Statistics in meteorology without tears

Part II: The three kinds of probabilities

8th Moscow lecture May 2016
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There are three types of probabilities: **the classical, the frequentist and the Bayesian**

1. **The classical applies to the probabilities when tossing of a die (1/6) or a coin (1/2).**

2. **The frequentist applies to analyses of historical observation sets (to derive e.g. climatologically based probabilities).**

3. **The Bayesian, subjective or degree of belief is used by e.g. to summarize or update one's preliminary assessment considering new available information.**

The classical definition of probability helps us play games and add probabilities.

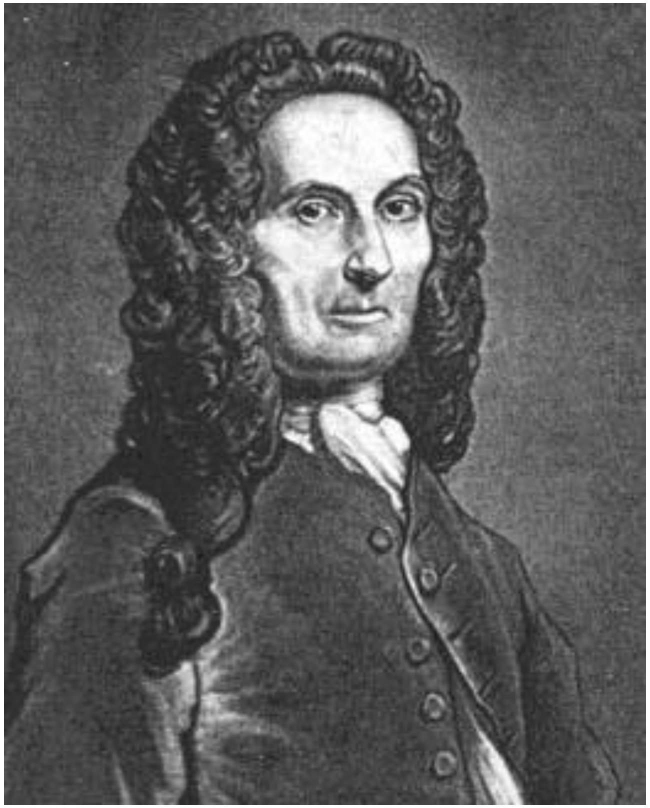
Probability theory grew out of the interest in gambling

But people have gambled since the last ice age or even before that – so why did it take so long for probability theory to develop?

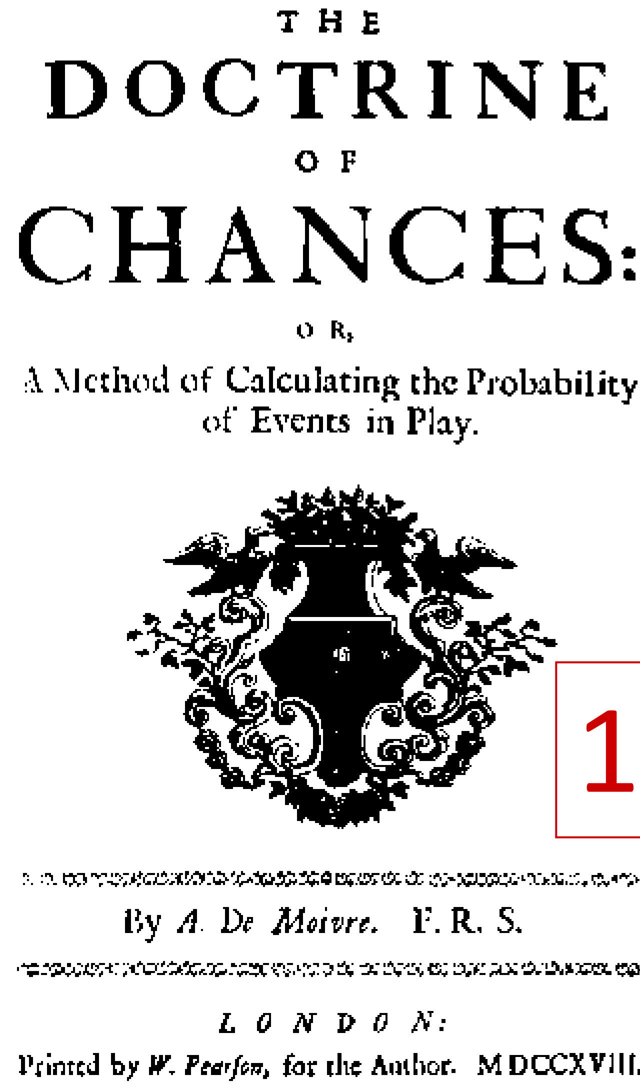
Why did this knowledge not "spill over" into science?

Because people did not have any perception of *randomness*

(except perhaps Cicero and some other Romans)



Abraham De Moivre
1667-1754



THE
DOCTRINE
OF
CHANCES:
OR,
A Method of Calculating the Probability of Events in Play.

By *A. De Moivre.* F. R. S.

LONDON:
Printed by *W. Pearson,* for the Author. MDCCXVIII.

1718

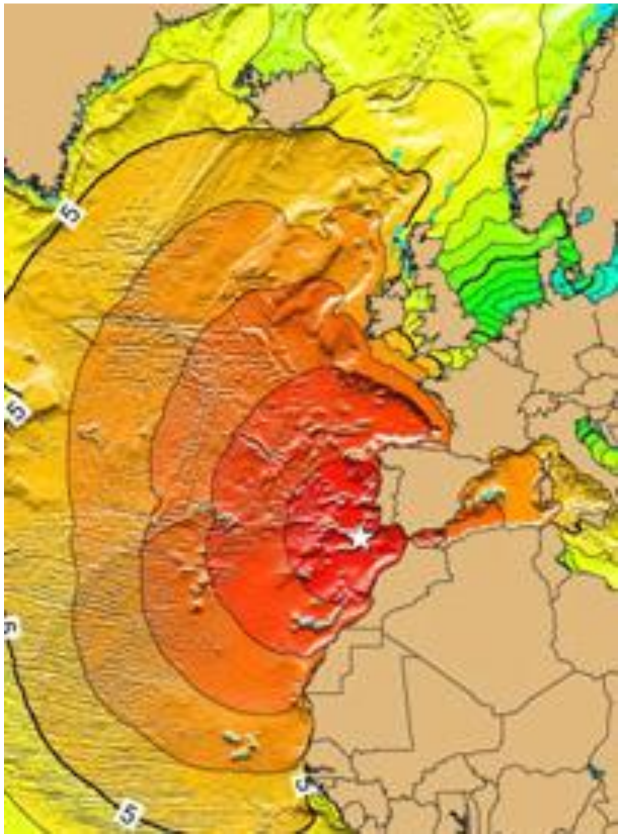
From causes to effects

Deduction

Direct probabilities

Combinatorics

The Lisbon earthquake and tsunami 1755



made people start doubt the existence of an all mighty God that decided everything.

