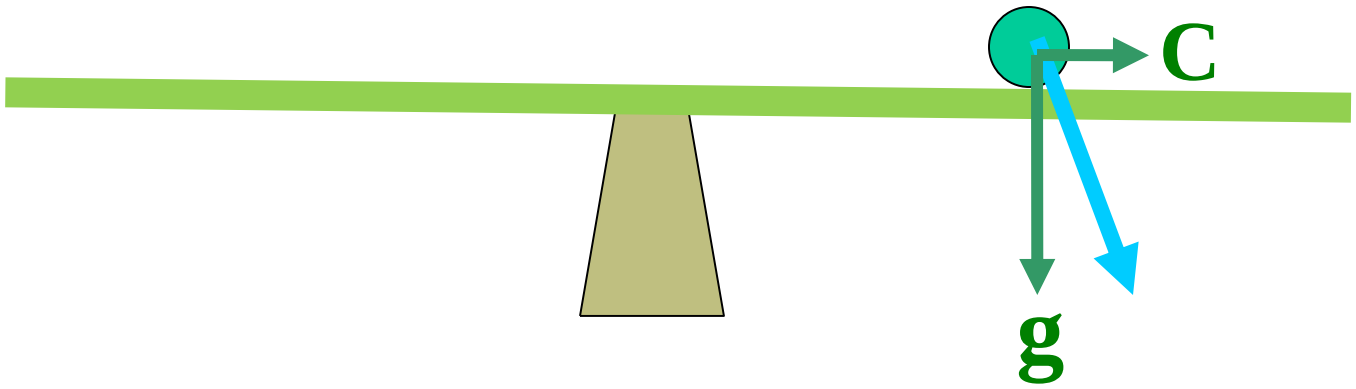
Applied on a flat rotating carousel all we have to do is

$$\left(\frac{d\mathbf{V}_r}{dt}\right)_r = \mathbf{g} - 2\boldsymbol{\Omega}\times\mathbf{V}_r - \boldsymbol{\Omega}\times(\boldsymbol{\Omega}\times\mathbf{R})$$

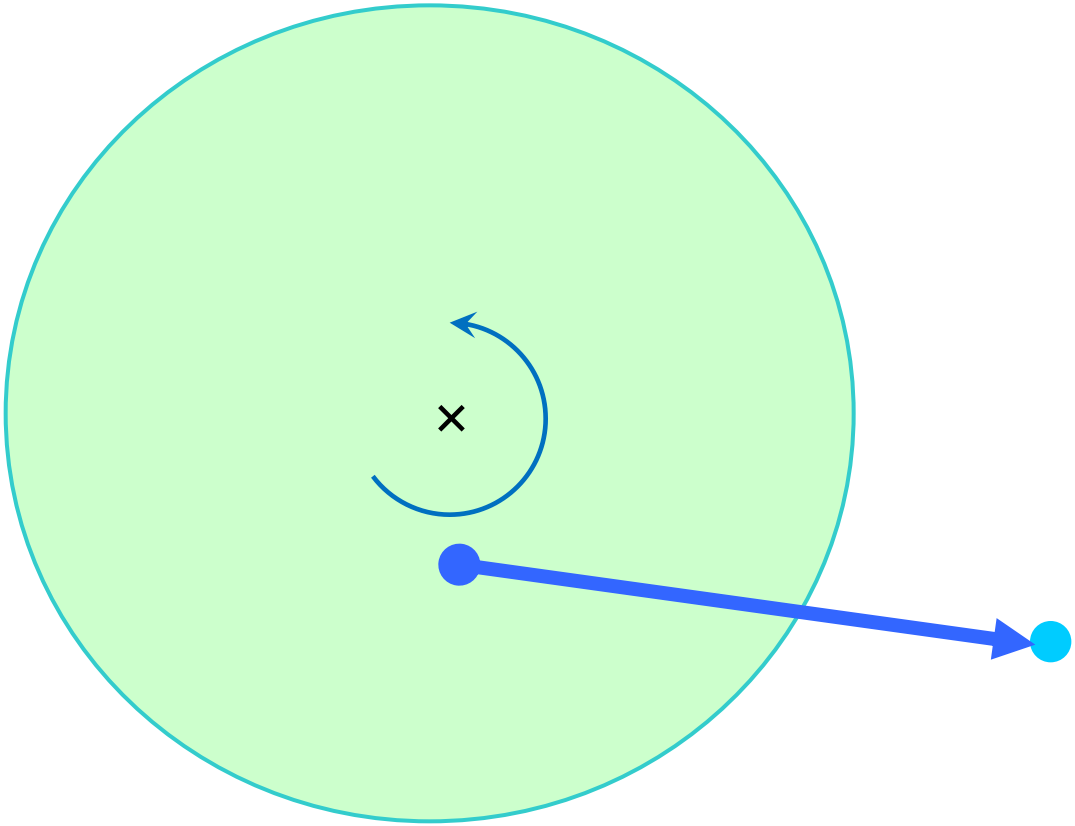
Gravity

R = distance to the rotational axis

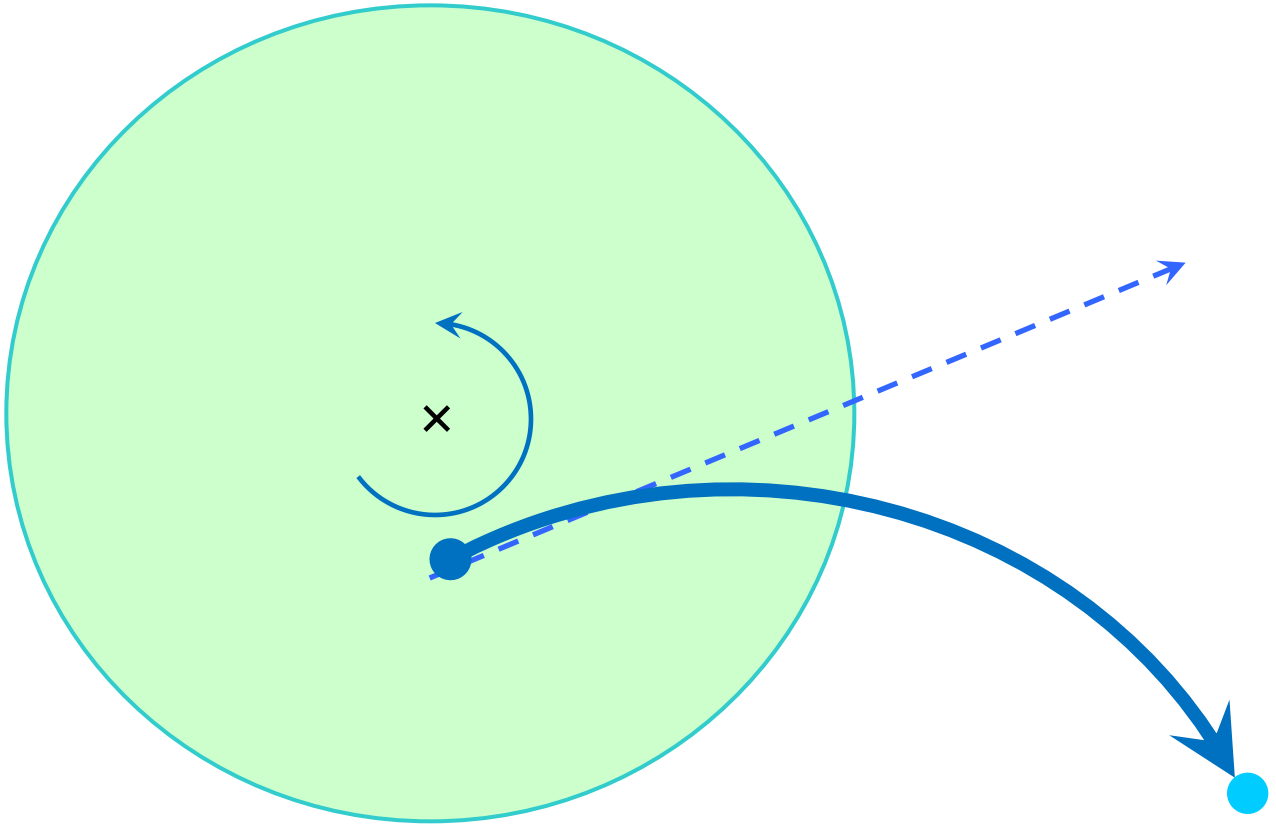
This is the equation for a moving body on a rotating carrousel where gravity and the centrifugal force are perpendicular



