3. A new look at the Coriolis force – *it is not an optical illusion!*

The scientific-mathematical basis for these lectures

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Notes and Correspondence
Is the Coriolis effect an 'optical illusion'?

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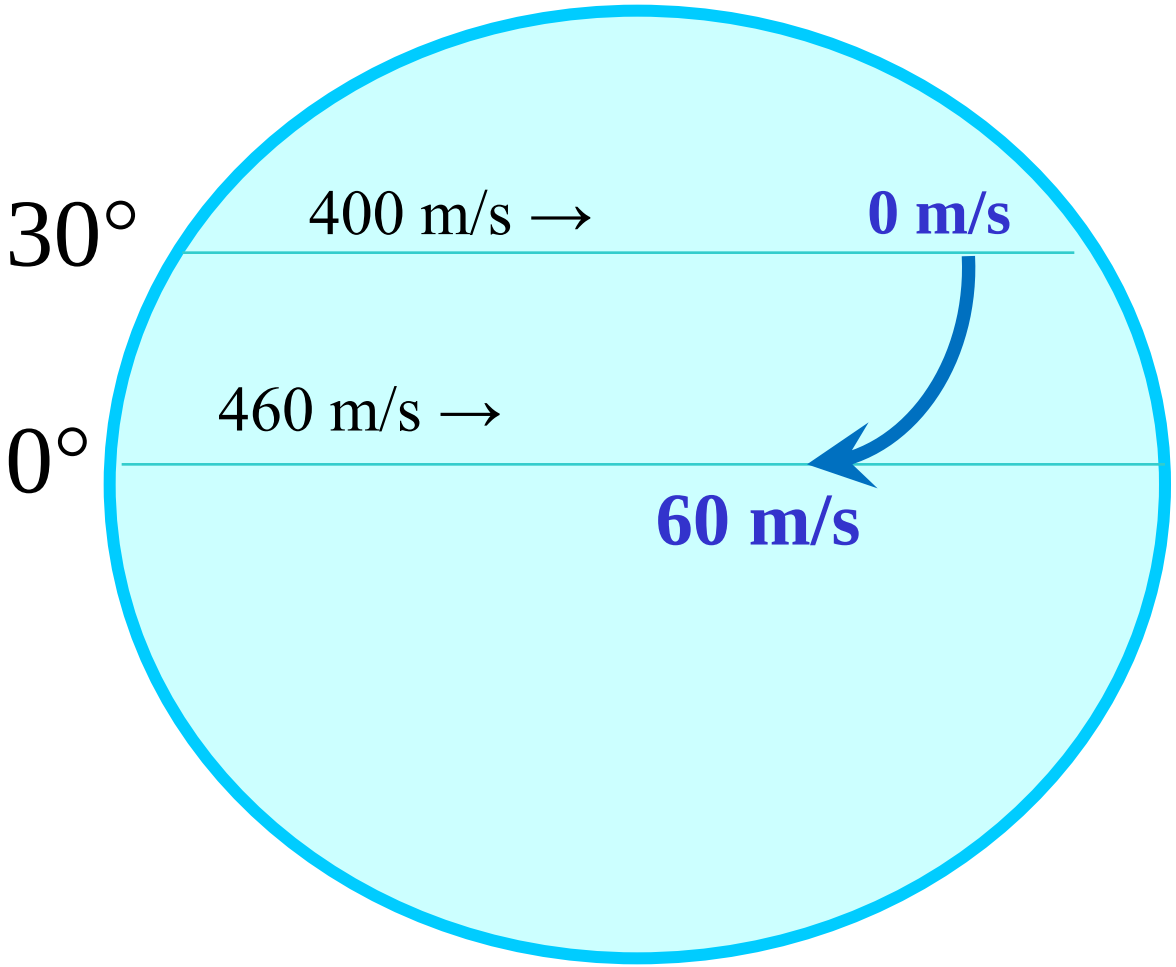
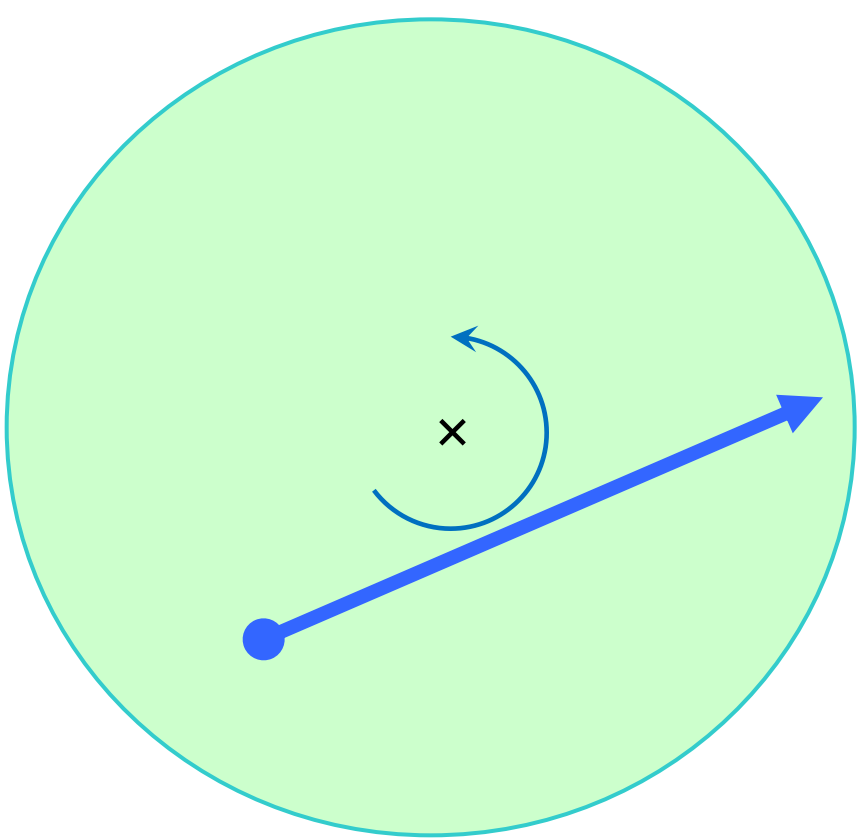
The difference between the derivations of the Coriolis effect on a rotating turntable and on the rotating Earth is discussed. In the latter case a real force, the component of the earth's gravitational attraction, non-parallel to the local vertical, plays a central role by balancing the centrifugal force. That a real force is involved leaves open, not only the question on the inertial nature of the 'inertial oscillations', but also the way we tend to physically conceptualize the terrestrial Coriolis effect.

This is not the Coriolis Effect!

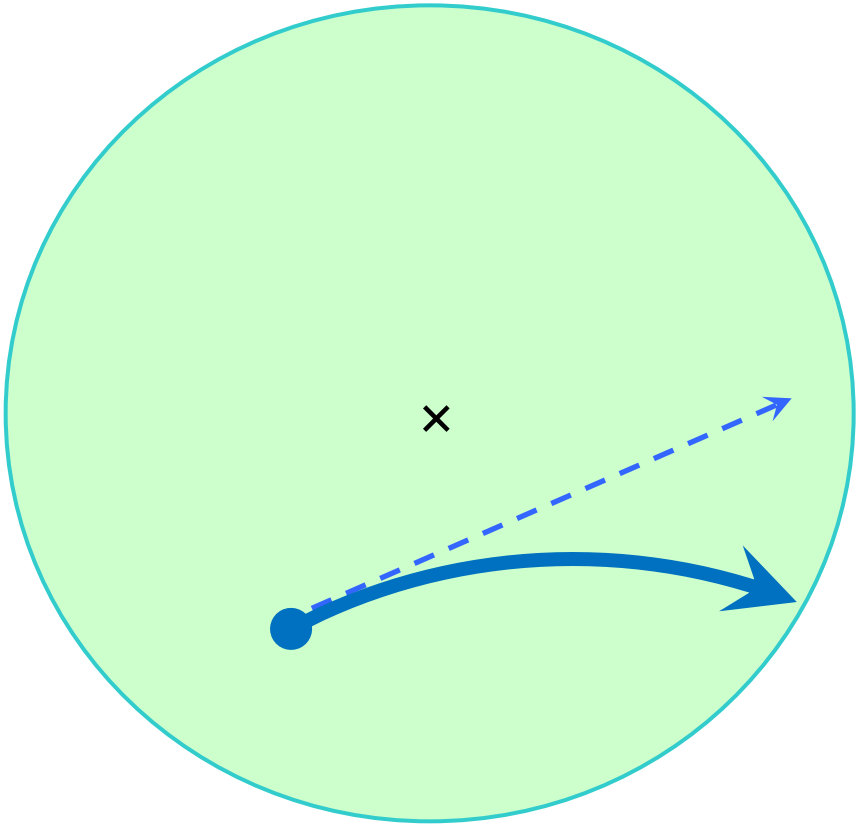


A popular, but very erroneous explanation of the Coriolis effect on a rotating planet, assuming the winds conserve their absolute velocity – **which they do not!**

