Dynamic meteorology without tears

Part II: The Coriolis Effect

The scientific-mathematical basis for these lectures

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Notes and Correspondence
Is the Coriolis effect an 'optical illusion'?

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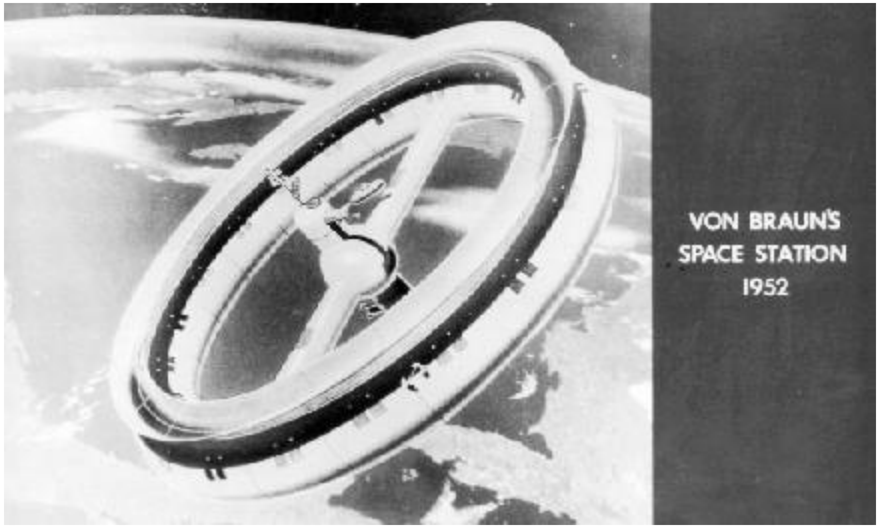
The difference between the derivations of the Coriolis effect on a rotating turntable and on the rotating Earth is discussed. In the latter case a real force, the component of the earth's gravitational attraction, non-parallel to the local vertical, plays a central role by balancing the centrifugal force. That a real force is involved leaves open, not only the question on the inertial nature of the 'inertial oscillations', but also the way we tend to physically conceptualize the terrestrial Coriolis effect.

The Coriolis force is said to be "fictitious" and unable to "do work", but this does not mean the Coriolis Effect is an "optical illusion"

In the 1950's and 1960's the Russians and Americans planned to create artificial gravity on their space stations by letting them rotate. This was nicely depicted in Stanley Kubrick's 1969 movie "2001 - A Space Odyssey":



https://www.youtube.com/watch?v=q3oHmVhviO8

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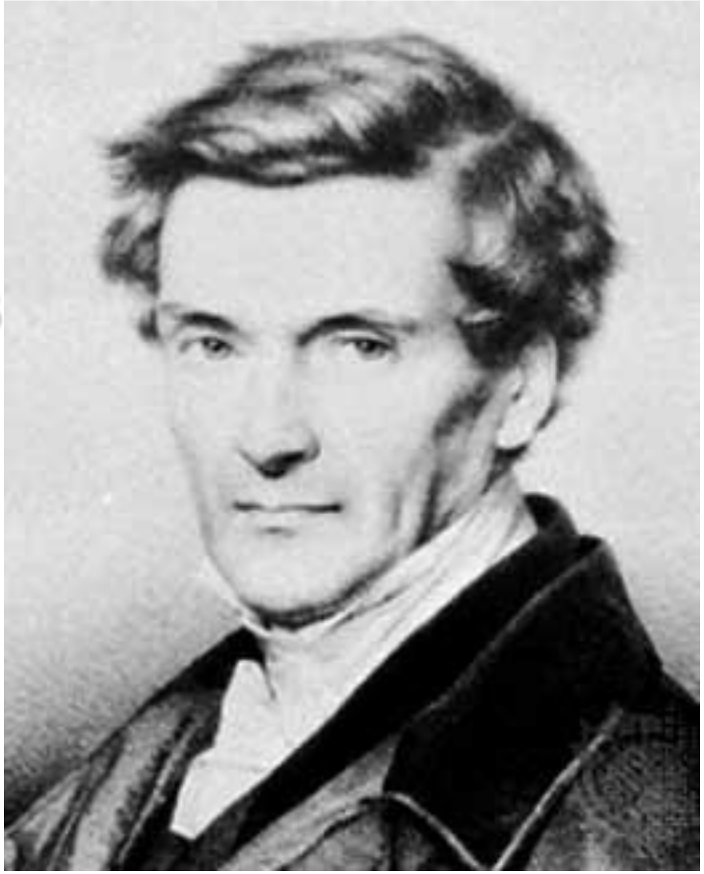
But the Coriolis Effect made this an impossibility – why?