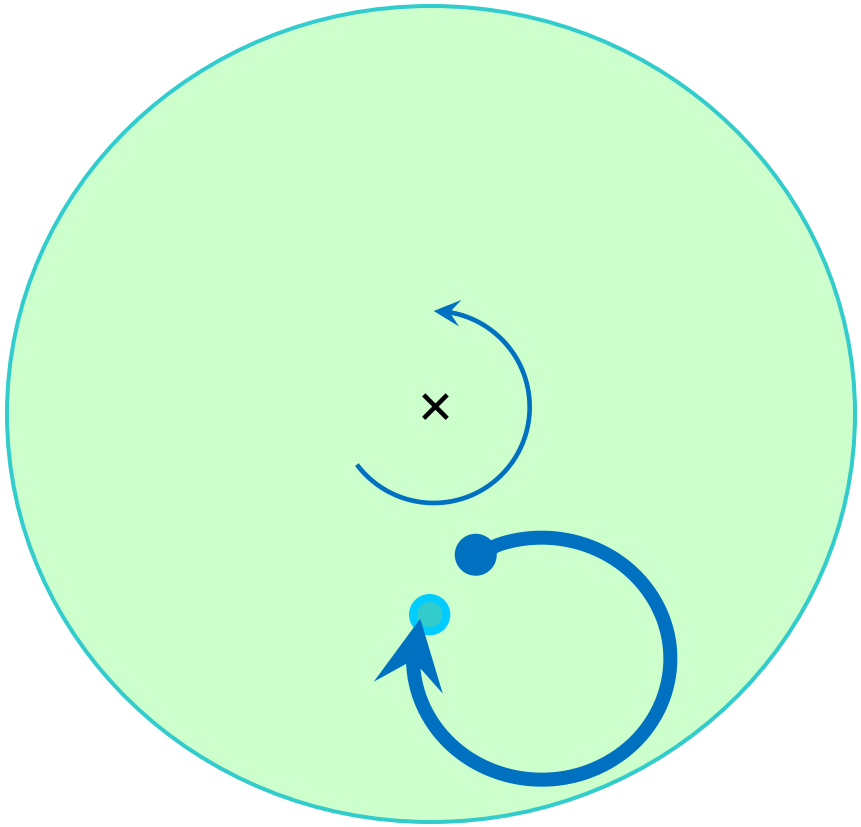
Seen from outside a body is moving out from a rotating carrousel ...



The deflection relative the rotating carrousel is described by the combined effect of the Coriolis and centrifugal forces



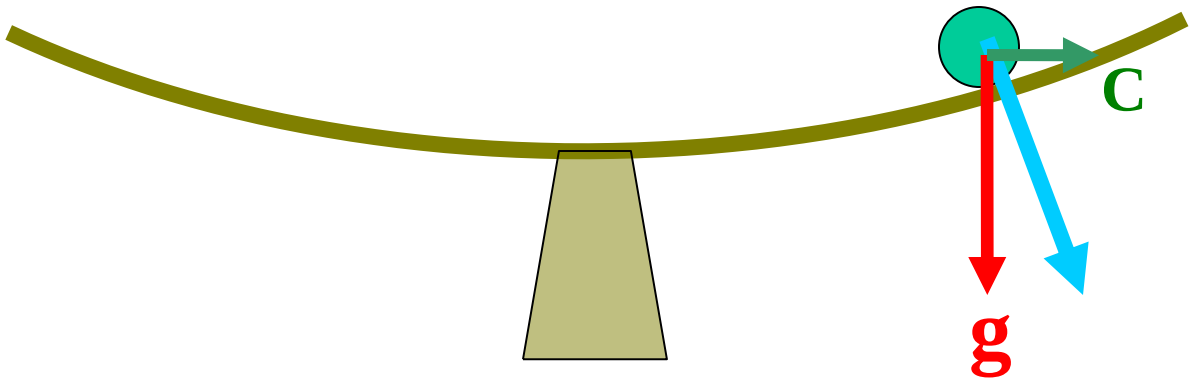
If only the Coriolis force had been active, and not the centrifugal forcem the relative path would have been a closed “inertia” circle



But how can we get rid of the centrifugal force?

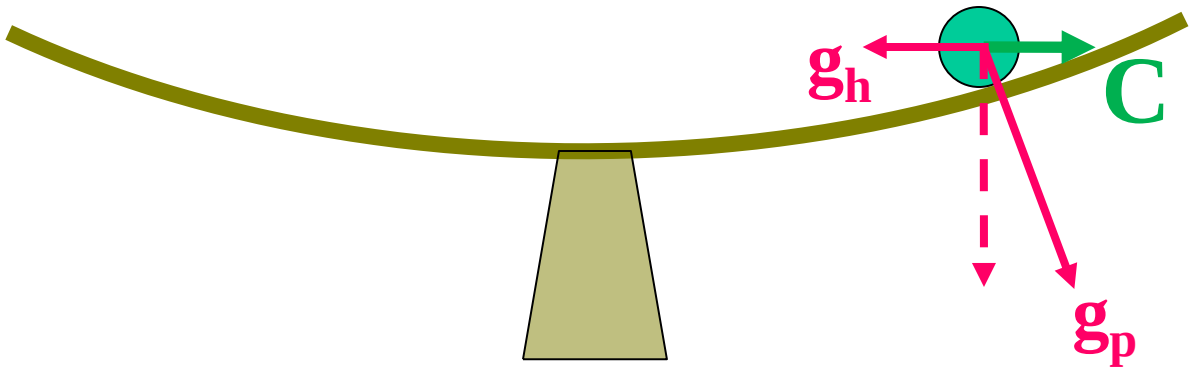
We can “get rid of” the centrifugal force by making the carrousel concave-

This makes the horizontal component of gravity cancel the centrifugal force

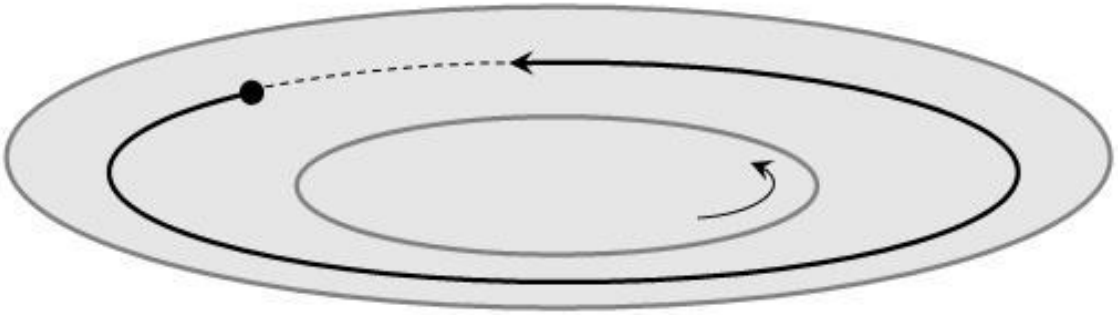


Resolve gravity g into one horizontal (h) and one perpendicular (p) vector.