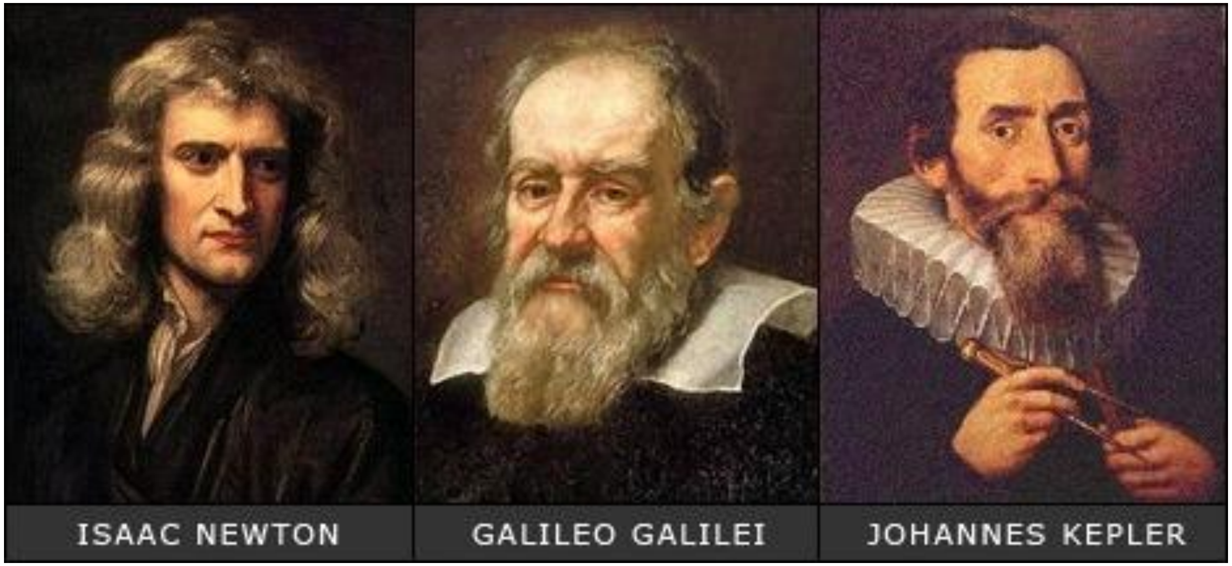
From 1750’s ideas about randomness in science

The frequentist definition of probabilities involves statistical calibration, climatological risk estimations and verification of probability forecasts.

$$BS = \frac{1}{N}\sum_{i=1}^{N}(p_i - o_i)^2$$

The Brier Score, use to verify probabilistic (weather) forecasts

Before the 1800's there was a poor understanding of randomness in measurement errors



1. Scientists had the routine to select their "best" measurement
2. They didn't understand that measurement errors add up and randomly cancel out
3. **They disliked averages of observations since these did not normally agree with measured values**

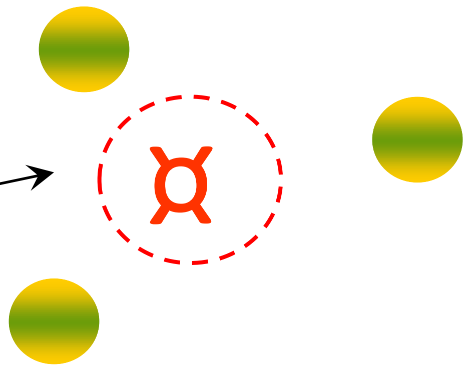
18th century view on observation errors

1. Astronomers in the 1600:s and 1700:s tried to find out <u>which</u> of their diverging observations was the “right” one

2. In the late 1700’ it was realized that that the observations should be <u>combined</u> **even if the result did not agree with any of the observations**

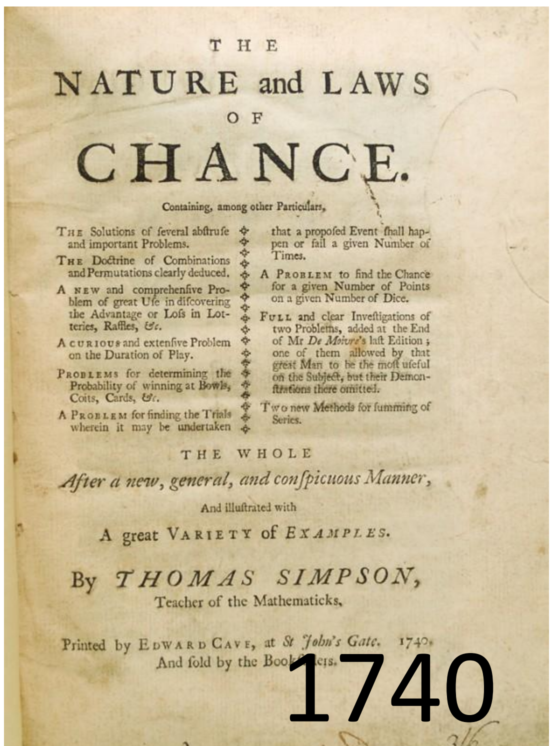
3. The first mathematical discussion on statistical inference

Where is Jupiter?



Thomas Simpson 1710-61

Mathematician



THE

NATURE and LAWS

OF

CHANCE.

Containing, among other Particulars,

The Solutions of ſeveral abſtruſe and important Problems.

The Doctrine of Combinations and Permutations clearly deduced.

A new and comprehenſive Problem of great Uſe in diſcovering the Advantage or Loſs in Lotteries, Raffles, &c.

A curious and extenſive Problem on the Duration of Play.

Problems for determining the Probability of winning at Bowls, Coits, Cards, &c.

A Problem for finding the Trials wherein it may be undertaken that a propoſed Event ſhall happen or fail a given Number of Times.

A Problem to find the Chance for a given Number of Points on a given Number of Dice.

Full and clear Inveſtigations of two Problems, added at the End of Mr De Moivre's laſt Edition; one of them allowed by that great Man to be the moſt uſeful on the Subject, but their Demonſtrations there omitted.

Two new Methods for ſumming of Series.

THE WHOLE

After a new, general, and conſpicuous Manner,

And illuſtrated with

A great Variety of Examples.

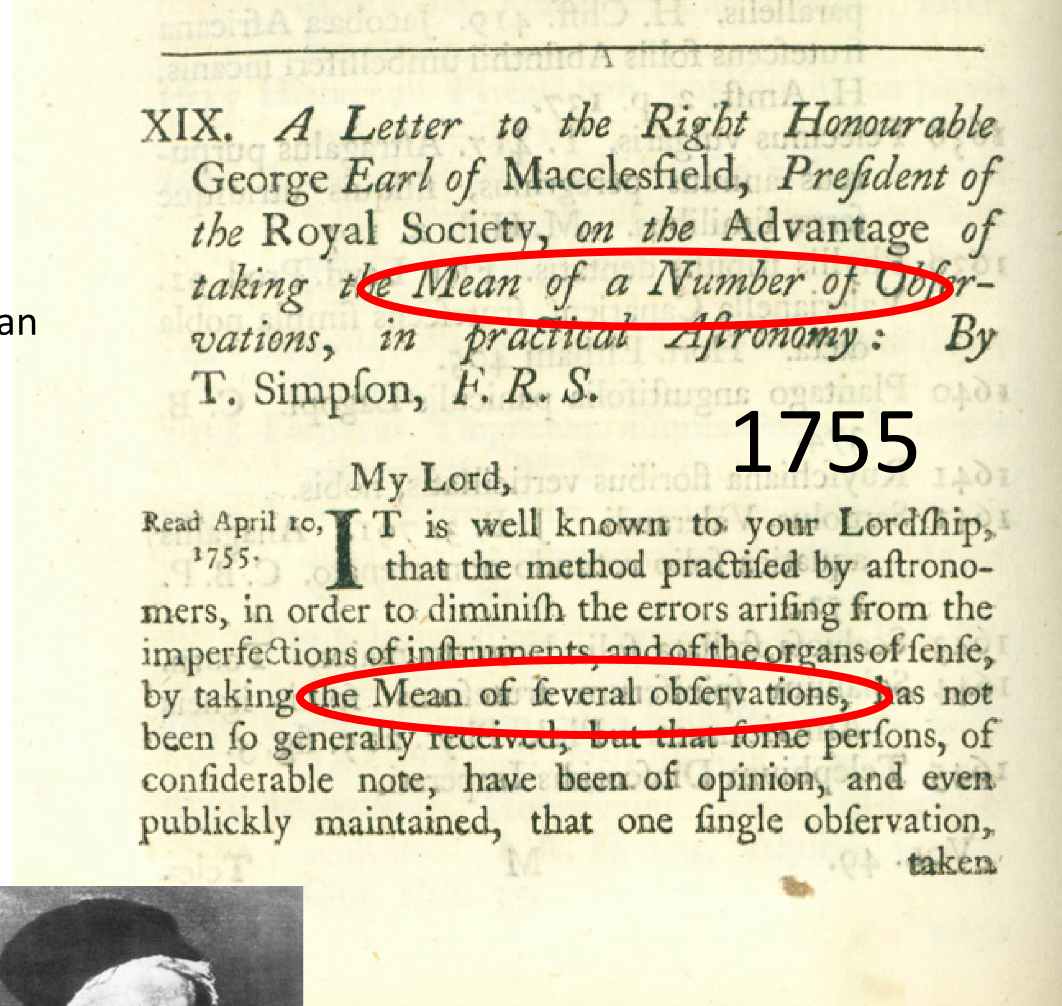
By *THOMAS SIMPSON,*

Teacher of the Mathematicks.