This is not only the Coriolis effect but also ***the centrifugal effect***

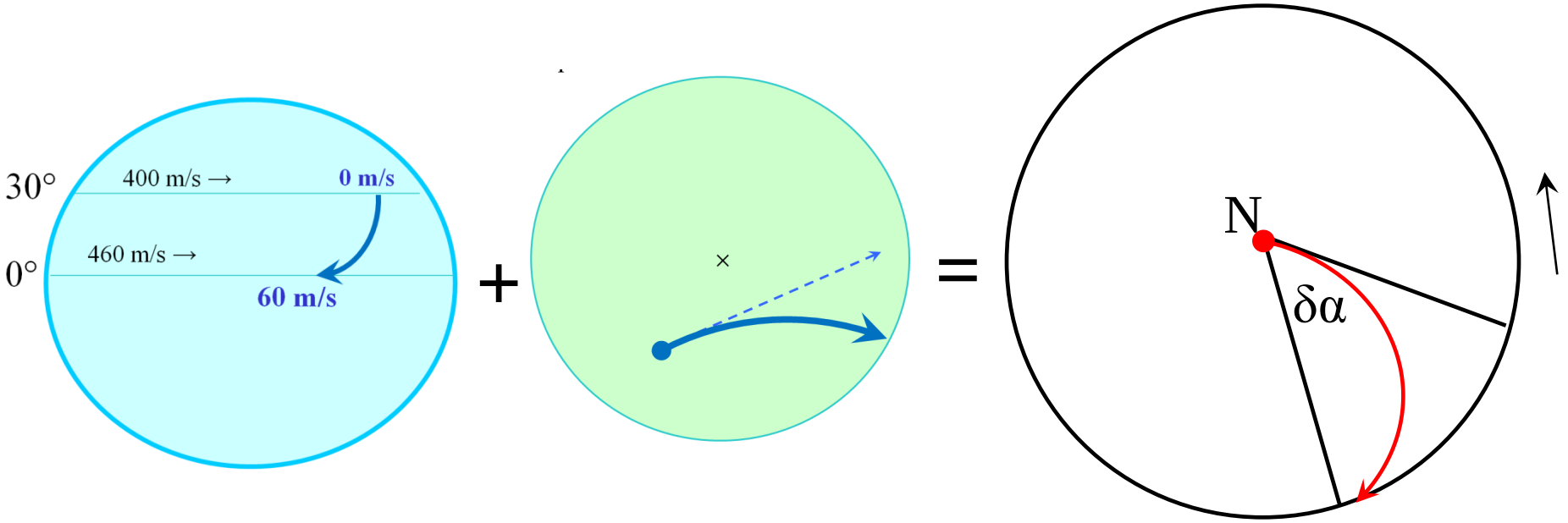


Seen from **outside** the carousel



Seen from **inside** the carousel

But if the two erroneous assumptions are mathematically combined, as is done in many textbooks, **one gets the correct result**



The derivation combines the previous erroneous assumptions: that the absolute velocity is conserved (left) and that on a rotating carousel only the Coriolis force is active (right)

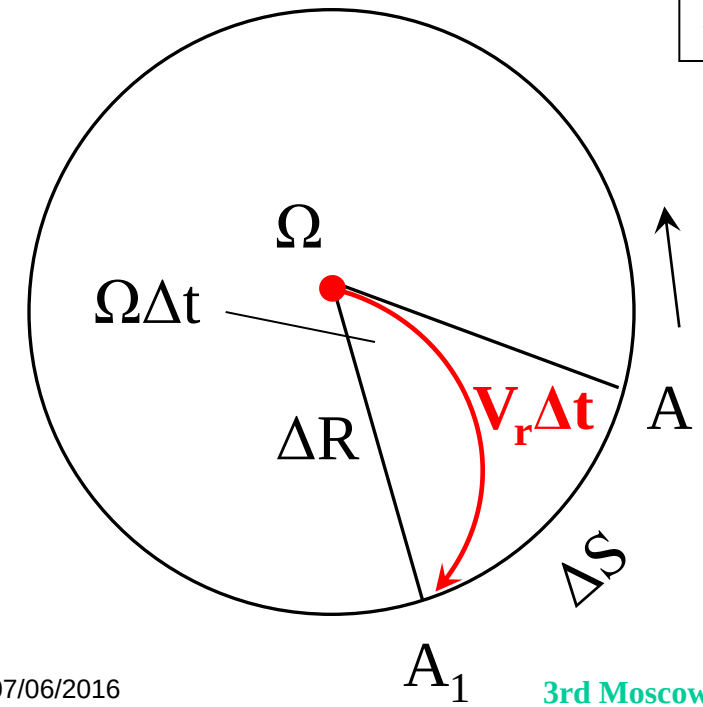
This very easily understood, but erroneous, derivation of the Coriolis force appears in two different versions

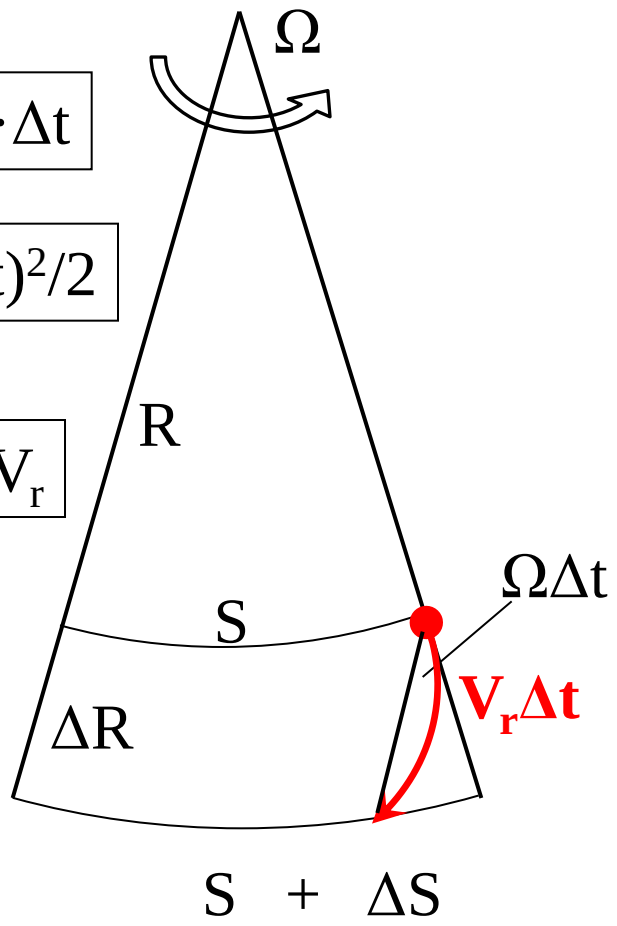
acc = sideways acceleration (Coriolis effect)

$\Delta S = \Omega \cdot \Delta t \cdot V_r \cdot \Delta t$

$\Delta S = acc \cdot (\Delta t)^2/2$

$acc = 2\Omega \cdot V_r$





One of them is found in at least two Russian textbooks in dynamic meteorology

Министерство высшего и среднего специального образования РСФСР

ЛЕНИНГРАДСКИЙ ГИДРОМЕТЕОРОЛОГИЧЕСКИЙ ИНСТИТУТ

И. И. МЕЛЬНИКОВА, В. М. РАДИКЕВИЧ

ДИНАМИЧЕСКАЯ МЕТЕОРОЛОГИЯ

(учебное пособие для океанологов)

Под редакцией профессора Д. Л. Лайхтмана

ЛЕНИНГРАД
1974

Отклоняющая сила вращения Земли (сила Кориолиса)

Отклоняющая сила вращения Земли представляет дополнительную инерционную силу, действующую на частичку воздуха, движущуюся относительно поверхности Земли. Сила Кориолиса (названа по имени французского механика Густава Гаспара Кориолиса, впервые рассчитавшего эту силу) возникает за счет вращения Земли. Если бы Земля не вращалась, то путь частицы воздуха от полюса до экватора был бы NA (рис. 3), в результате вращения Земли частица попадает в точку A_1, $NA_1 = c \cdot dt$ (где c — скорость частицы). За время dt Земля повернулась на угол $\delta\alpha = \omega dt$.

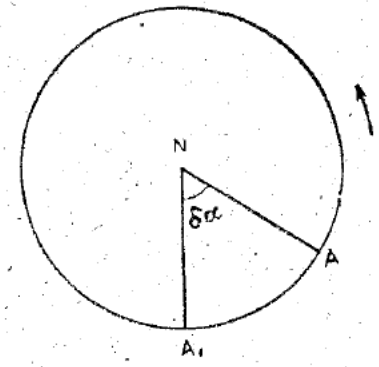


Рис. 3. Траектория движения частицы от полюса к экватору

Для малых dt мало $\delta\alpha$ и можно считать

$$AA_1 = NA_1 \cdot \delta\alpha = c\omega (dt)^2.$$

С другой стороны, для равномерно-ускоренного движения

$$AA_1 = \frac{1}{2} a \cdot (dt)^2,$$

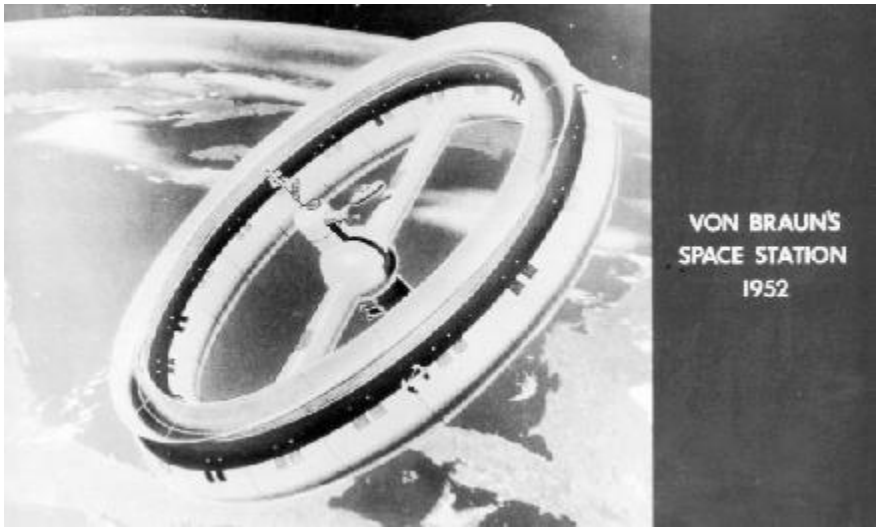
где a — ускорение за счет вращения Земли или ускорение Кориолиса.