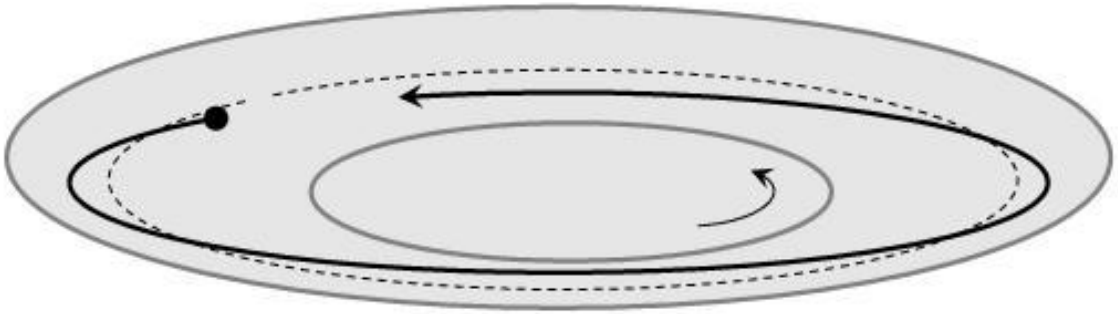
The "horizontal" component of gravity balances the centrifugal force



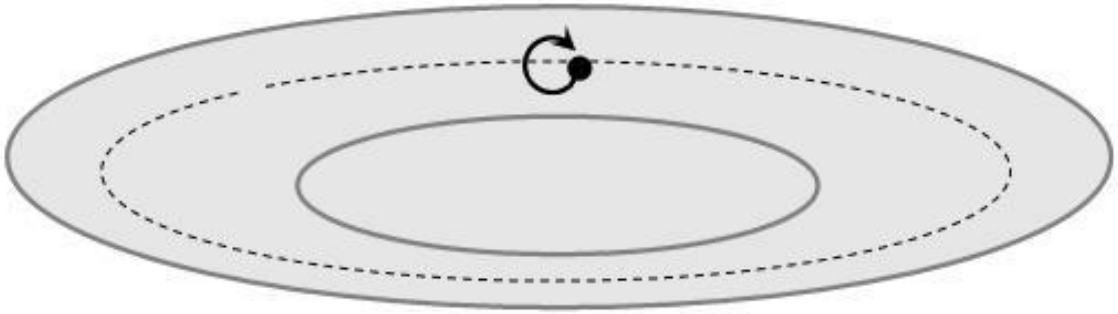
The motion seen from outside



The small body is not perturbed

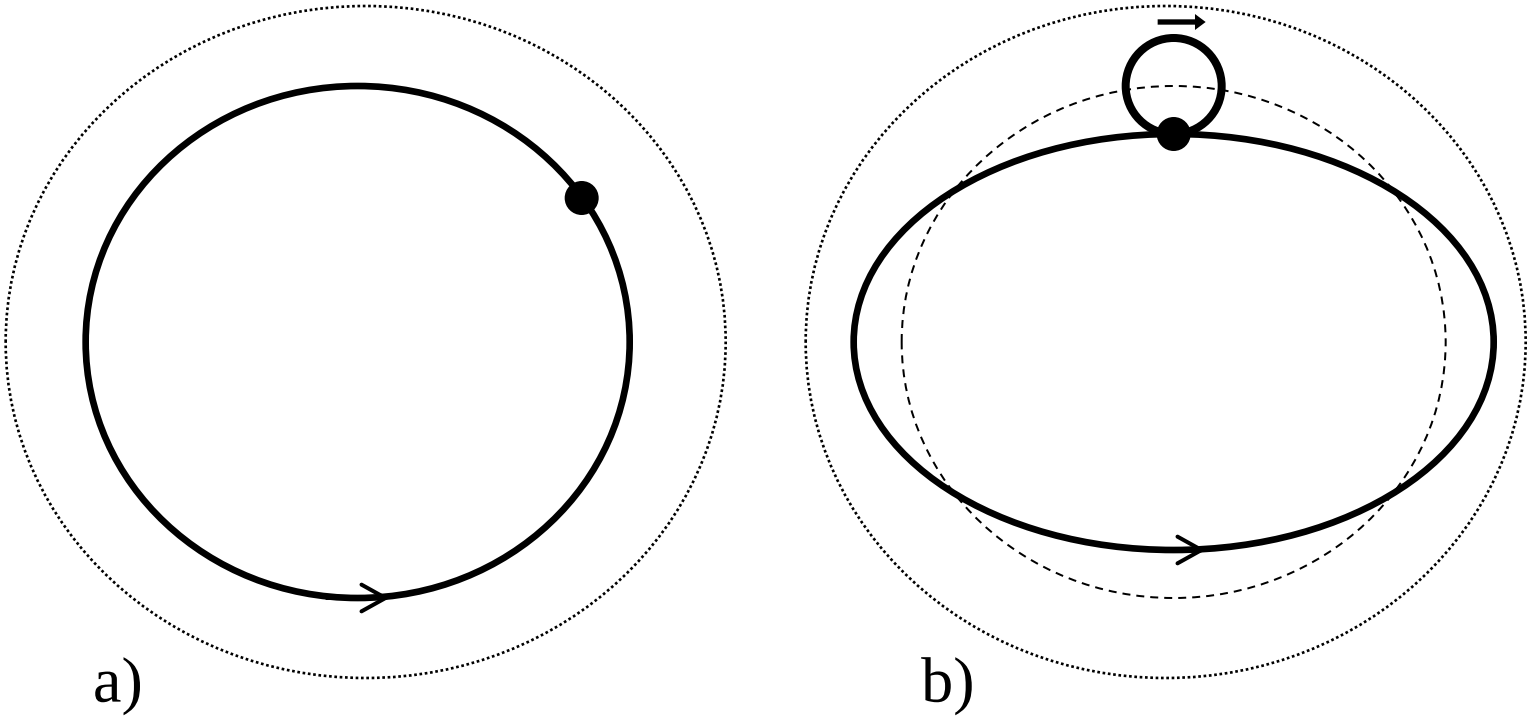


The small body is perturbed

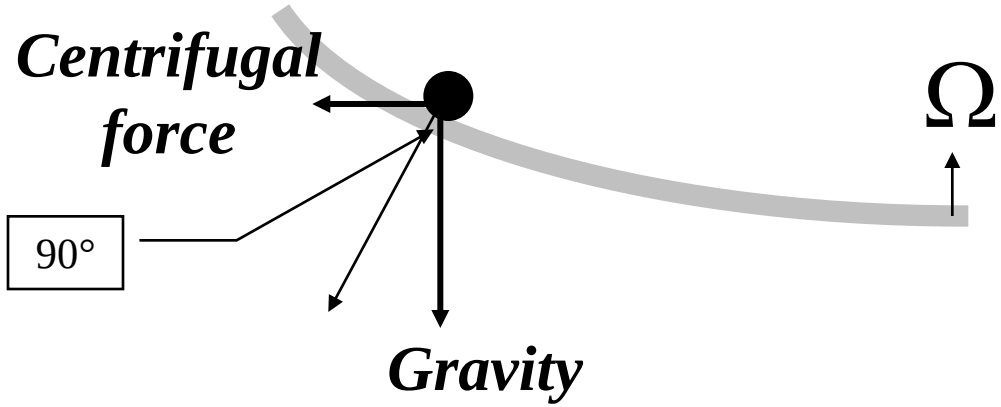


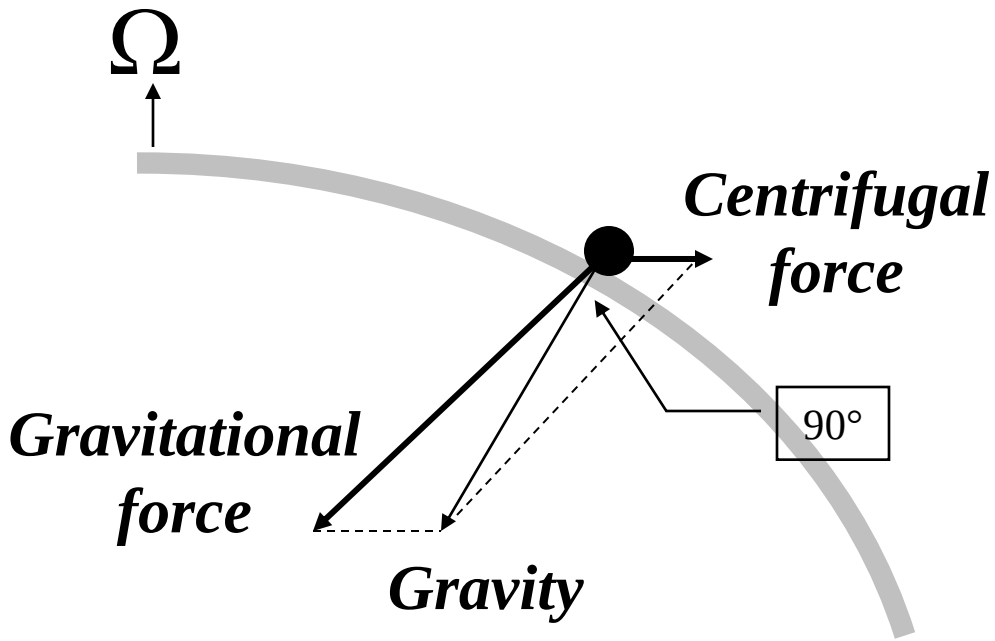
The same seen from “inside” the rotating parabola

The same seen from above



So now we know how to “get rid of” the centrifugal force on a carousel, but what about the earth?