Printed by Edward Cave, at St John's Gate. 1740.

And ſold by the Bookſellers.

1740



XIX. *A Letter to the Right Honourable* George *Earl of* Macclesfield, *Preſident of the* Royal Society, *on the* Advantage *of taking the Mean of a Number of Obſervations, in practical Aſtronomy:* By T. Simpſon, *F. R. S.*

1755

My Lord,

Read April 10, 1755.

IT is well known to your Lordſhip, that the method practiſed by aſtronomers, in order to diminiſh the errors ariſing from the imperfections of inſtruments, and of the organs of ſenſe, by taking the Mean of ſeveral obſervations, has not been ſo generally received, but that ſome perſons, of conſiderable note, have been of opinion, and even publickly maintained, that one ſingle obſervation, taken

Only accepted 50-60 years later thanks to the works by Lagrange and Gauss

The subjective or Bayesian probabilities measure our degree of belief

A Bayesian approach avoids over-confident probabilities such as Concorde before 2000 being the world’s safest air plane

accidents

$$\frac{0}{100\,000^{h}} < \frac{1}{1\,000\,000^{h}}$$

Concorde — Some other airline

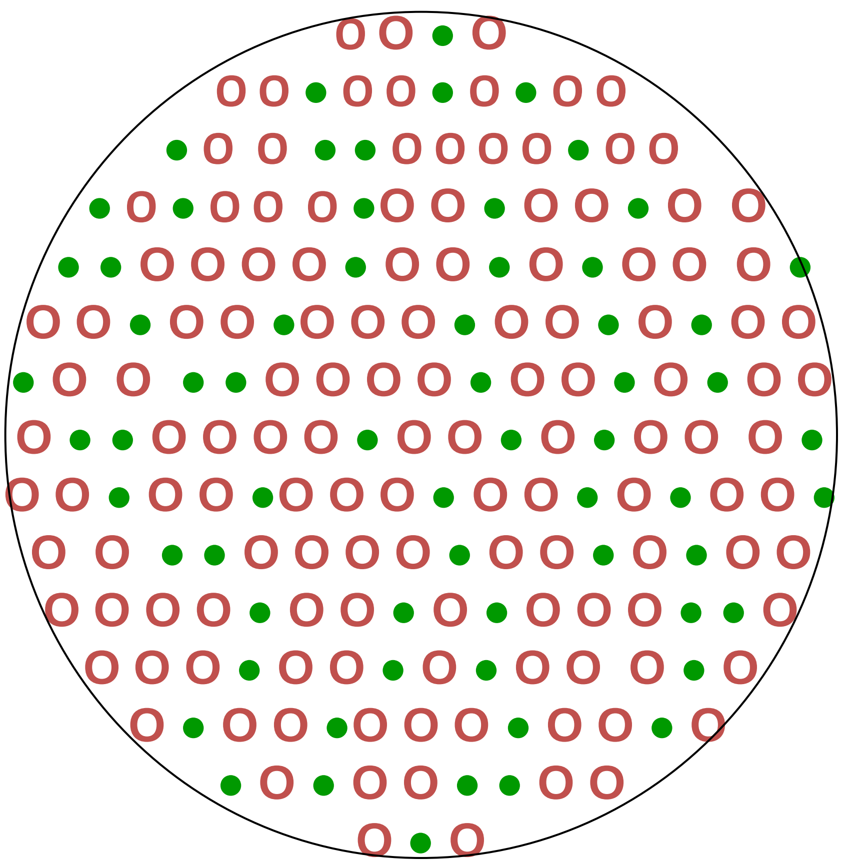
. . .after the 2000 crash the most unsafe

$$\frac{1}{100\,000^{h}} > \frac{1}{1\,000\,000^{h}}$$

A Bayesian would **not**, before 2000, have regarded Concord as the world’s safest airplane

Classical probabilities

rain 30% **dry 70%**



Ensemble of ∞ independent NWP

Selecting three balls yields

● ● ● 3% of cases

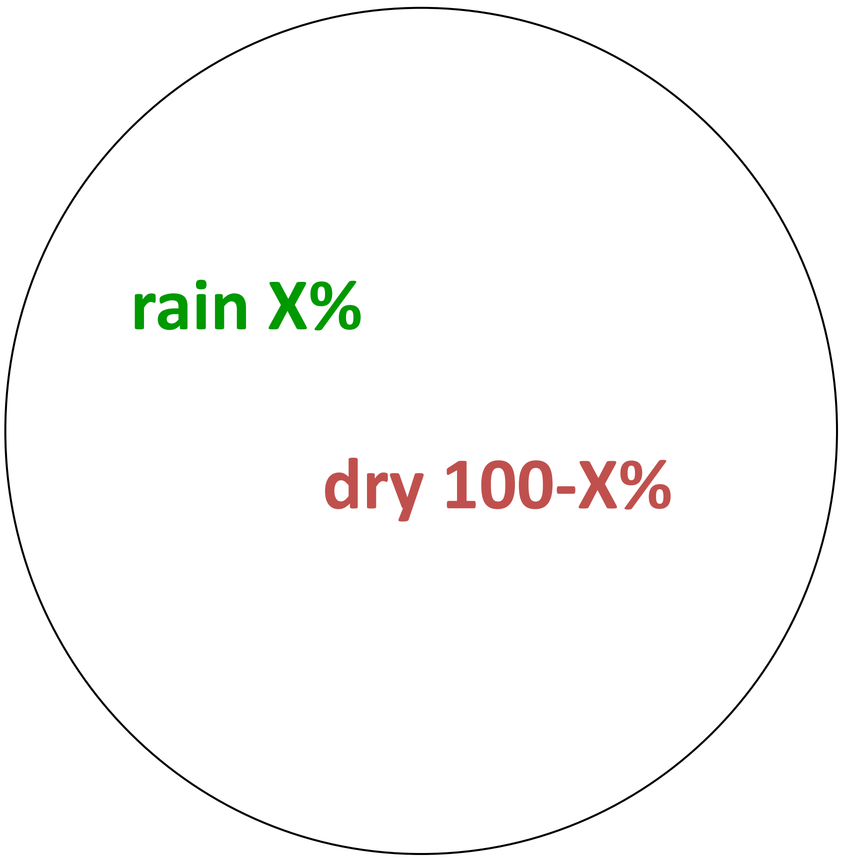
● ● o 19% - " -

● o o 44% - " -

o o o 34% - " –

. . . with a risk of 56% of misrepresentation

Inverse or Bayesian probabilities:



Selecting three balls and getting

● ● o

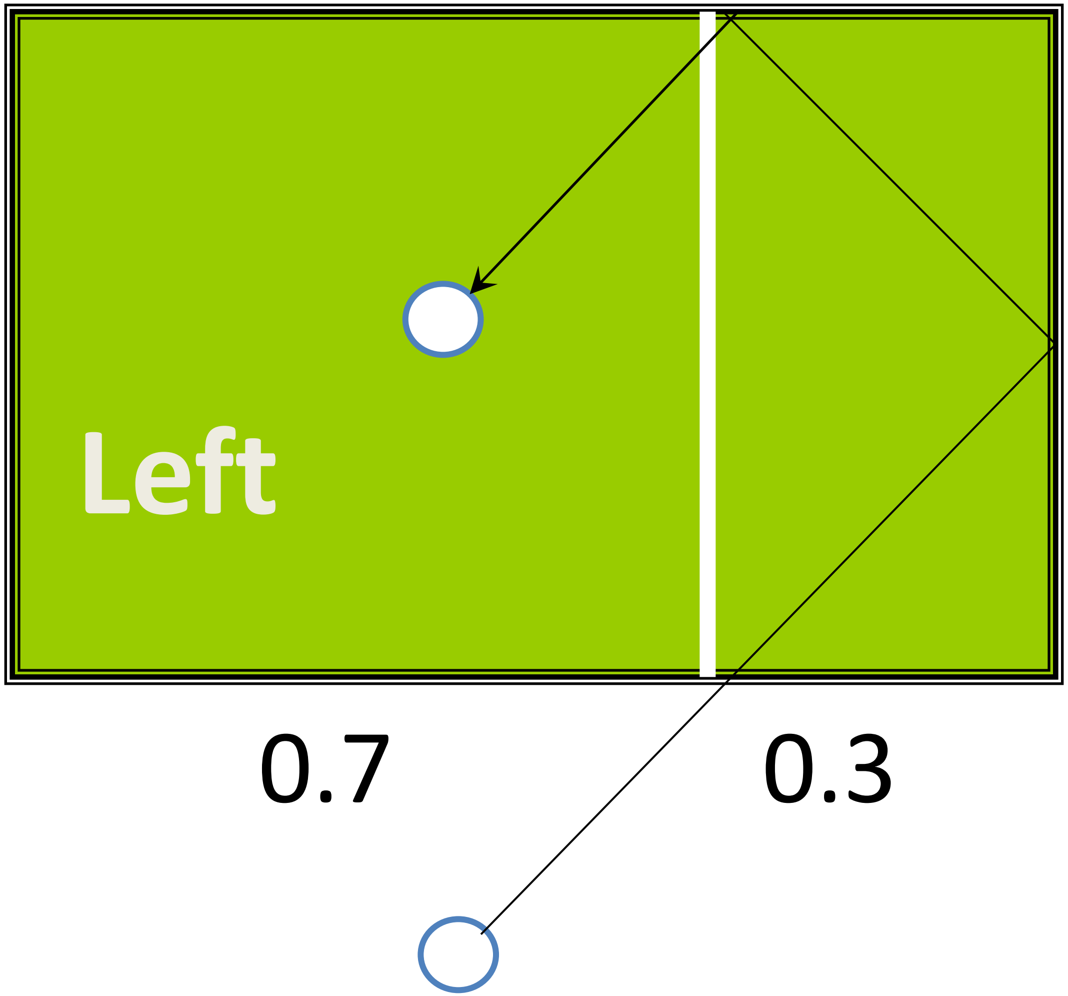
What does that tell us about the proportions

X and 100 – X?

Ensemble of ∞ independent NWP

Bayes's billiard table experiment

Bayes's experiment as it would have been set up by (de Moivre) a classicist:



Throwing 3 balls

Prob(RRR)= 3%

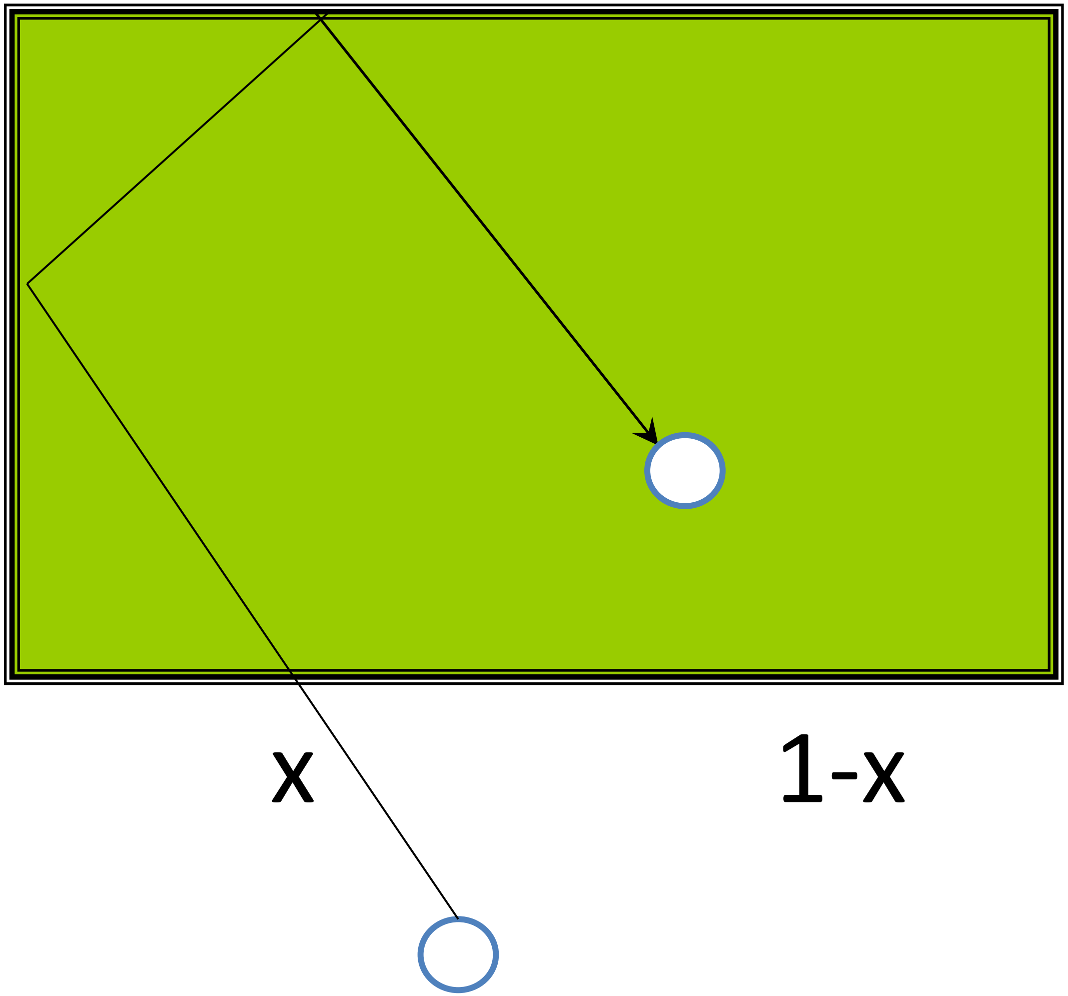
Prob(RRL,RLR,LRR)=19%

Prob(RLL,LRL,LLR)=44%

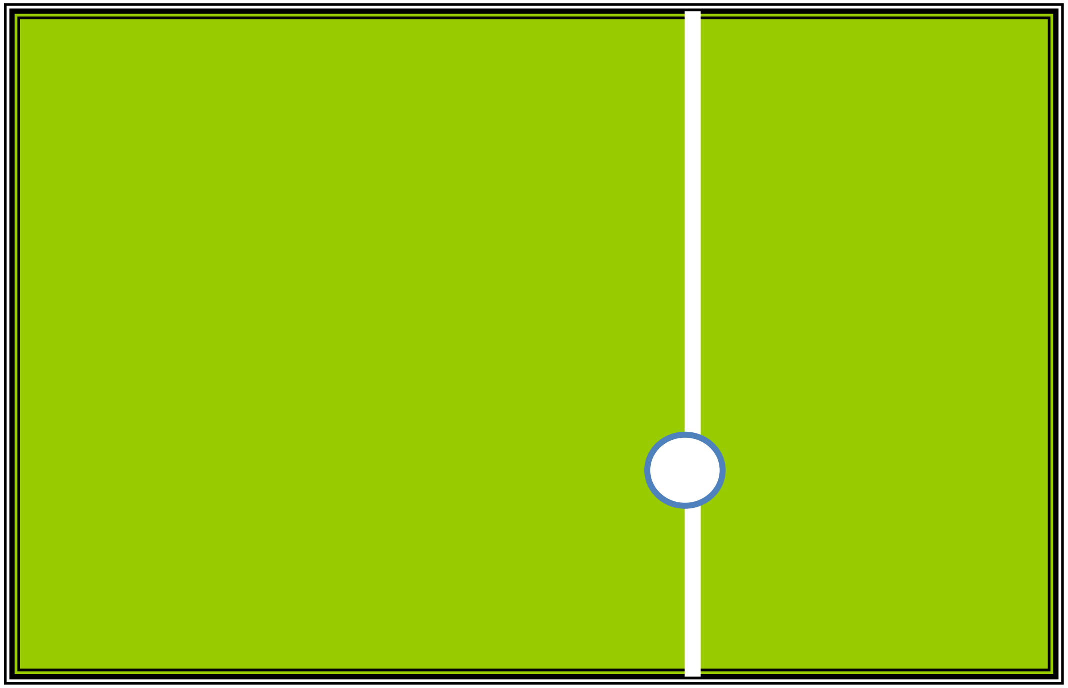
Prob(LLL)=34%

Thomas Bayes' experiment

(defining the white line)