Из сравнения выражений для AA_1 получаем

$$a = 2\omega \cdot c, \qquad (2.2.4)$$

The Coriolis force is said to be “fictitious” and unable to “do work”, but this does not mean the Coriolis Effect is an “optical illusion”

In the 1950’s and 1960’s the Russians and Americans planned to create artificial gravity on their space stations by letting them rotate. This was nicely depicted in Stanley Kubrick’s 1969 movie “2001 - A Space Odyssey”:

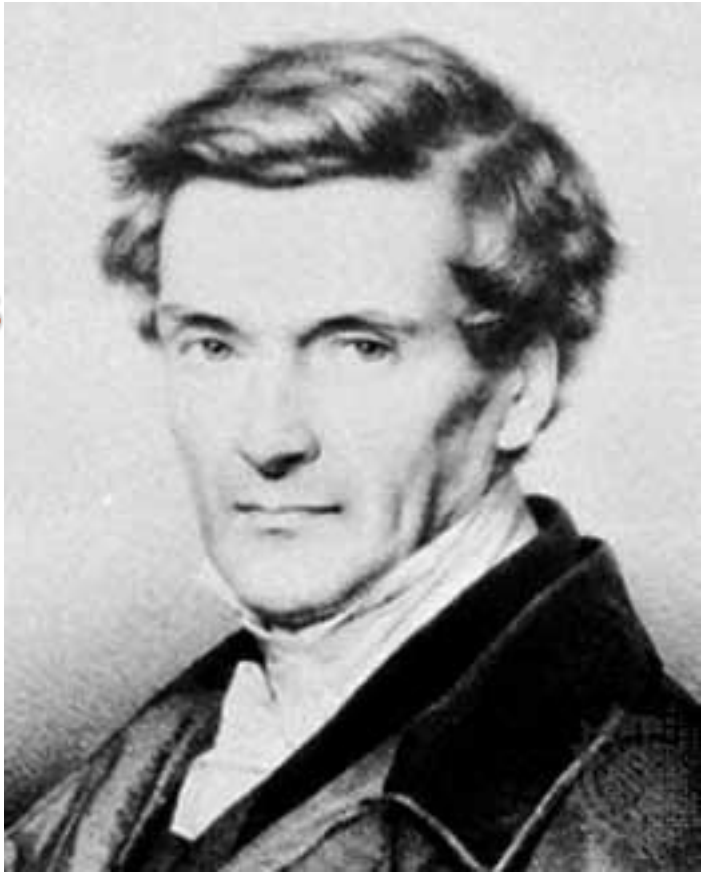


https://www.youtube.com/watch?v=q3oHmVhviO8

https://www.youtube.com/watch?v=1wJQ5UrAsIY&ebc=ANyPxKo4CqF8_xFhOGFvxKcYafafA0yy4qJOLEyy9E-Ar-6ou7TNub_e9DNKLtfamKKTqQ_HhYpnX_z5ZZG8mZpbPrLBqQgTkA

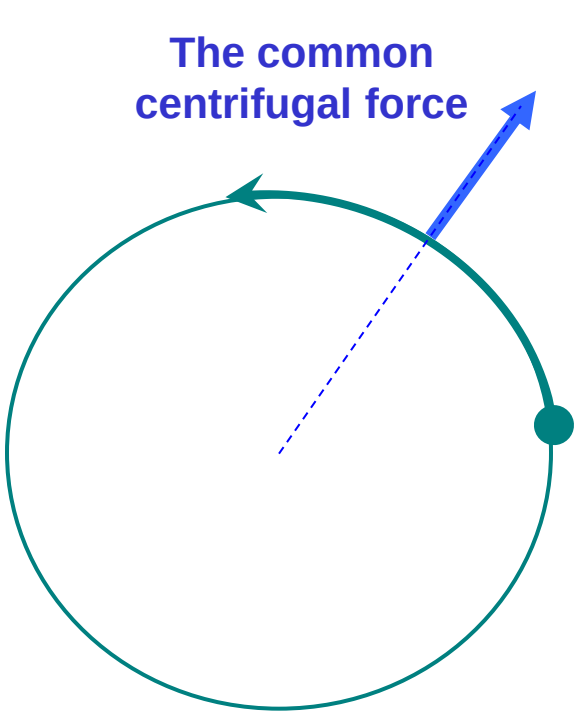
The answer is in the original 1835 Coriolis paper

Gaspard Gustave Coriolis 1784-1843

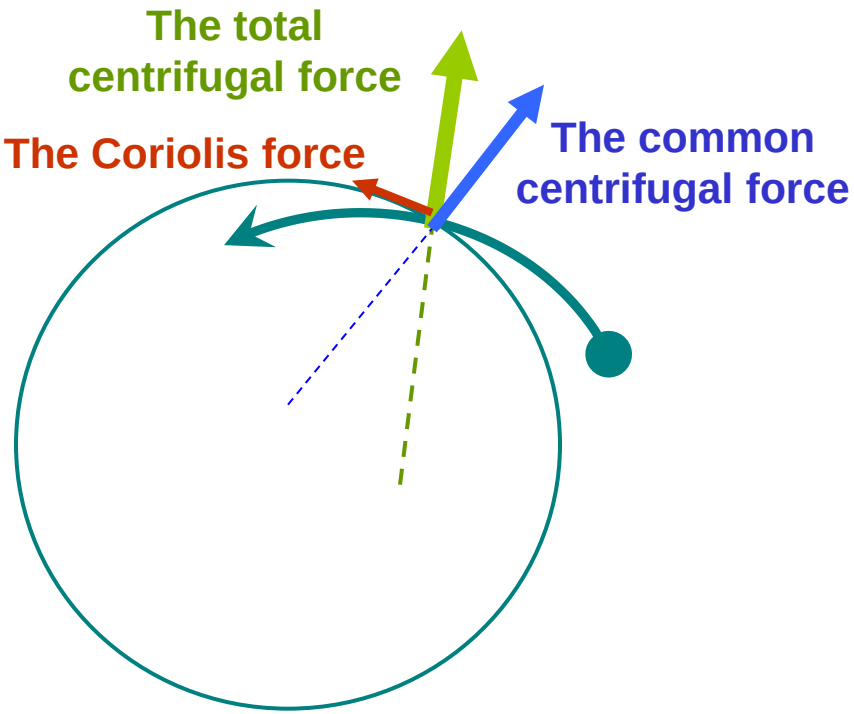


Coriolis was interested, neither in the atmosphere nor in the oceans – ***but in machines***

Coriolis was interested in how the centrifugal effect acted on moving parts in rotating machines



A stationary object within the rotating system



An object moving (inwards) in the rotating system

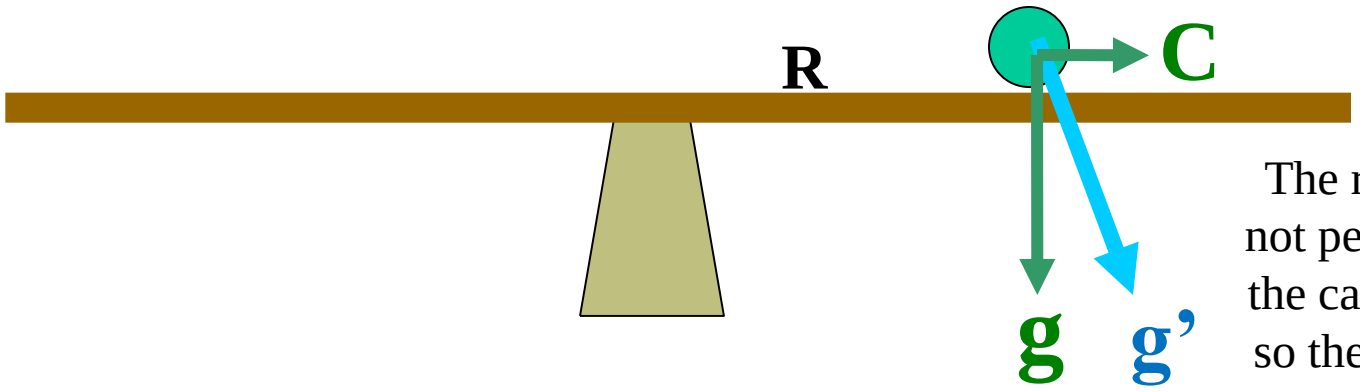
Coriolis's force was the "extra force" that had to be added to the common centrifugal force to get the total centrifugal force

Applied on a flat rotating carousel at a distance **R**

$$\left(\frac{d\mathbf{V}_r}{dt}\right)_r = \mathbf{g} - 2\boldsymbol{\Omega}\times\mathbf{V}_r - \boldsymbol{\Omega}\times(\boldsymbol{\Omega}\times\mathbf{R})$$

Gravity

R = distance to the rotational axis

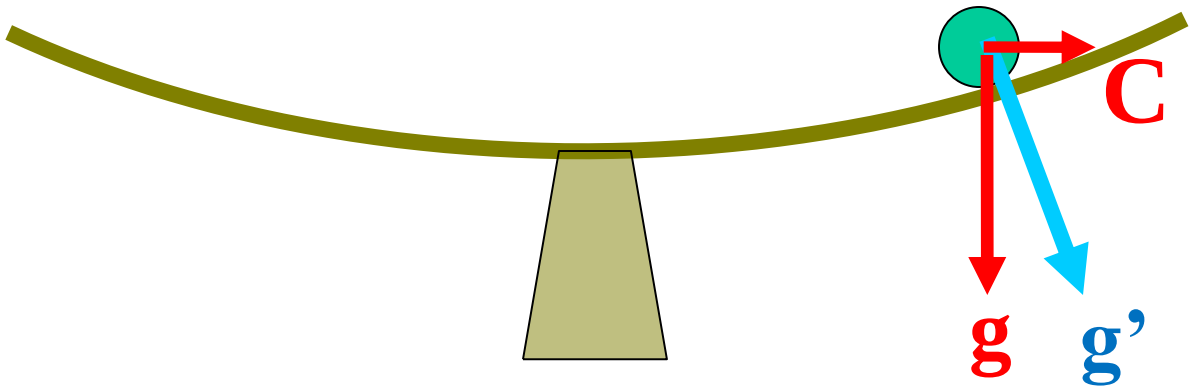


The resultant **g'** is not perpendicular to the carousel surface so the object glides outward