Exactly in the same way, by making use of the shape of the earth

Let us go further where we left Coriolis (1835) in the previous lecture

The Coriolis force (per unit mass)

The centrifugal acceleration

$$\left(\frac{d\mathbf{V}_r}{dt}\right)_r = \frac{d\mathbf{V}}{dt} - 2\boldsymbol{\Omega}\times\mathbf{V}_r - \boldsymbol{\Omega}\times(\boldsymbol{\Omega}\times\mathbf{r})$$

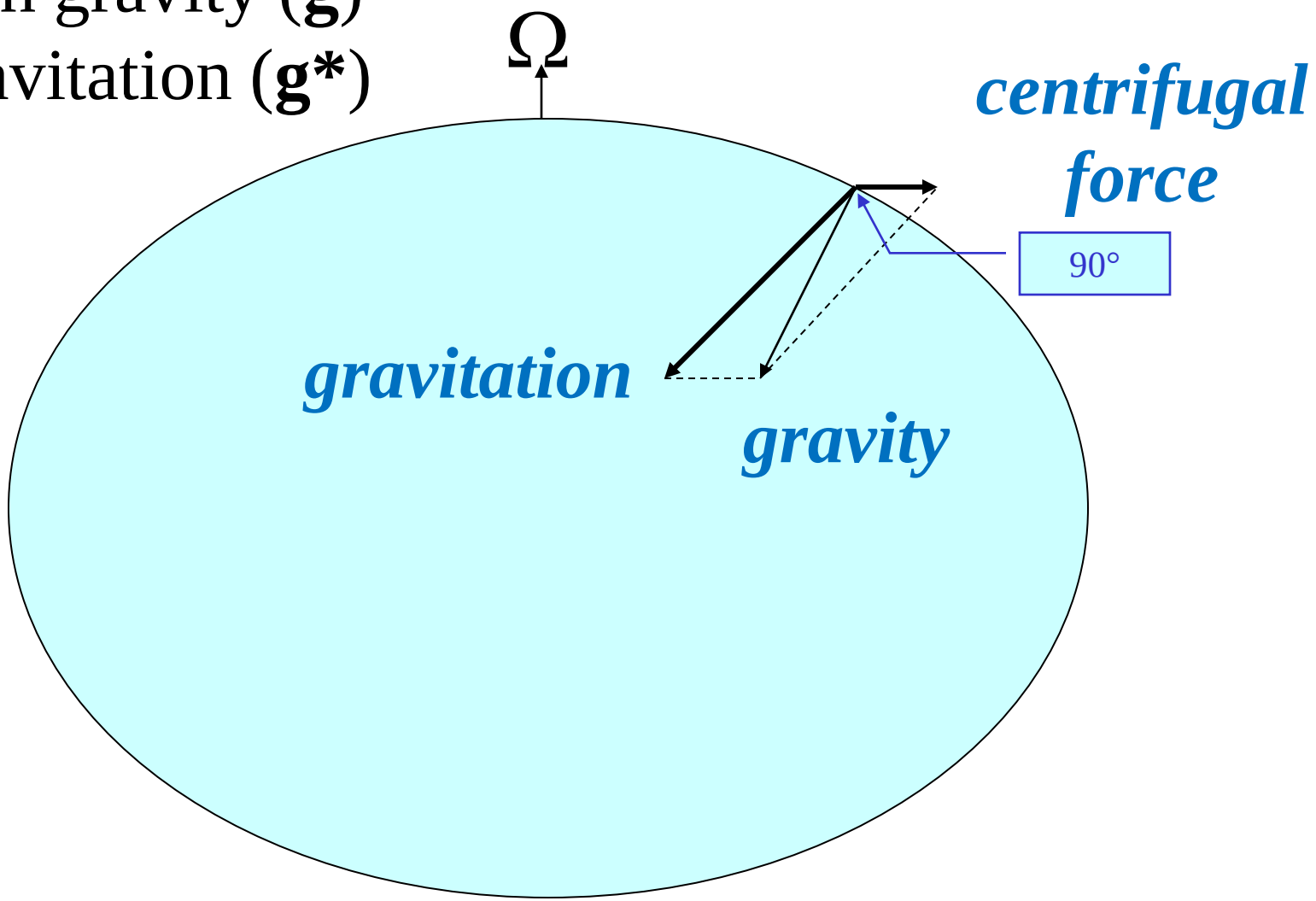
Applied on the rotating earth all we have to do is

$$\left(\frac{d\mathbf{V}_r}{dt}\right)_r = \mathbf{g}^* - 2\boldsymbol{\Omega}\times\mathbf{V}_r - \boldsymbol{\Omega}\times(\boldsymbol{\Omega}\times\mathbf{R})$$