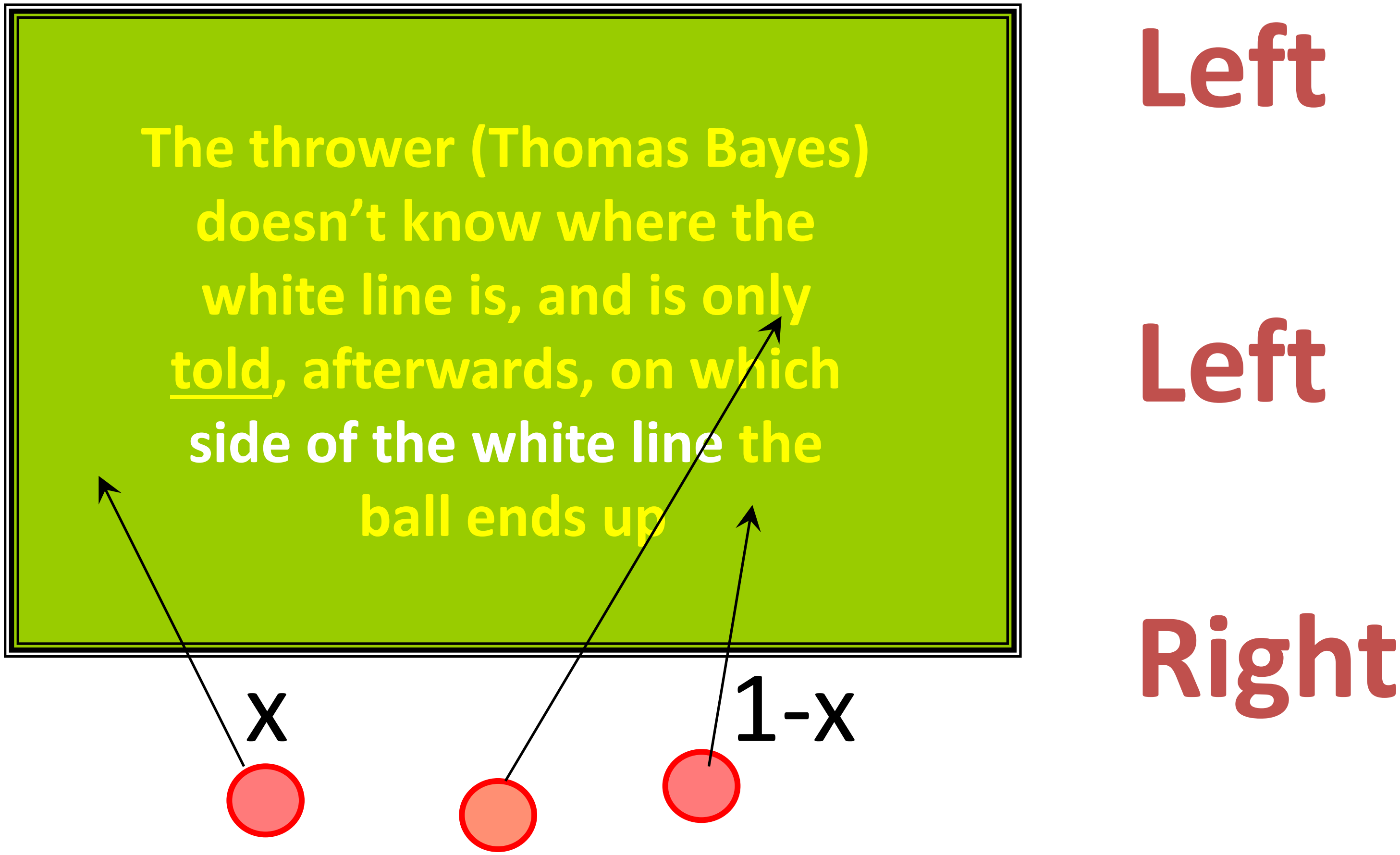


Thomas Bayes' experiment
(defining the white line)



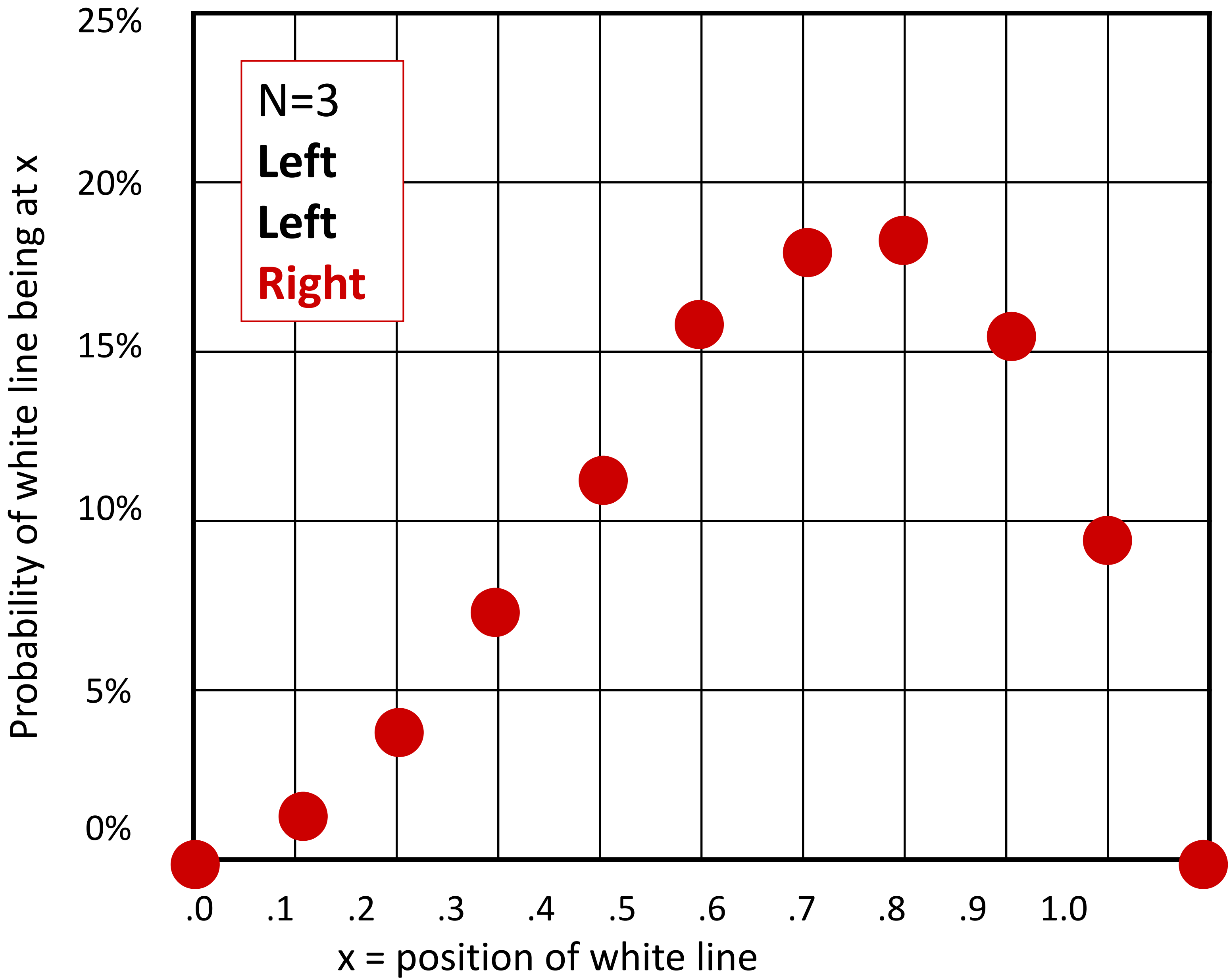
x 1-x

Thomas Bayes’ experiment



This can be solved by using the non-controversial "Bayes Rule"

$$\text{prob}(\mathbf{A}|\mathbf{B}) = \frac{\text{prob}(\mathbf{A})\cdot\text{prob}(\mathbf{B}|\mathbf{A})}{\text{prob}(\mathbf{B})}$$