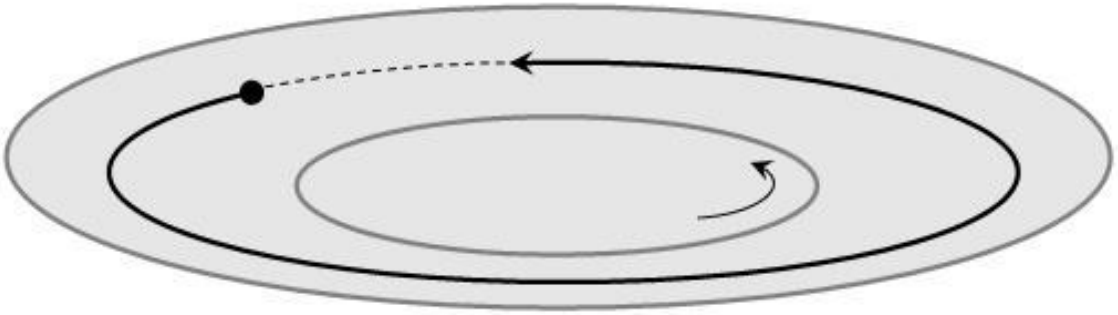
By making the carrousel concave we "get rid of" the centrifugal force by combining it with gravity

$$\mathbf{g}' = \mathbf{g} - \boldsymbol{\Omega} \times (\boldsymbol{\Omega} \times \mathbf{R})$$

$$\left(\frac{d\mathbf{V}_r}{dt} \right)_r = \mathbf{g}' - 2\boldsymbol{\Omega} \times \mathbf{V}_r$$

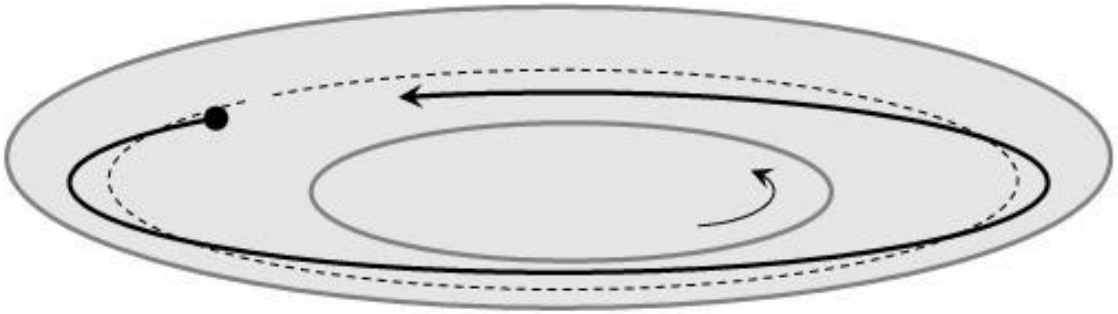


The resultant **g'** is now perpendicular to the carousel surface so the object does not glide outward