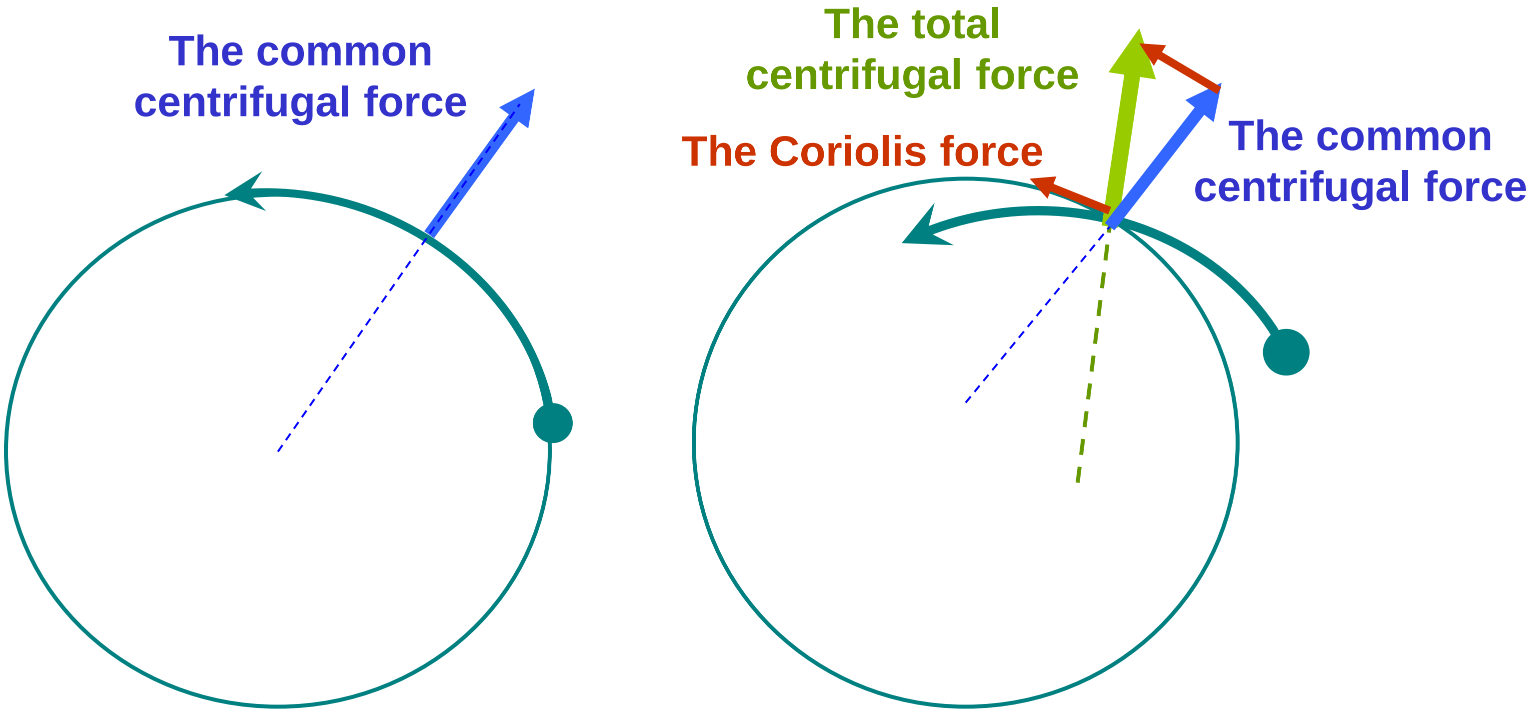
The answer is in the original 1835 Coriolis paper

Gaspard Gustave Coriolis 1784-1843



Coriolis was interested, neither in the atmosphere nor in the oceans – ***but in machines***

Coriolis was interested in how the centrifugal effect acted on moving parts in rotating machines



A stationary object within the rotating system

An object moving (inwards) in the rotating system

Coriolis's force was the "extra force" that had to be added to the common centrifugal force to get the total centrifugal force

The general equation for relative motion in a rotating system

The Coriolis force (per unit mass)