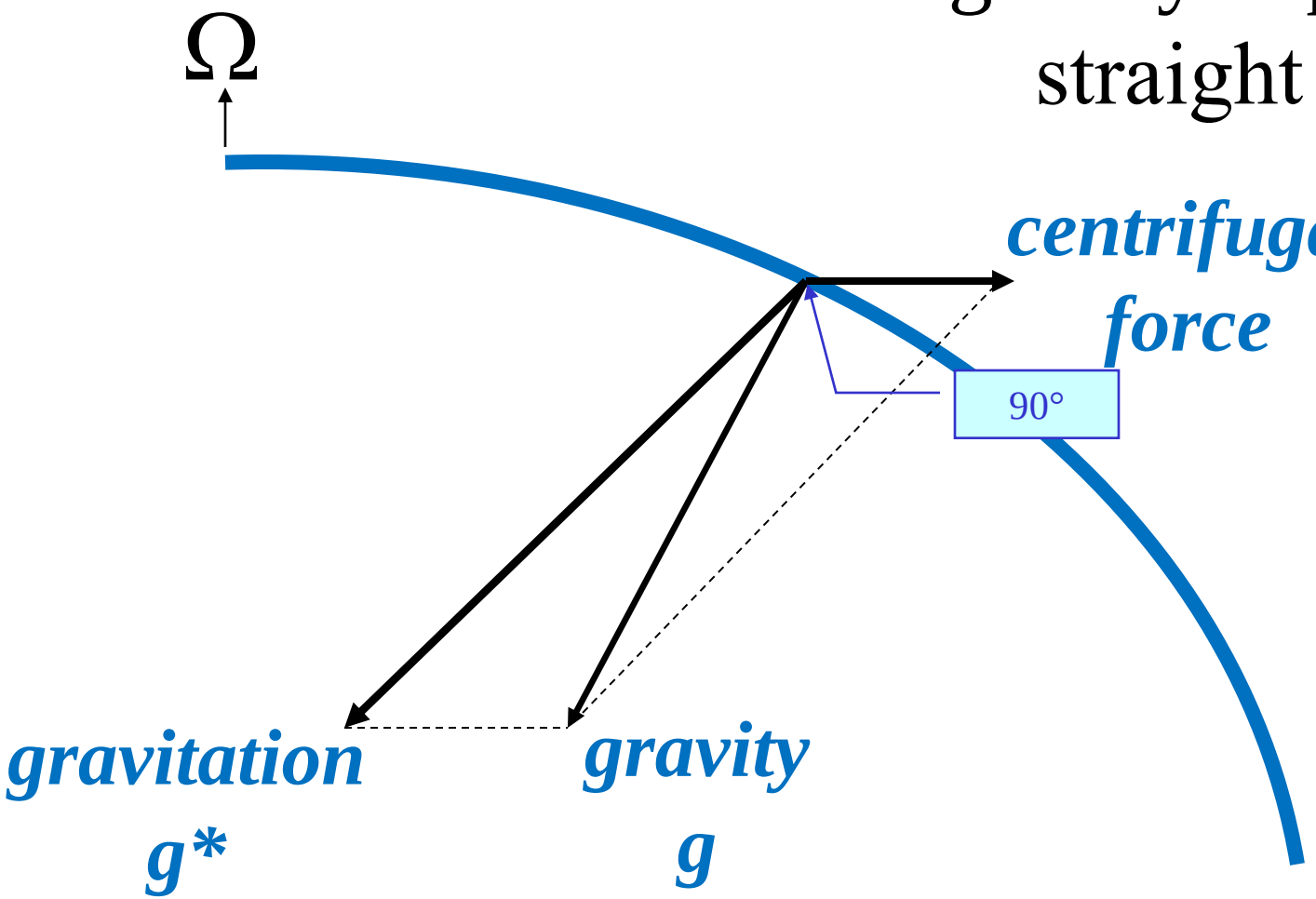
The earth's gravitational attraction

R = distance to the earth's rotational axis

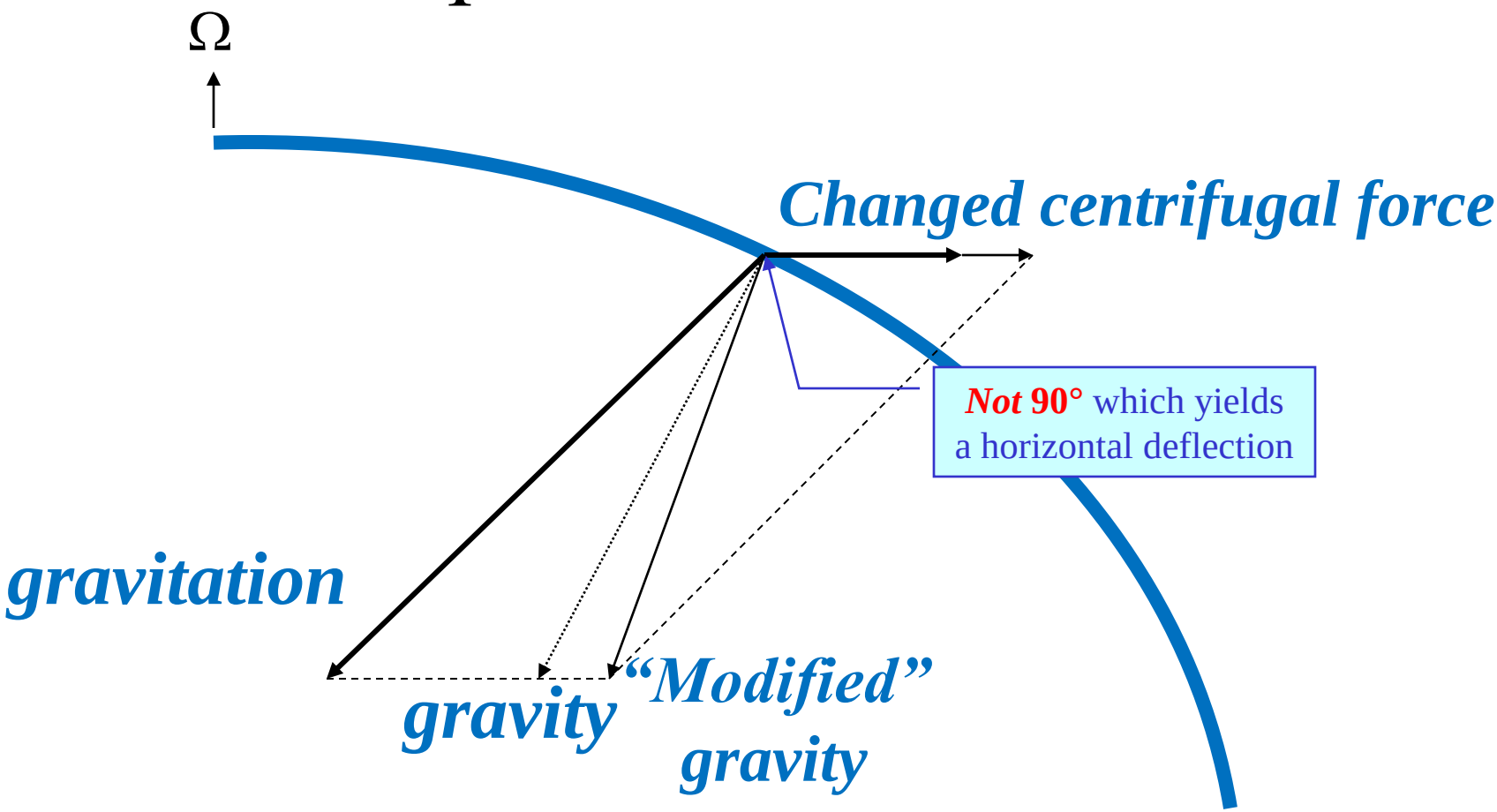
The crucial difference between gravity (**g**) and gravitation (**g***)



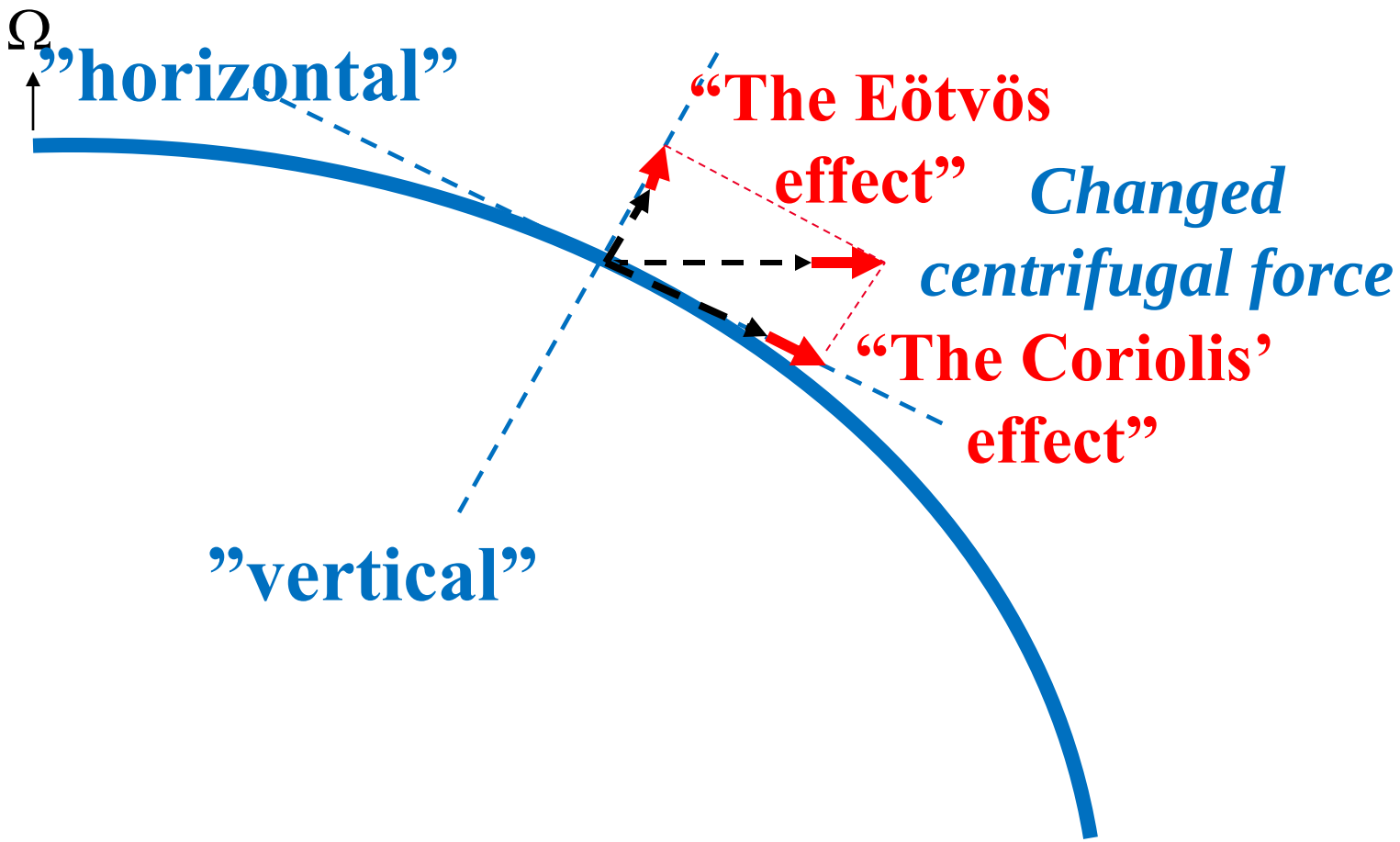
The force of gravity is pointing straight “down”