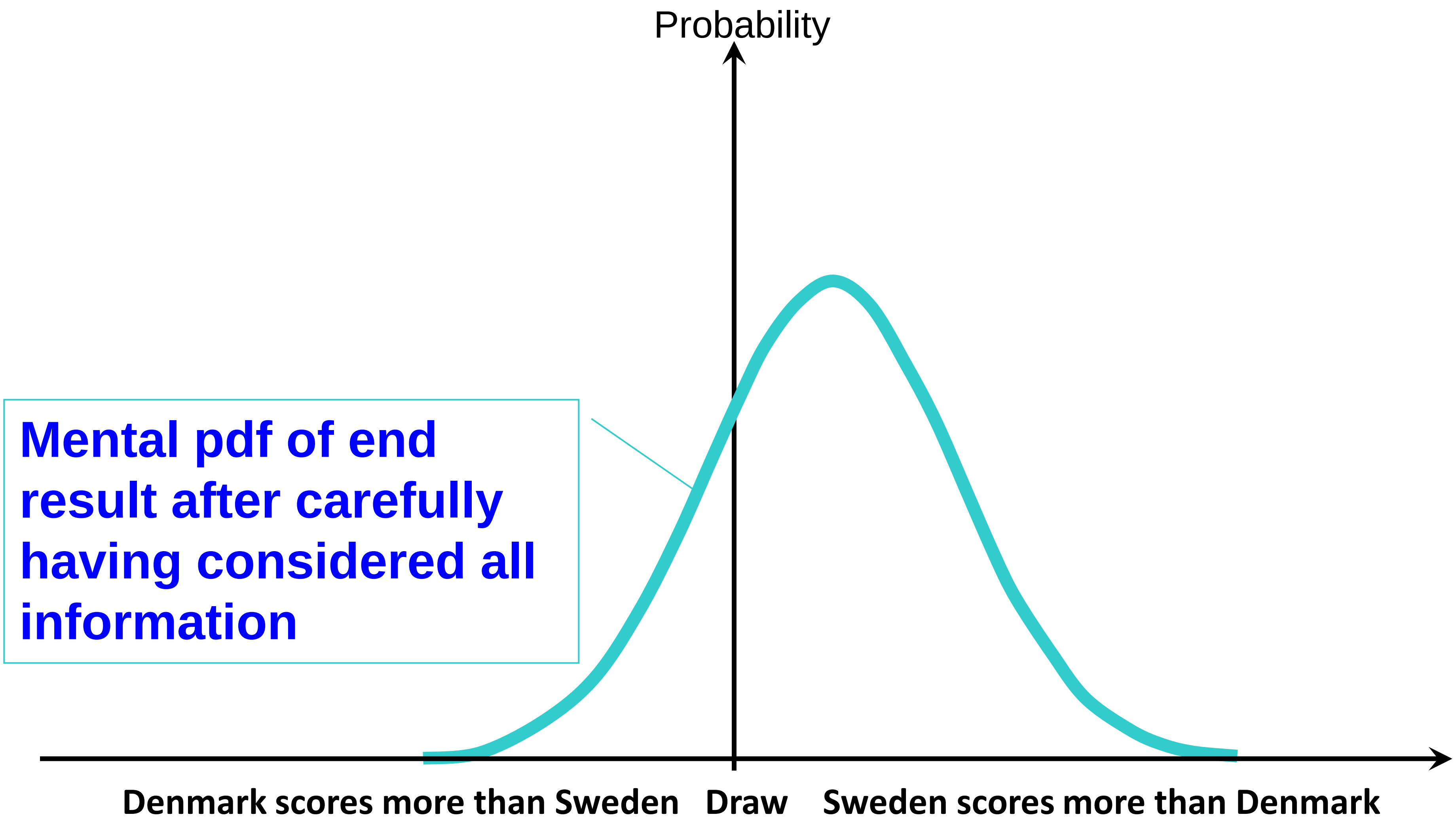
N=3
Left
Left
Right
25%
20%
15%
10%
5%
0%
Probability of white line being at x
.0
.1
.2
.3
.4
.5
.6
.7
.8
.9
1.0
x = position of white line