The centrifugal acceleration

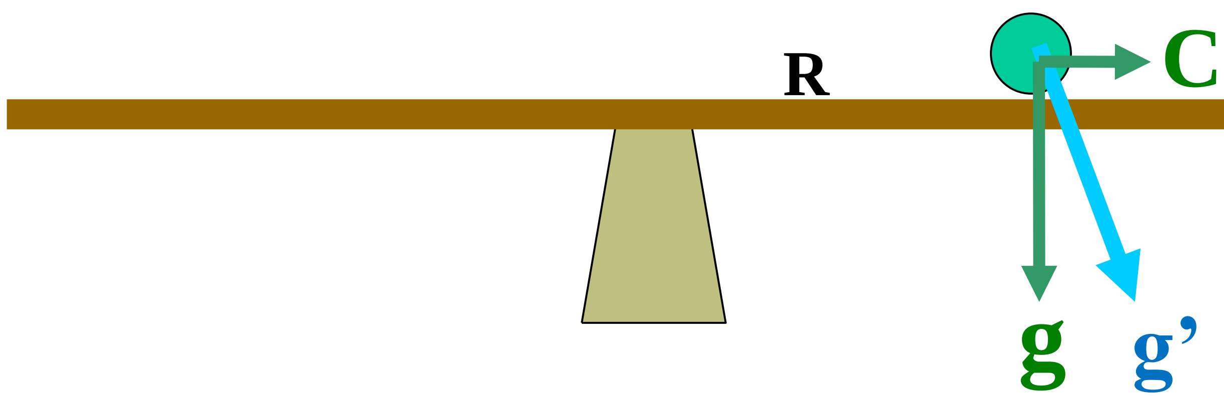
$$\left(\frac{d\,\mathbf{V}_r}{dt}\right)_r = \frac{d\,\mathbf{V}}{dt} - 2\boldsymbol{\Omega}\times\mathbf{V}_r - \boldsymbol{\Omega}\times(\boldsymbol{\Omega}\times\mathbf{r})$$

Applied on a flat rotating carousel at a distance **R**

$$\left(\frac{d\mathbf{V}_r}{dt}\right)_r = \mathbf{g} - 2\mathbf{\Omega}\times\mathbf{V}_r - \mathbf{\Omega}\times(\mathbf{\Omega}\times\mathbf{R})$$

Gravity

R = distance to the rotational axis

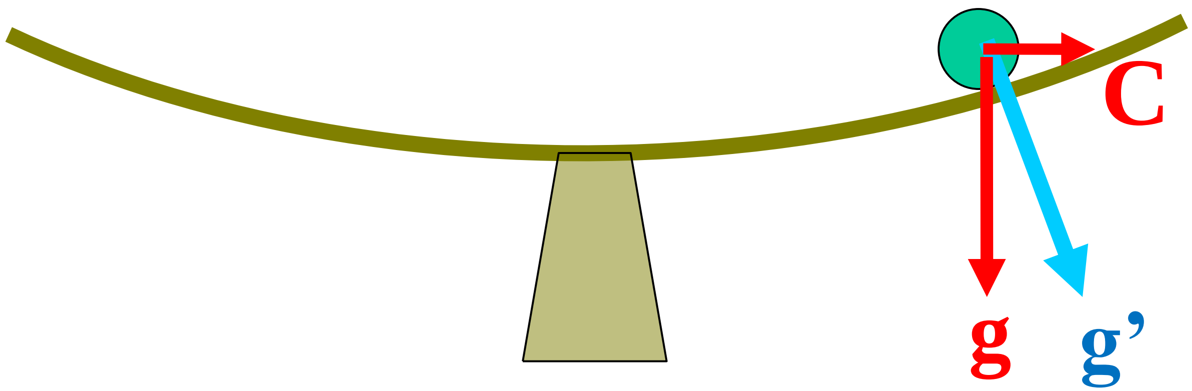


The resultant **g’** is not perpendicular to the carousel surface so the object glides outward

By making the carrousel concave we "get rid of" the centrifugal force by combining it with gravity

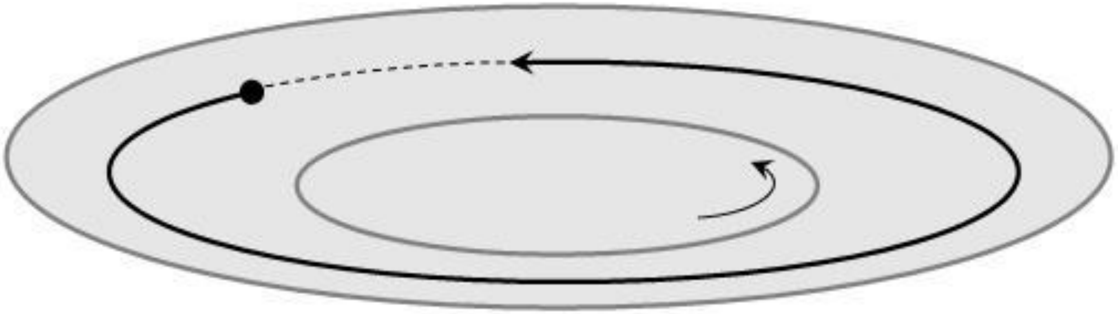
$$\mathbf{g}' = \mathbf{g} - \mathbf{\Omega} \times (\mathbf{\Omega} \times \mathbf{R})$$

$$\left(\frac{d\,\mathbf{V}_r}{dt}\right)_r = \mathbf{g}' - 2\mathbf{\Omega} \times \mathbf{V}_r$$