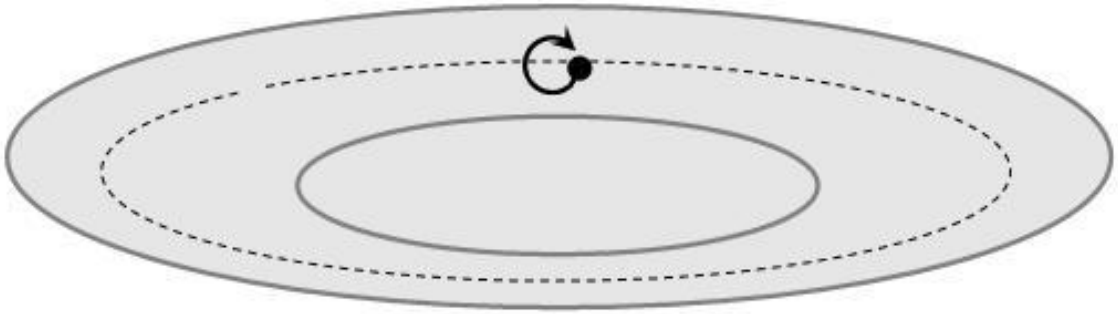


The motion seen from outside

The small body is not perturbed

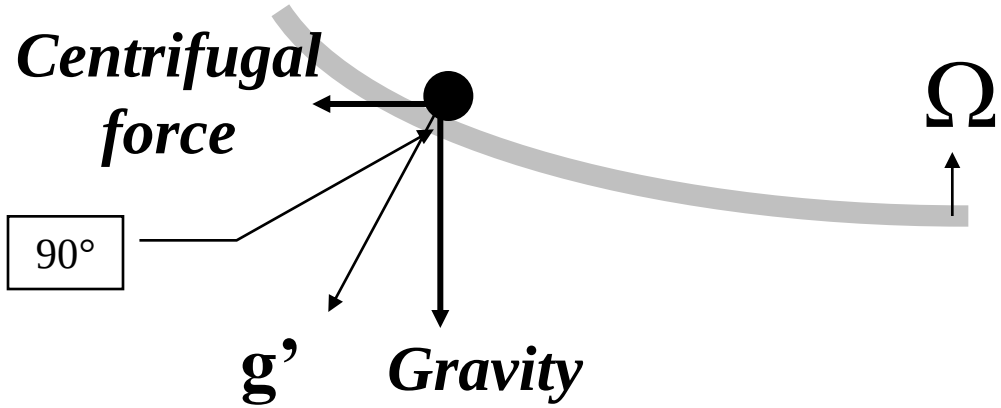


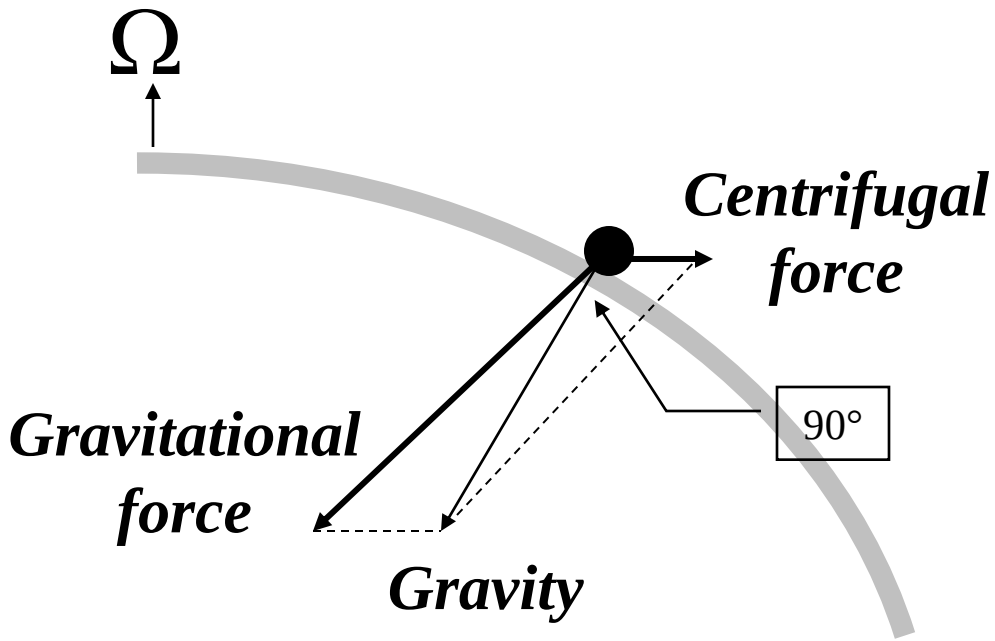
The small body is perturbed



The same seen from “inside” the rotating parabola

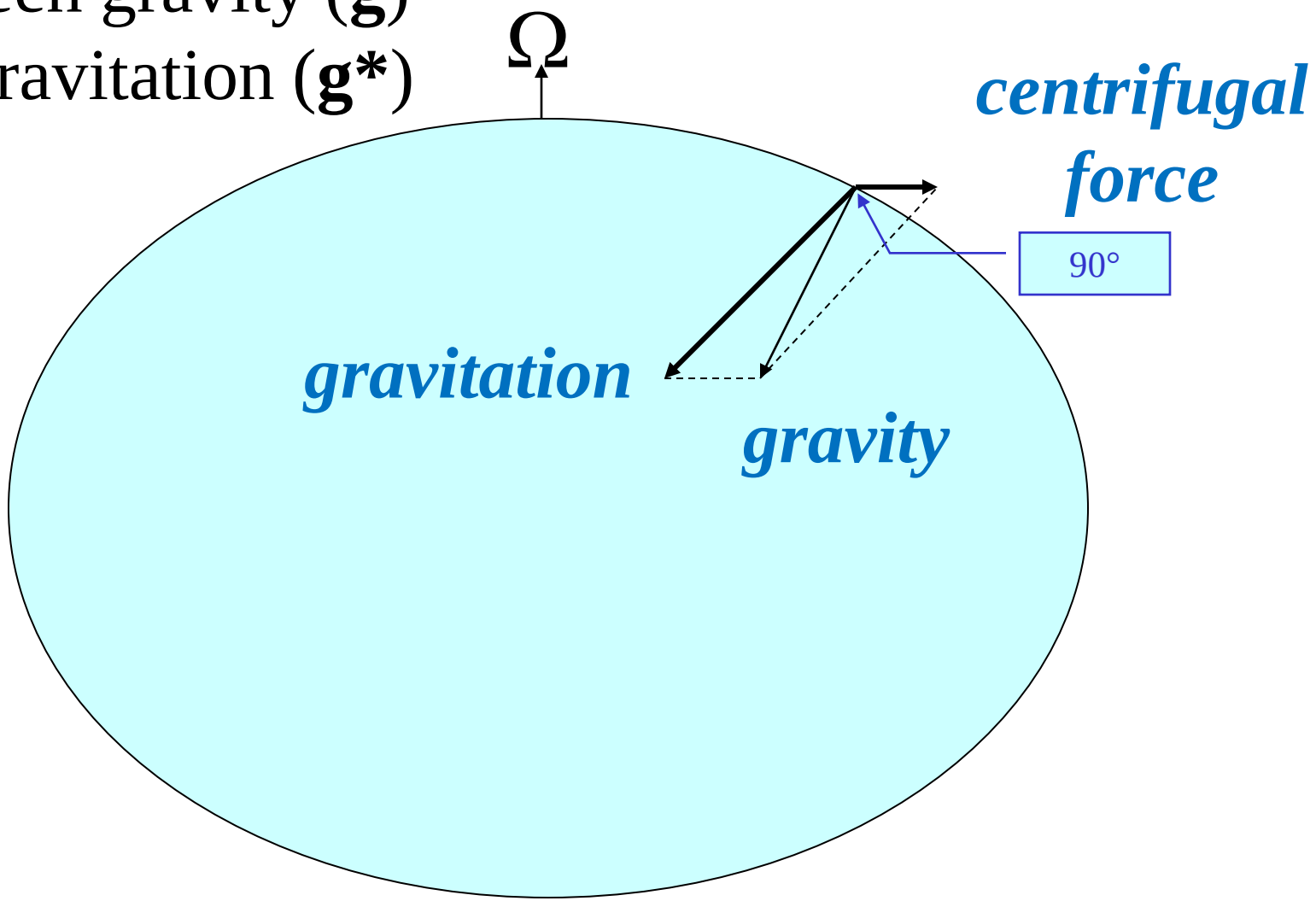
So now we know how to “get rid of” the centrifugal force on a carousel, but what about the earth?





Exactly in the same way, with the shape of the earth having an important role

The crucial difference between gravity (**g**) and gravitation (**g***)

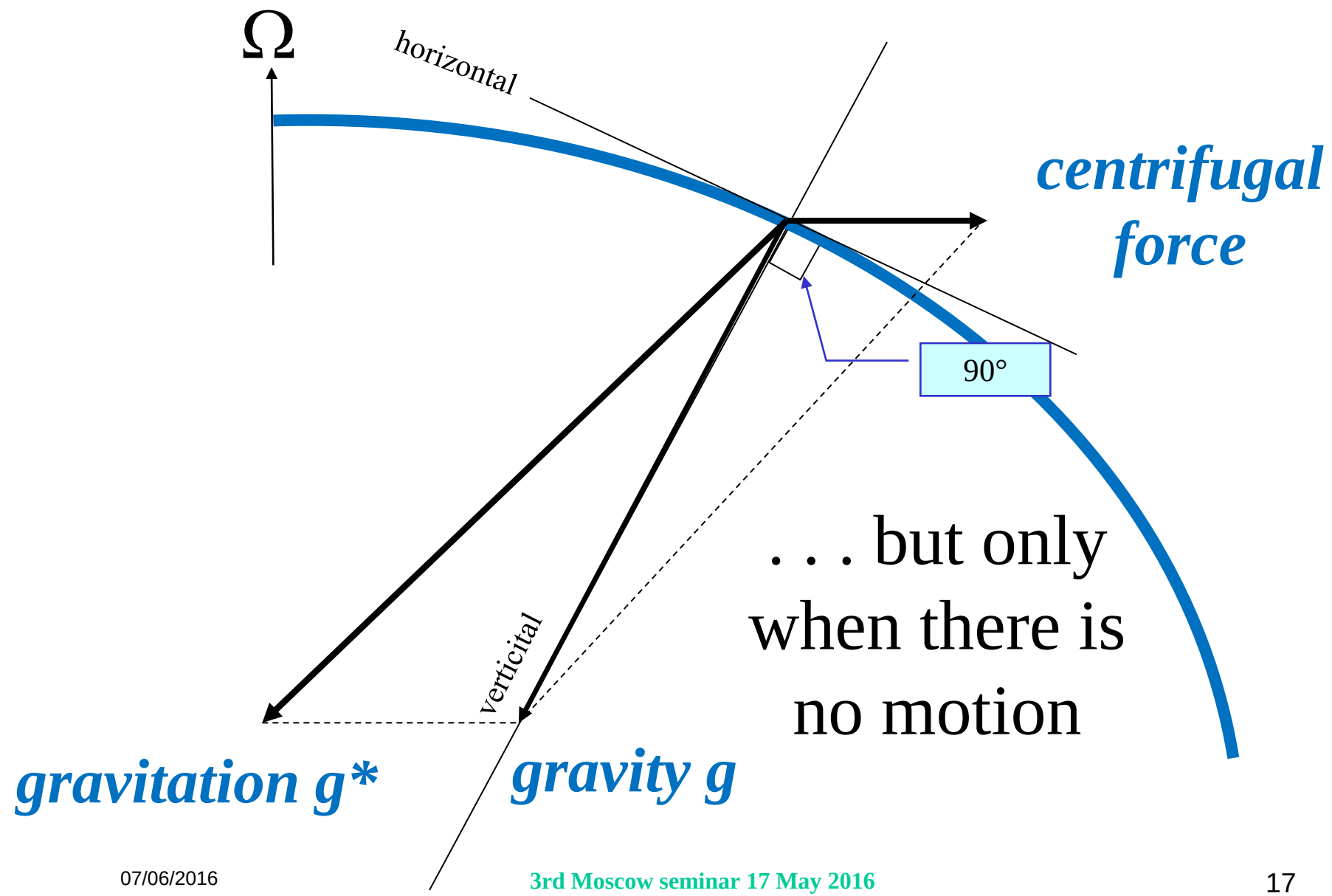


We combine the gravitational attraction (**g***) with the centrifugal force into gravity (**g**)

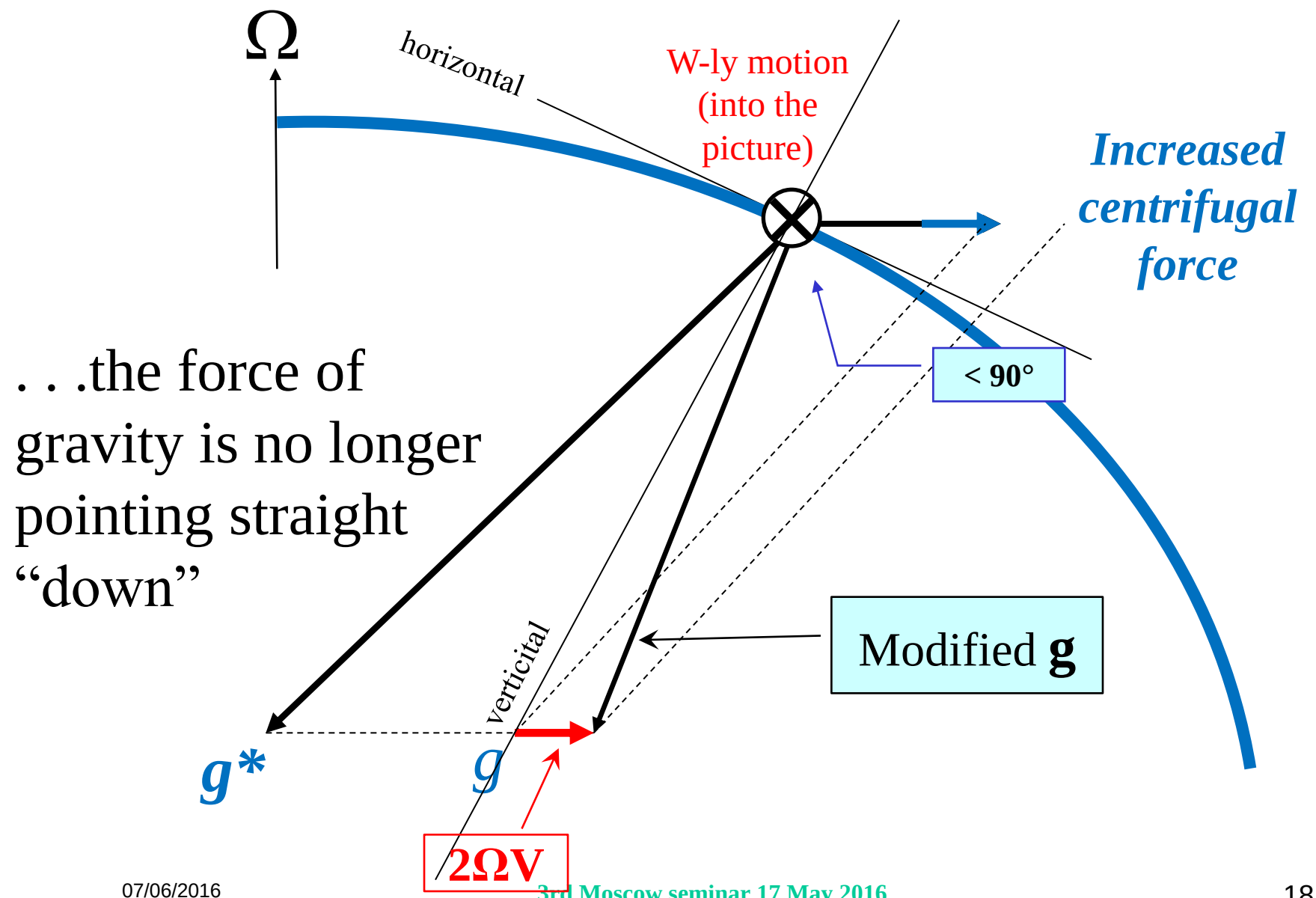
$$\left(\frac{d\mathbf{V}_r}{dt}\right)_r = \mathbf{g}^* - 2\boldsymbol{\Omega}\times\mathbf{V}_r - \boldsymbol{\Omega}\times(\boldsymbol{\Omega}\times\mathbf{R})$$

$$\left(\frac{d\mathbf{V}_r}{dt}\right)_r = \mathbf{g} - 2\boldsymbol{\Omega}\times\mathbf{V}_r$$