Updating of subjective probabilities

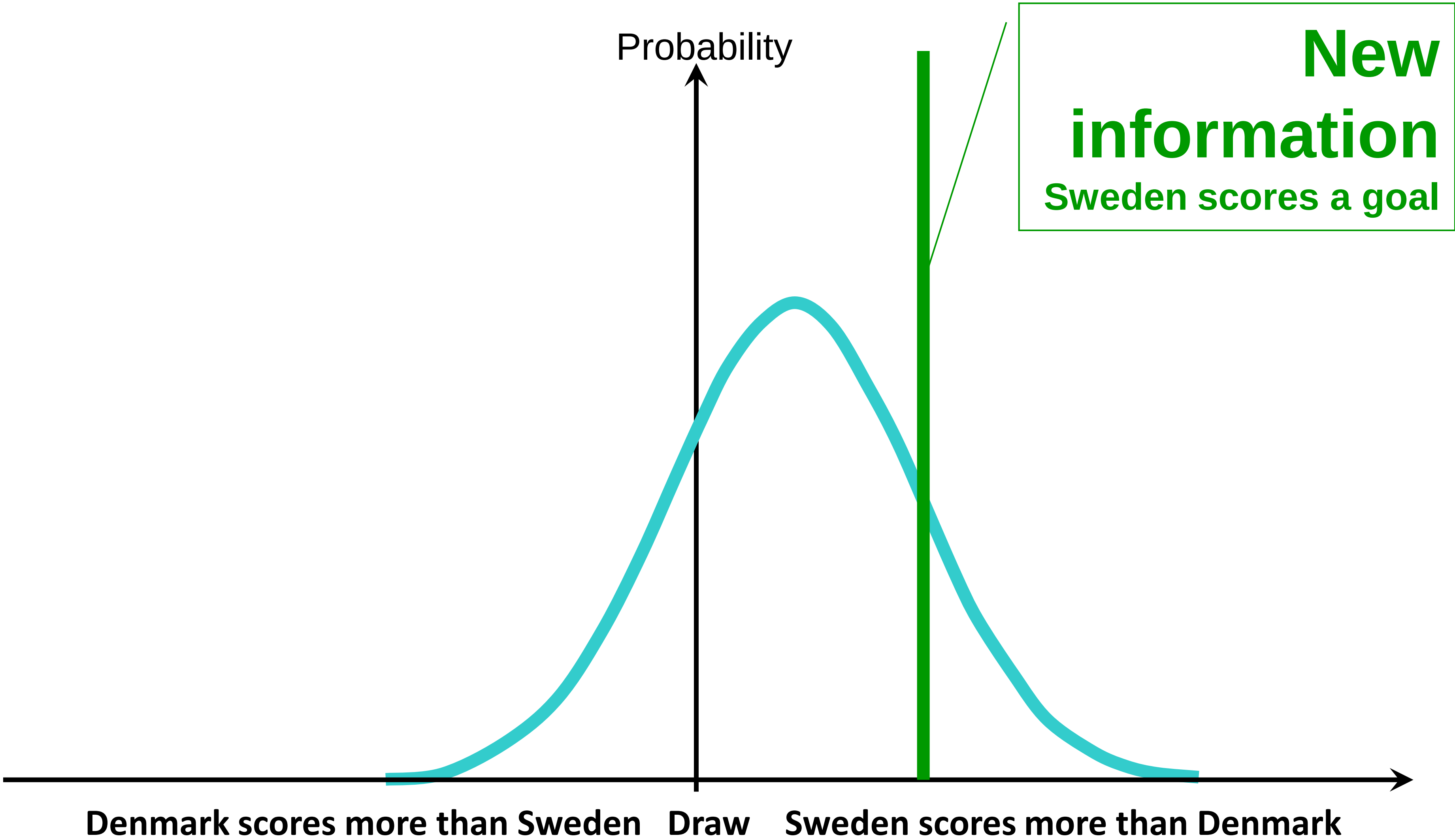
Denmark-Sweden football

After 78 minutes: 0 - 1

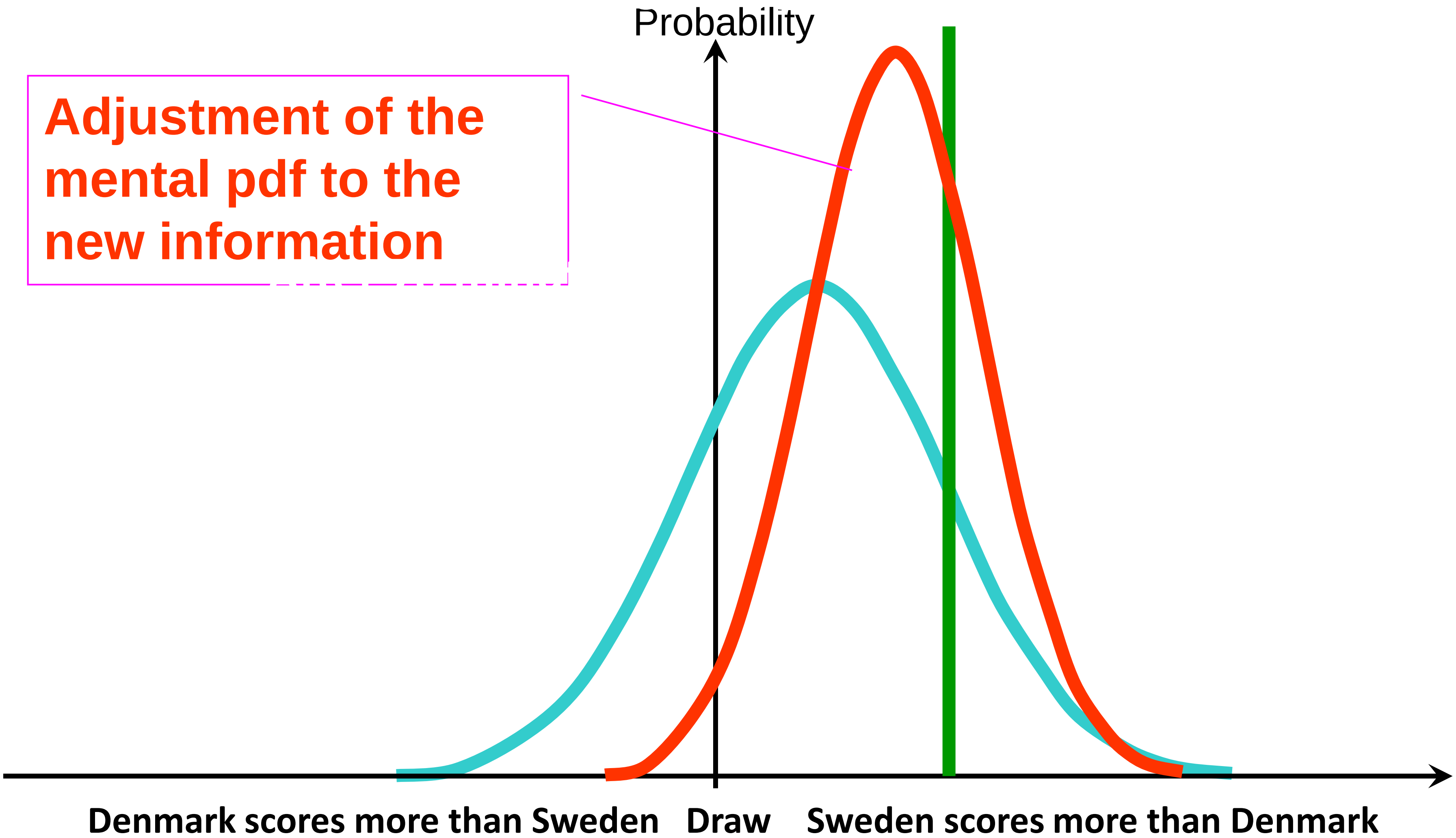
Updating of subjective probabilities



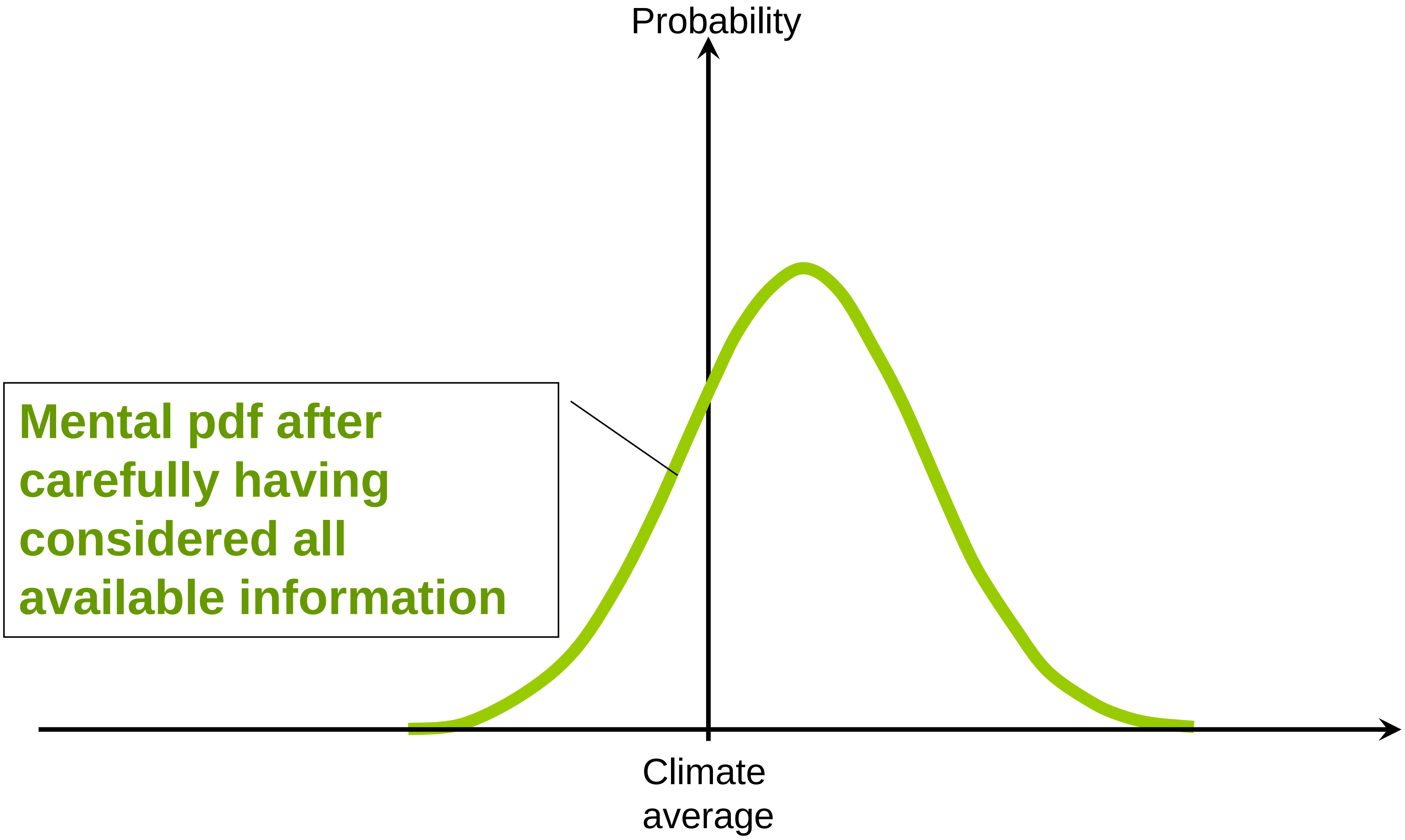
Updating of subjective probabilities