… except when there is motion



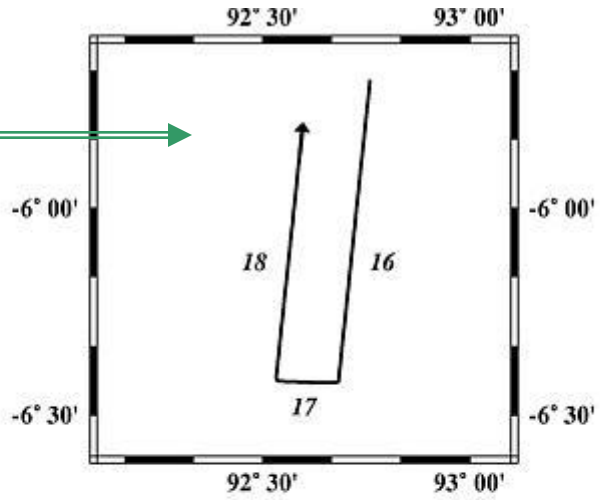
The sibling to the Coriolis Effect



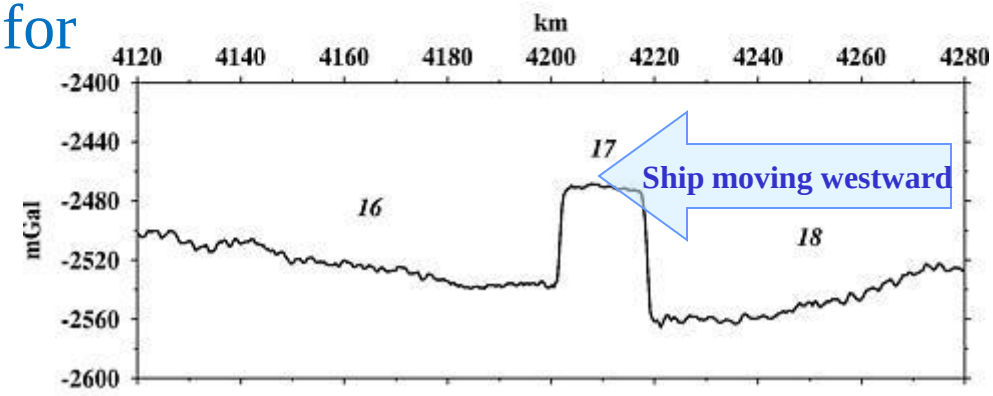
The Eötvös effect: We become lighter, weight less, when we move eastward, heavier when we move westward

Example of the Eötvös effect

Results from a French research vessel moving around in the Indian Ocean



The gyration between profiles 16 and 17 produces a 60 mGal increase in the gravity values which is not associated with any bathymetric variations. Gravity values are more noisy for NS profiles than for EW ones. (Helene Hebert, personal communication)



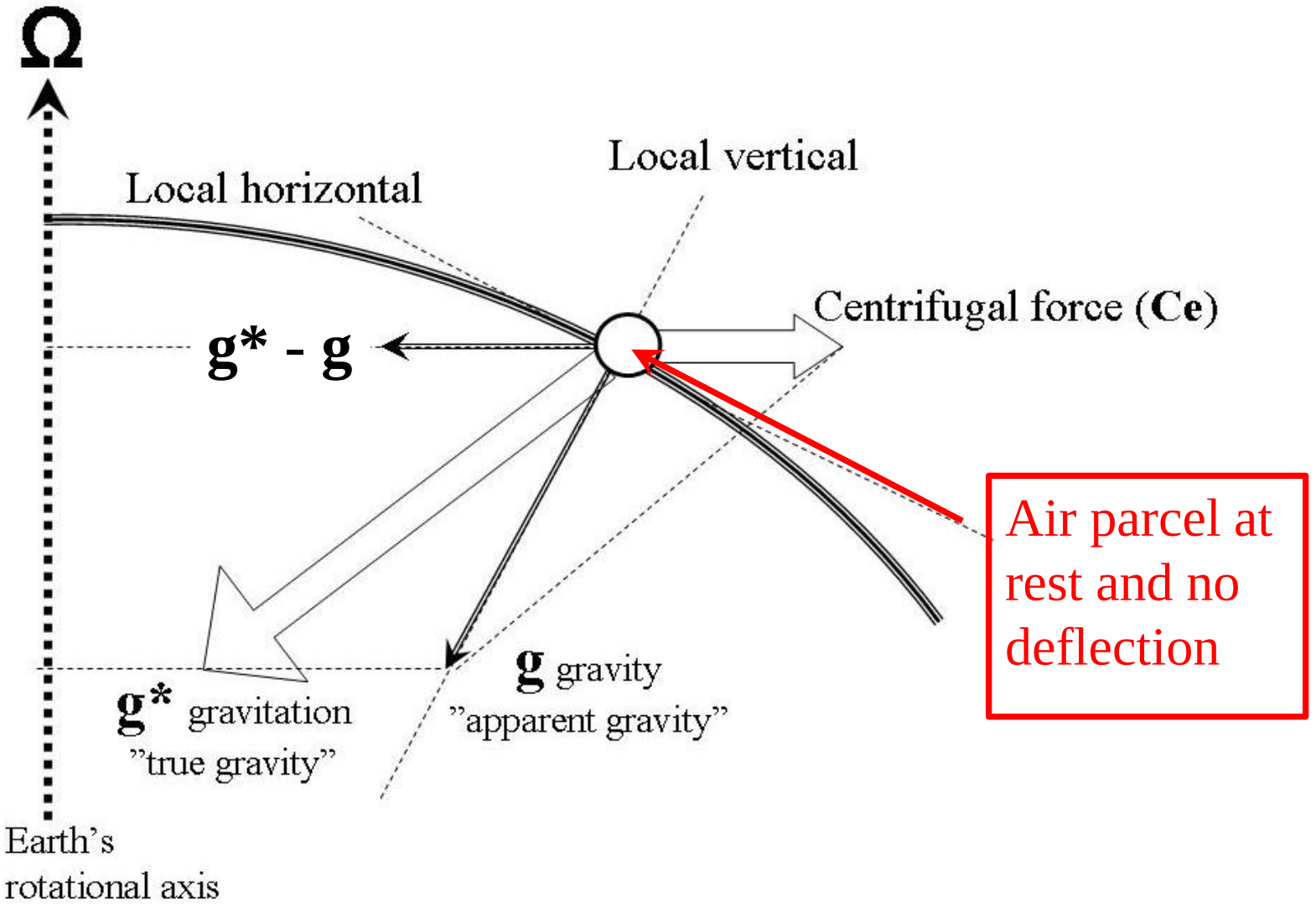
http://www.geologie.ens.fr/~hebert/THESE/CHAP2/FIGURES/fig1.html)

Controversial question: Is it the Coriolis force $\mathbf{-2\Omega \times V_r}$ that is deflecting the winds and the ocean currents?