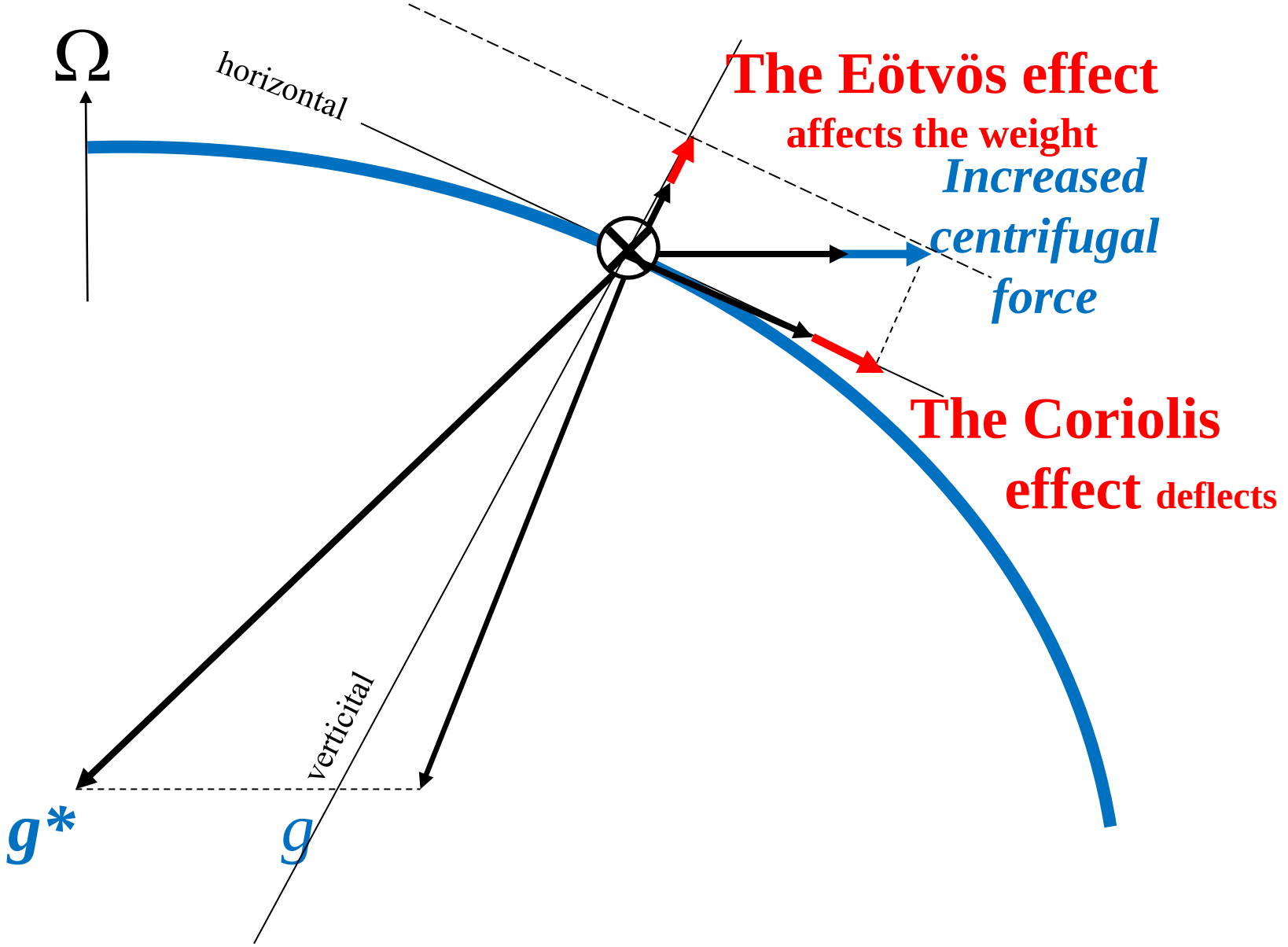
The force of gravity is pointing straight “down”



When there is motion . . .

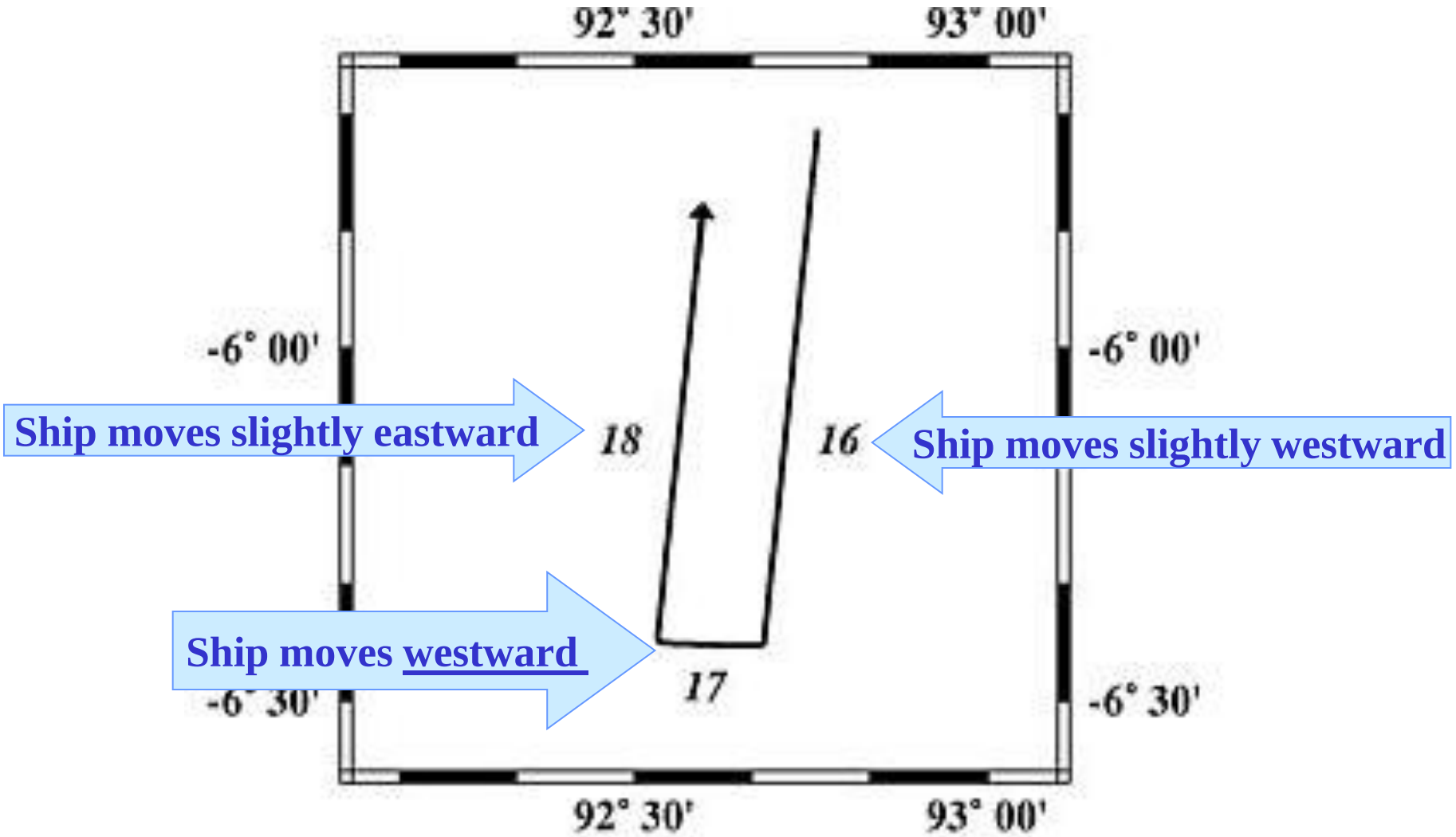


. . .the force of gravity is no longer pointing straight "down"


Ω
horizontal
The Eötvös effect
affects the weight
Increased
centrifugal
force
The Coriolis
effect deflects
verticital
g*
g