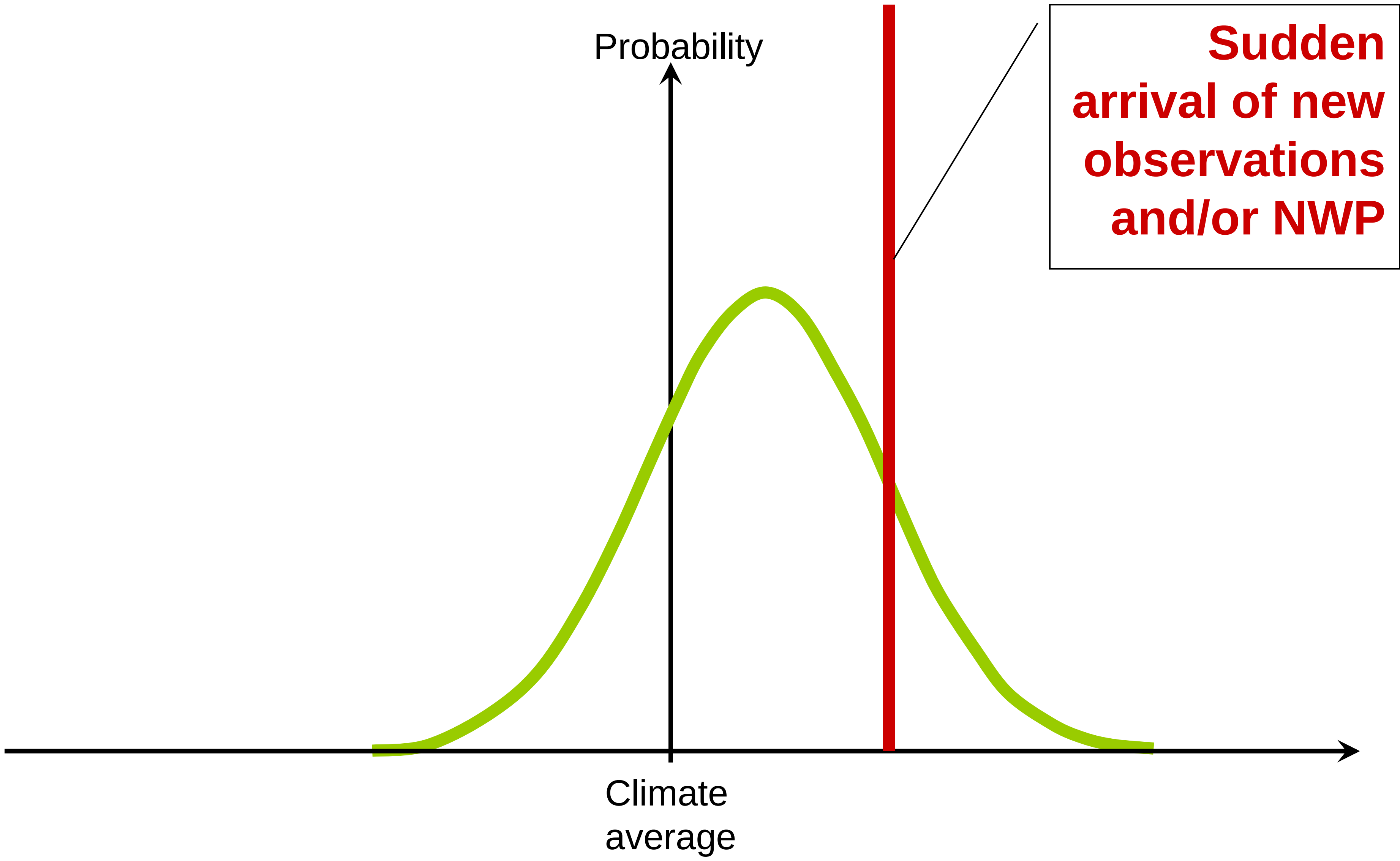
Updating of subjective probabilities



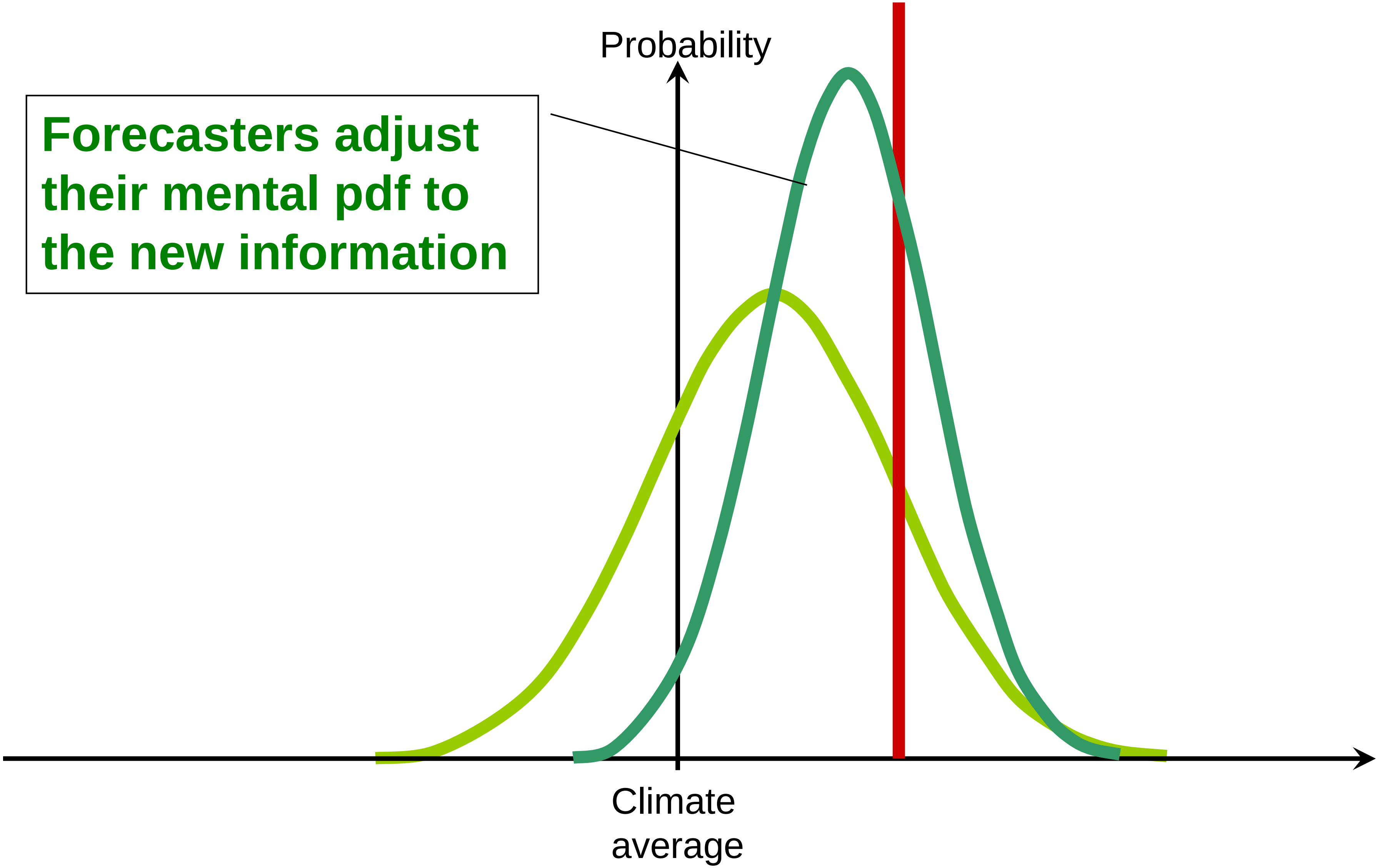
Intuitive Bayesianism among weather forecasters



Intuitive Bayesianism among weather forecasters



Intuitive Bayesianism among weather forecasters



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