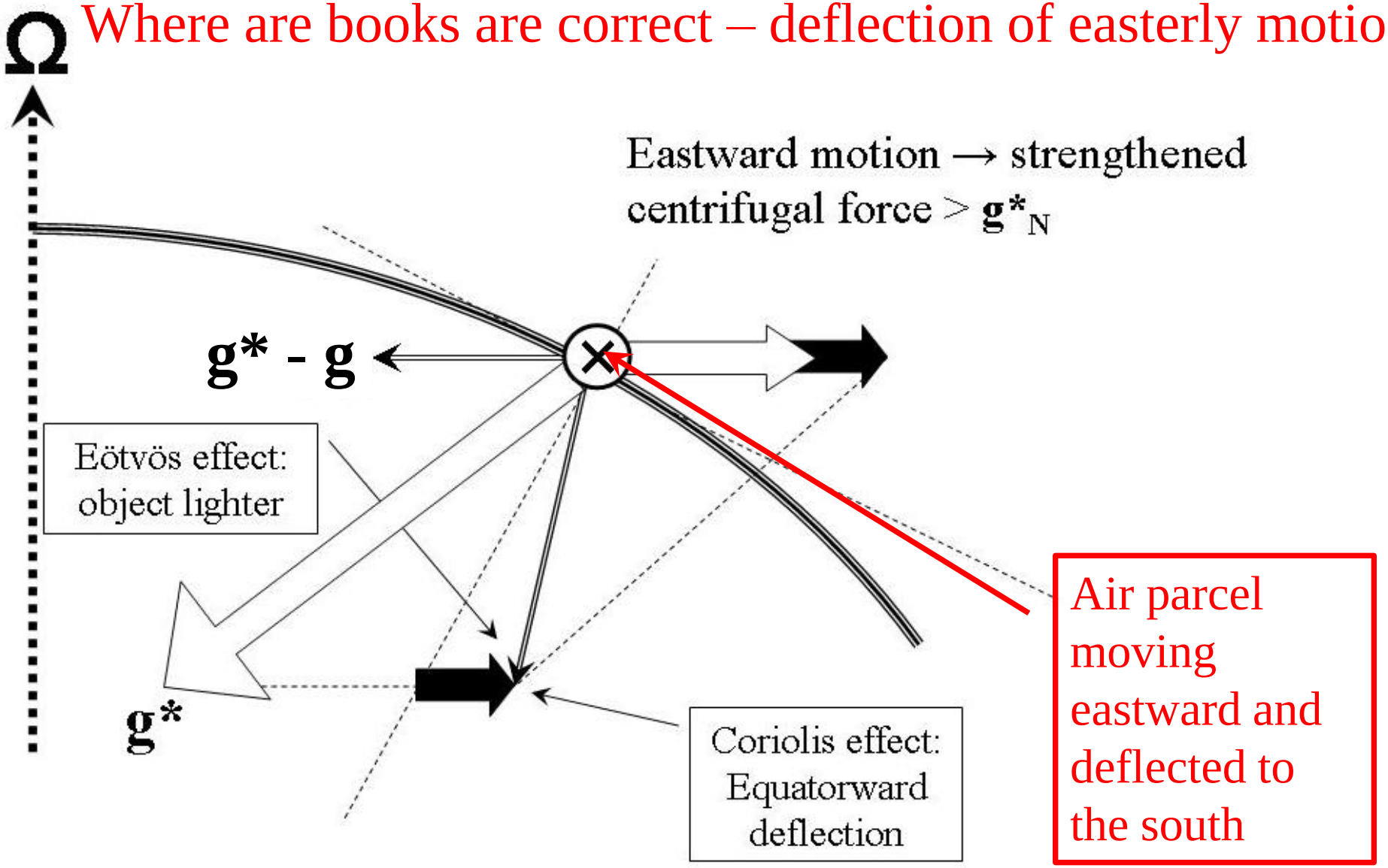
This ”indisputable truth” was in fact questioned by Dr Dale Durran in a correspondence to the Bull. AMS in 1993.

But few noticed that he had done this!

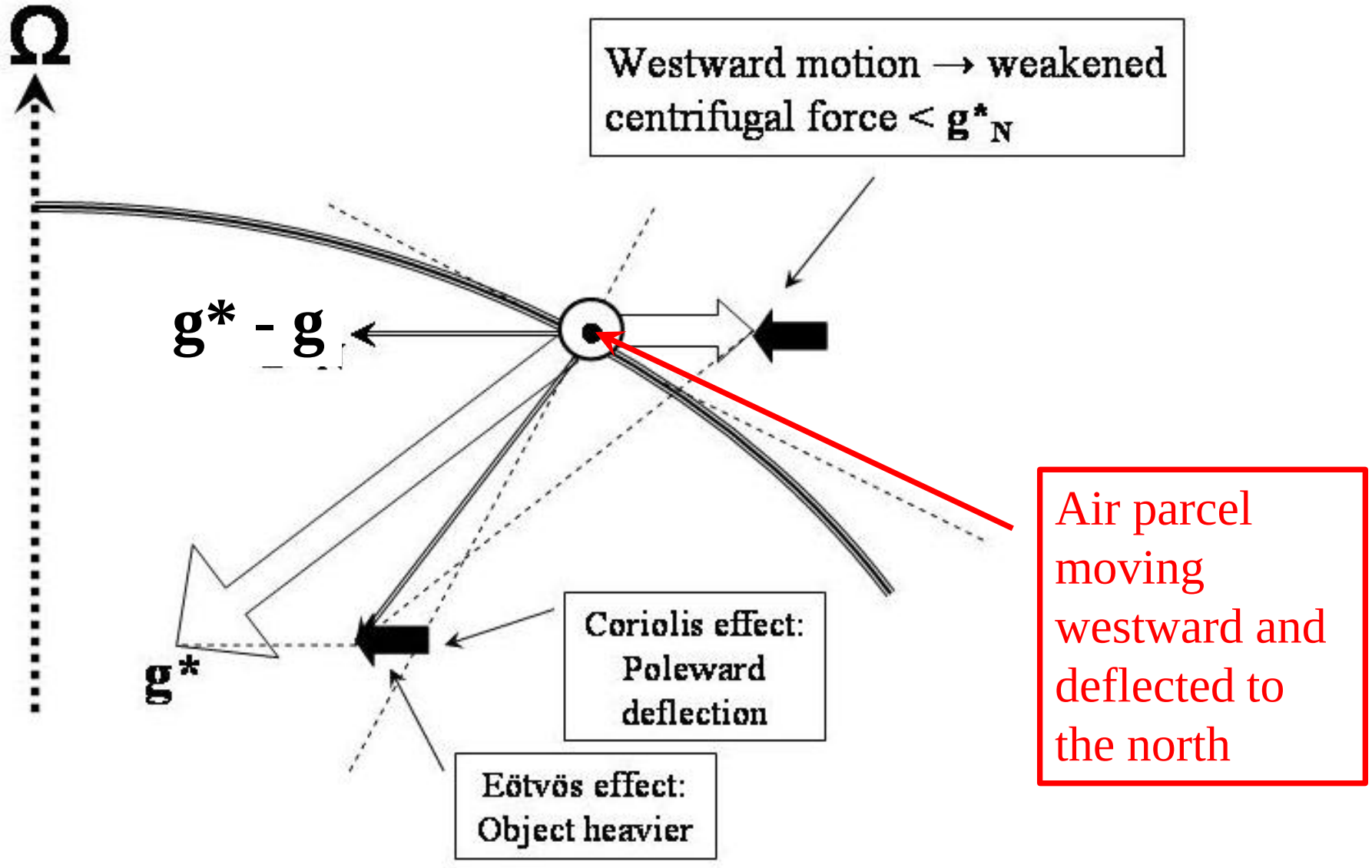
So I repeated the question in my QJRMS Note of 2015



Where are books are correct – deflection of easterly motion



Where most books get silent – deflection of westerly motion



We call

Gravitation + centrifugal force = gravity

So why not

Gravitation + centrifugal force + Coriolis force = **Durran force**

It is a real force!

But what about $\mathbf{-2\ \Omega \times V_r}$ which definitely is a fictitious force?

Going back to our original equation

$$\left(\frac{d\mathbf{V}_r}{dt}\right)_r = \mathbf{g}^* - 2\boldsymbol{\Omega}\times\mathbf{V}_r - \boldsymbol{\Omega}\times(\boldsymbol{\Omega}\times\mathbf{R})$$

$$\left(\frac{d\mathbf{V}_r}{dt}\right)_r = \mathbf{g} - 2\boldsymbol{\Omega}\times\mathbf{V}_r$$

But for 2-D horizontal motion we only have

$$\left(\frac{d\mathbf{V}_r}{dt}\right)_r = -2\boldsymbol{\Omega}\times\mathbf{V}_r$$

The other forces have disappeared mathematically, **but not physically**

The Coriolis force $\mathbf{-2\,\Omega \times V_r}$ is indeed fictitious and cannot do work etc

This term can mathematically, describe (kinematically) horizontal motion over the surface of a rotating planet. But the two other terms, describing the **dynamic** forces involved have disappeared only mathematically, but not physically