Example of the Eötvös effect

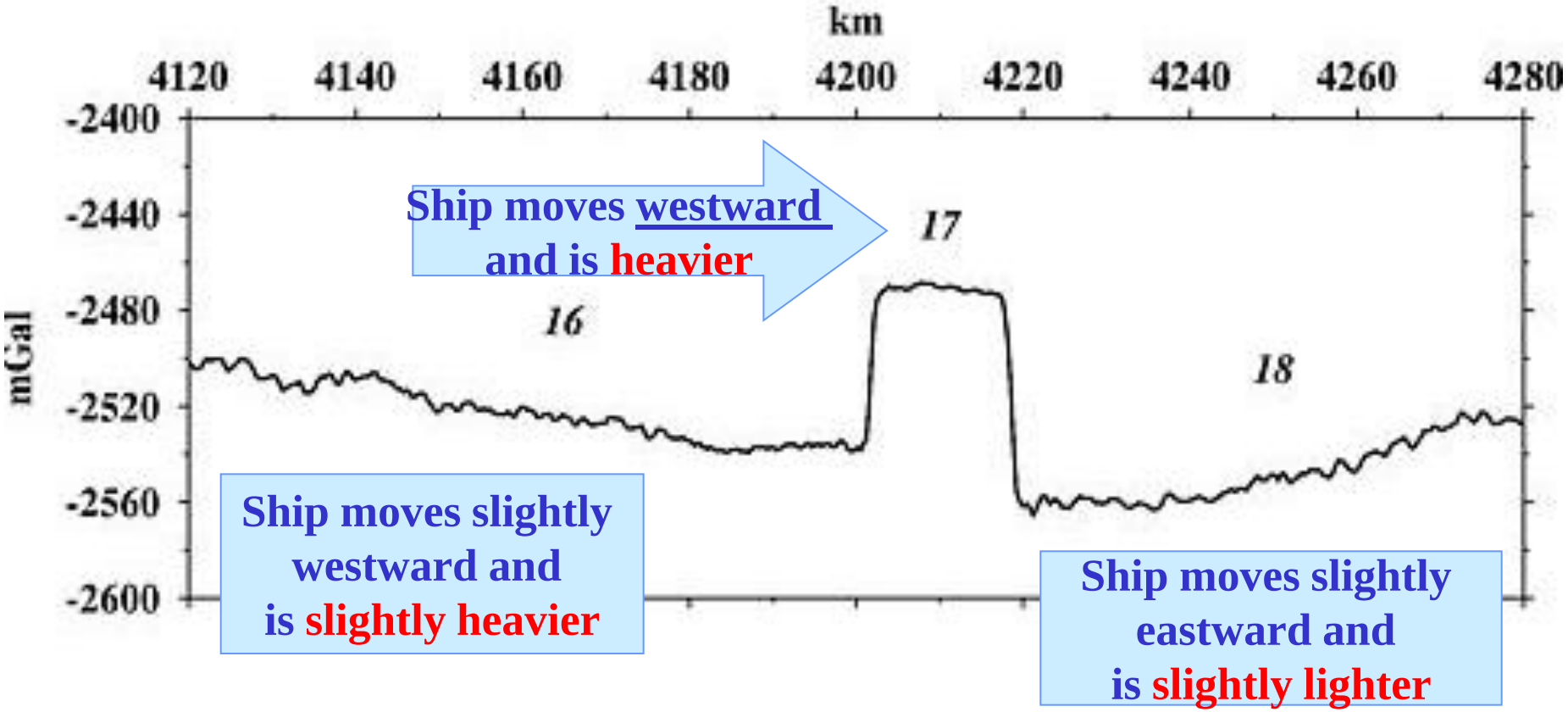
The **weight** of a French research vessel sailing in the Indian Ocean



http://www.geologie.ens.fr/~hebert/THESE/CHAP2/FIGURES/fig1.html)

Example of the Eötvös effect

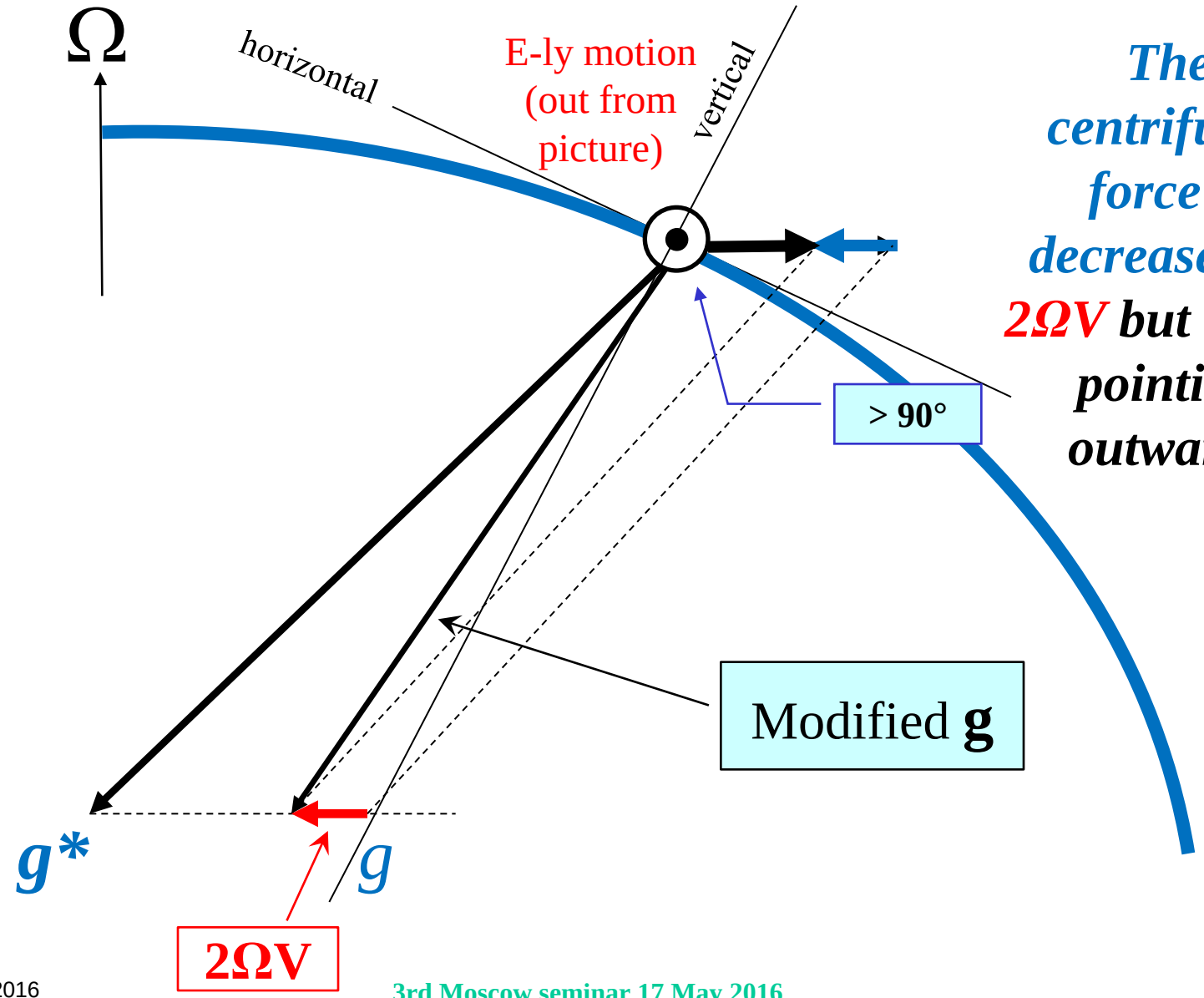
The weight of a French research vessel sailing in the Indian Ocean



The Eötvös effect only affects west-east motions

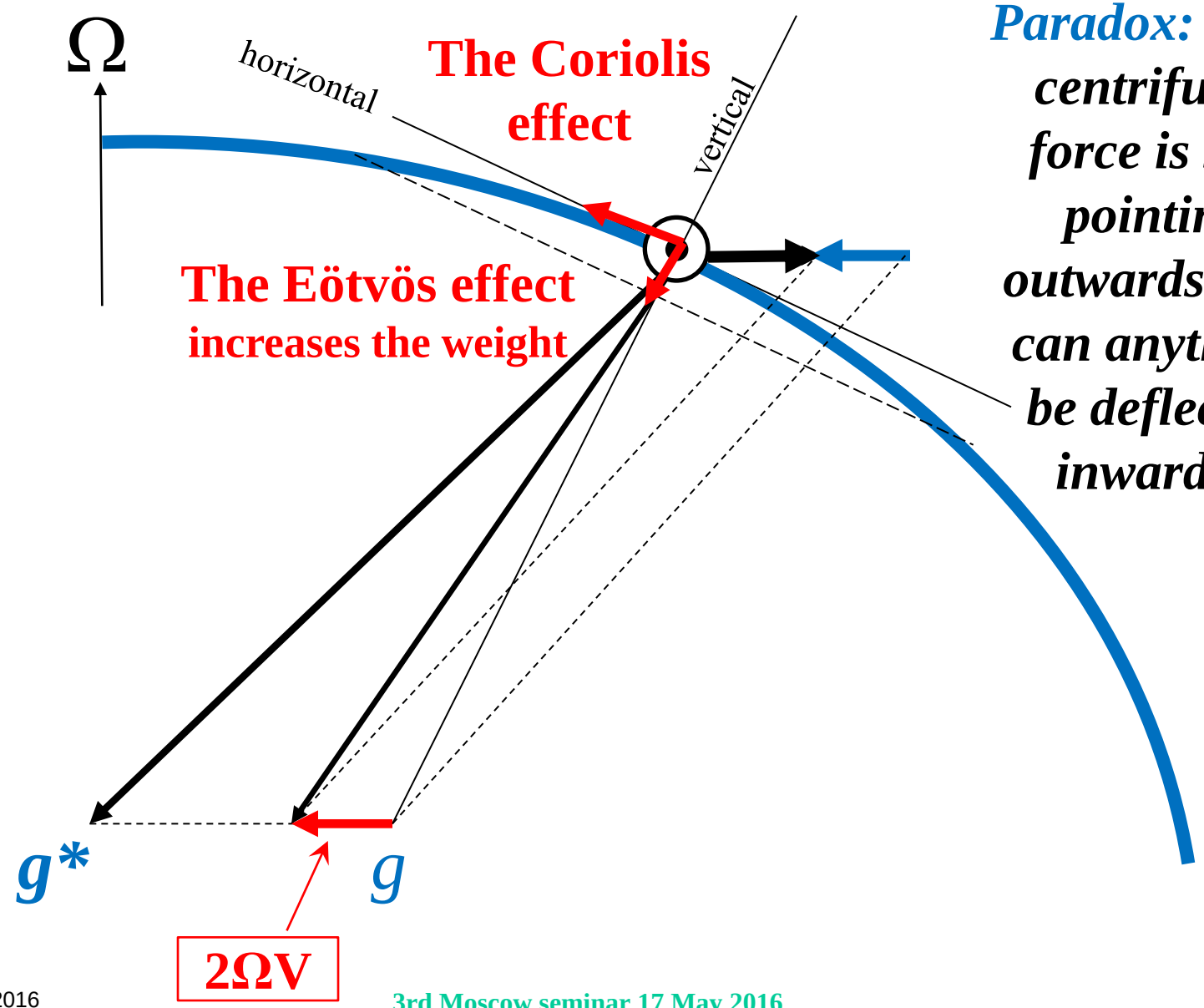
http://www.geologie.ens.fr/~hebert/THESE/CHAP2/FIGURES/fig1.html)

For <u>westward</u> motion the centrifugal force weakens



The centrifugal force is decreased by 2ΩV but is still pointing outwards

The **gravitational force** pulls the motion inwards



Paradox: ***if the centrifugal force is still pointing outwards how can anything be deflected inwards?***

Although the horizontal trajectory of a mass element can be **kinematically** described by

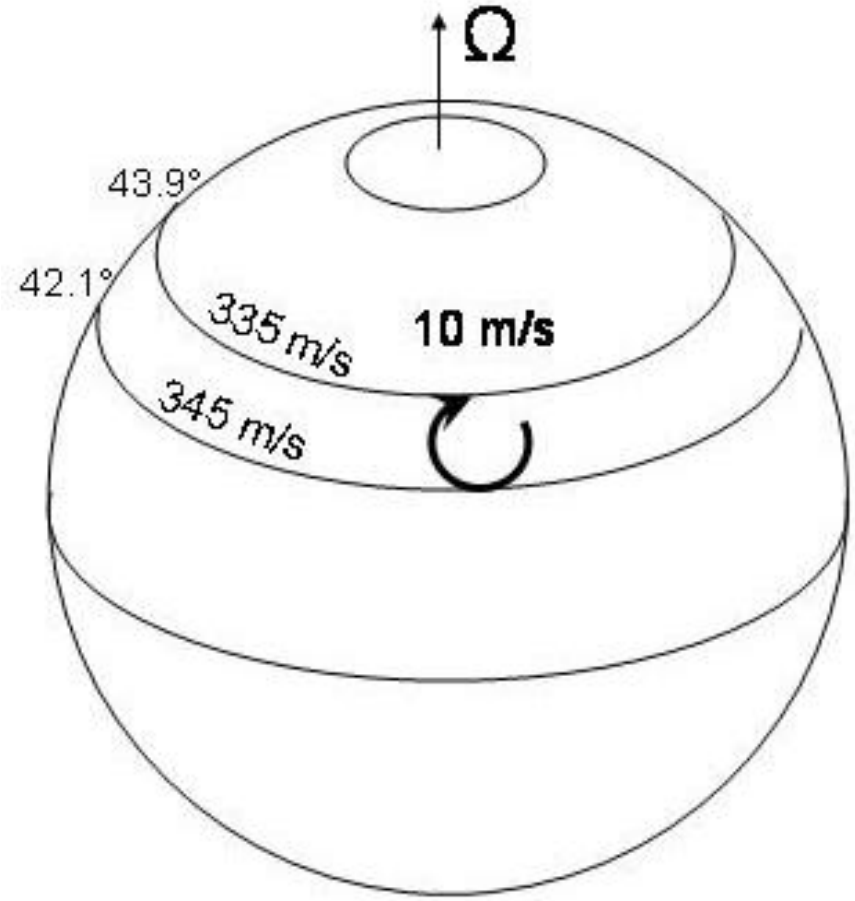
$$\left(\frac{d\,\mathbf{V}_r}{dt}\right)_h = \left(-2\boldsymbol{\Omega}\times\mathbf{V}_r\right)_h$$

...it can **dynamically** only be understood by considering all the three physical forces involved, of which one is a real force

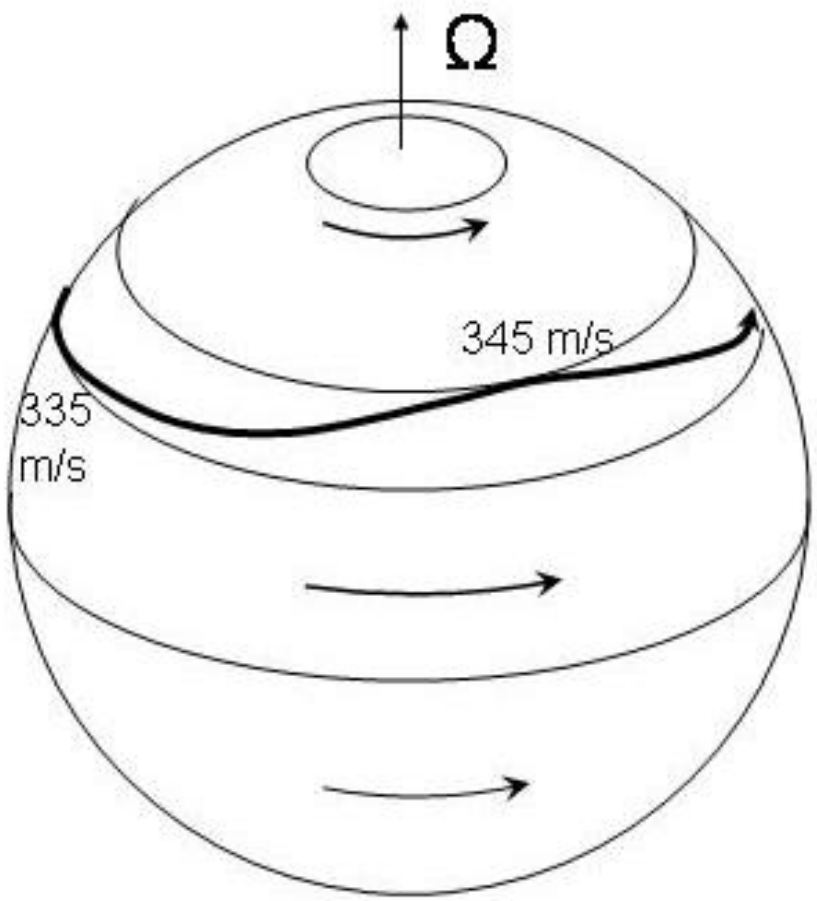
$$\left(\frac{d\,\mathbf{V}_r}{dt}\right)_r = \mathbf{g}^* - 2\boldsymbol{\Omega}\times\mathbf{V}_r - \boldsymbol{\Omega}\times(\boldsymbol{\Omega}\times\mathbf{R})$$

Q: ***Wouldn't a real force accelerate the absolute motion?***

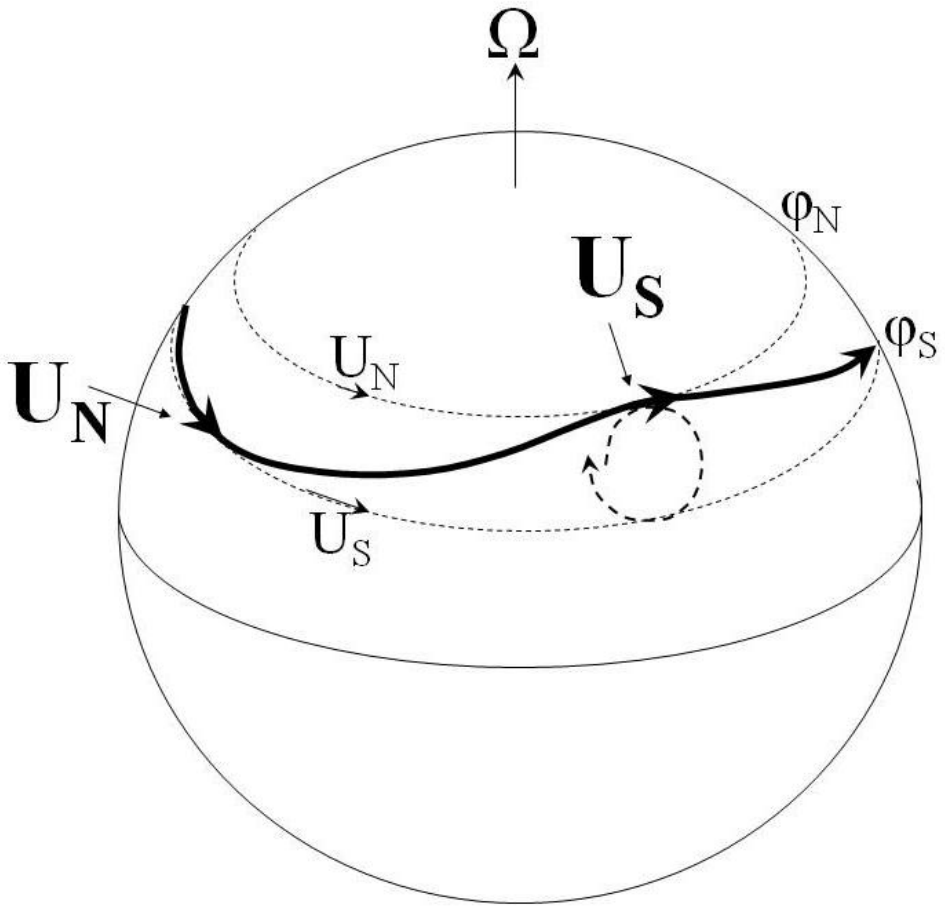
An inertia circle motion has constant relative velocity



The same motion in an absolute frame of reference



The absolute velocities $\mathbf{U_n}$ and $\mathbf{U_s}$ of an object performing an inertia oscillation (seen in an earth bound frame of reference) vary between the absolute velocities $\mathbf{U_n}$ and $\mathbf{U_s}$ of the opposite bounding latitudes



If the absolute velocities vary then also a **real force** is present, not only the **fictitious Coriolis force**

END