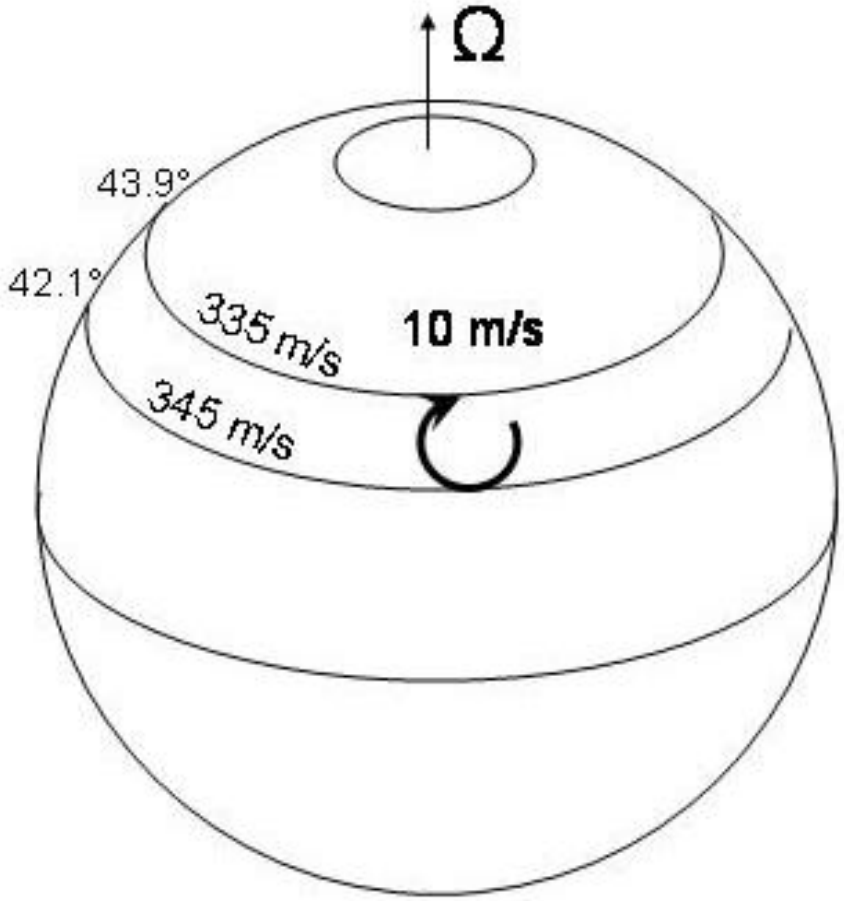
But if real forces are involved the absolute speed (kinetic energy) must change for a body moving over the earth's surface!

What do we know about that?

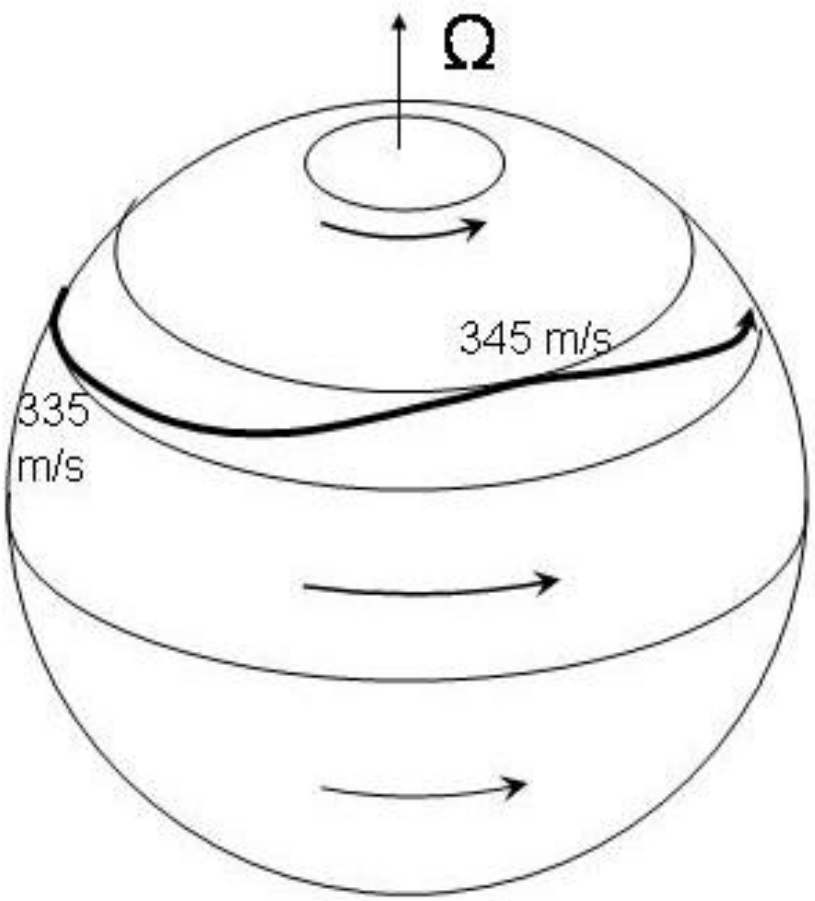
Not much because the "Hadley Principle" is so firmly engrained in our minds that we assume absolute velocity is conserved!

But it is not.

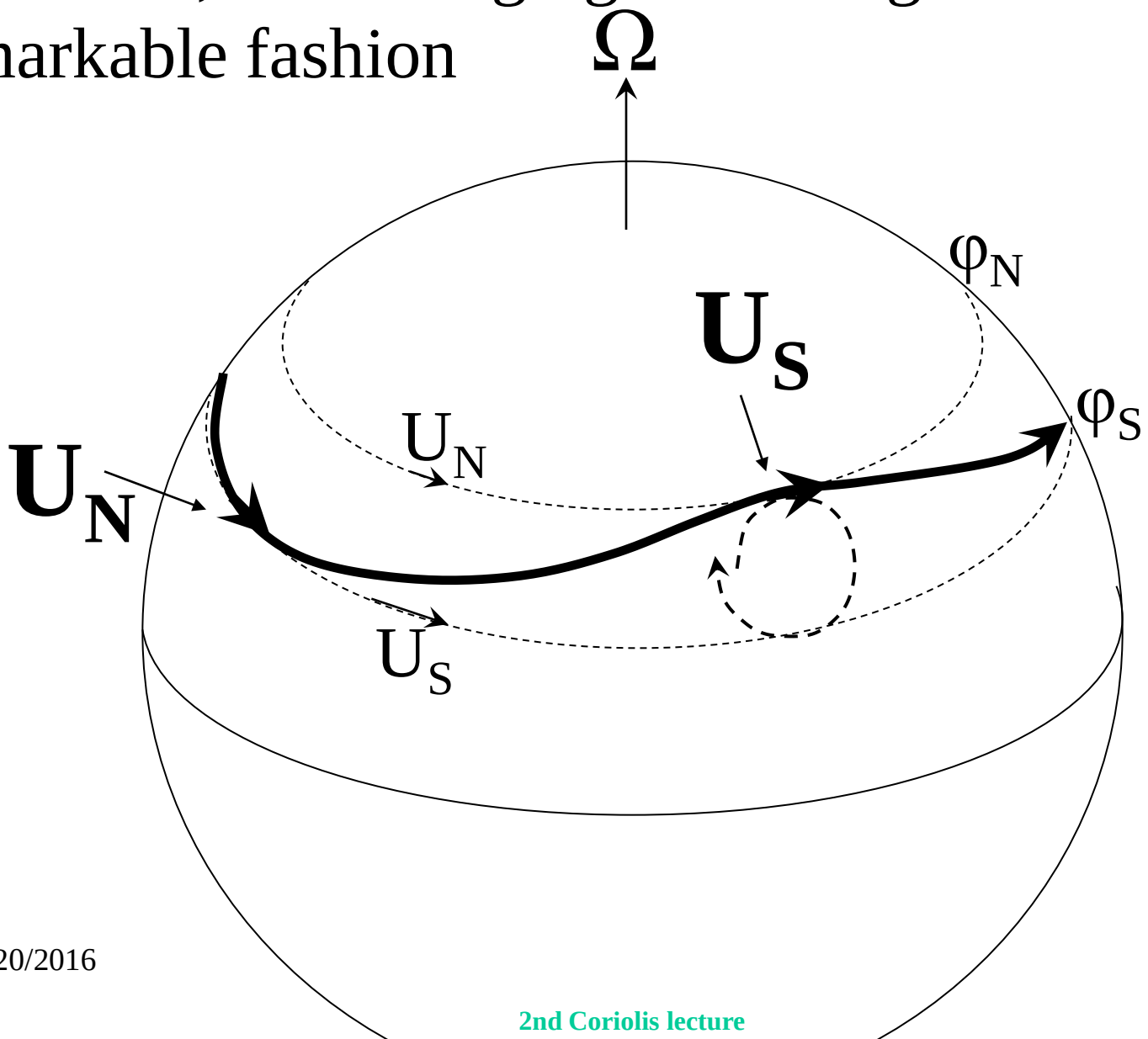
You saw this image earlier. It can be easily calculated from the equation for the inertia circle



Now here comes the same but watched from outside the earth



The absolute velocity (kinetic energy) is not conserved, it is changing and doing so in a remarkable fashion



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