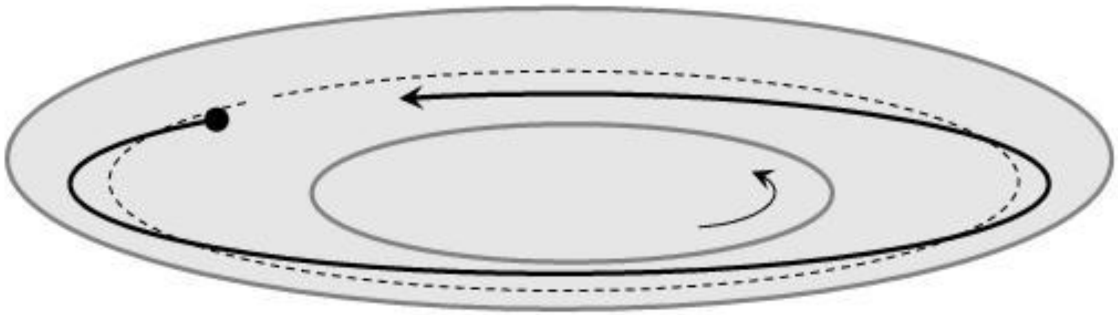


The resultant **g'** is now perpendicular to the carousel surface so the object does not glide outward

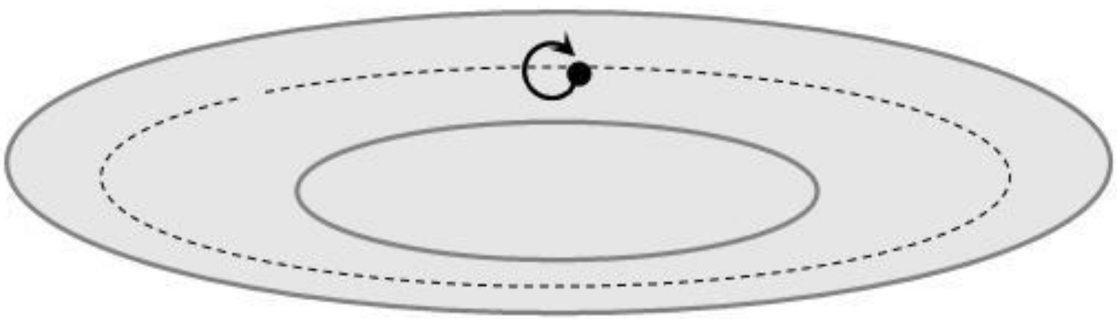
The motion seen from outside



The small body is not perturbed

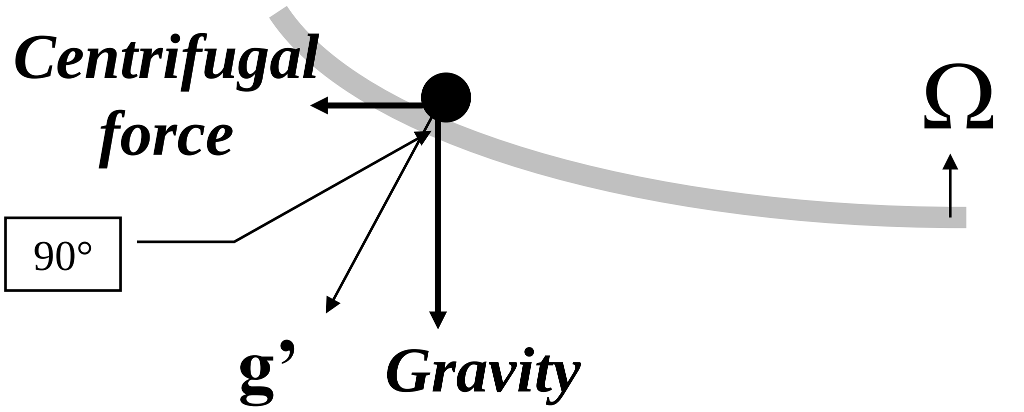


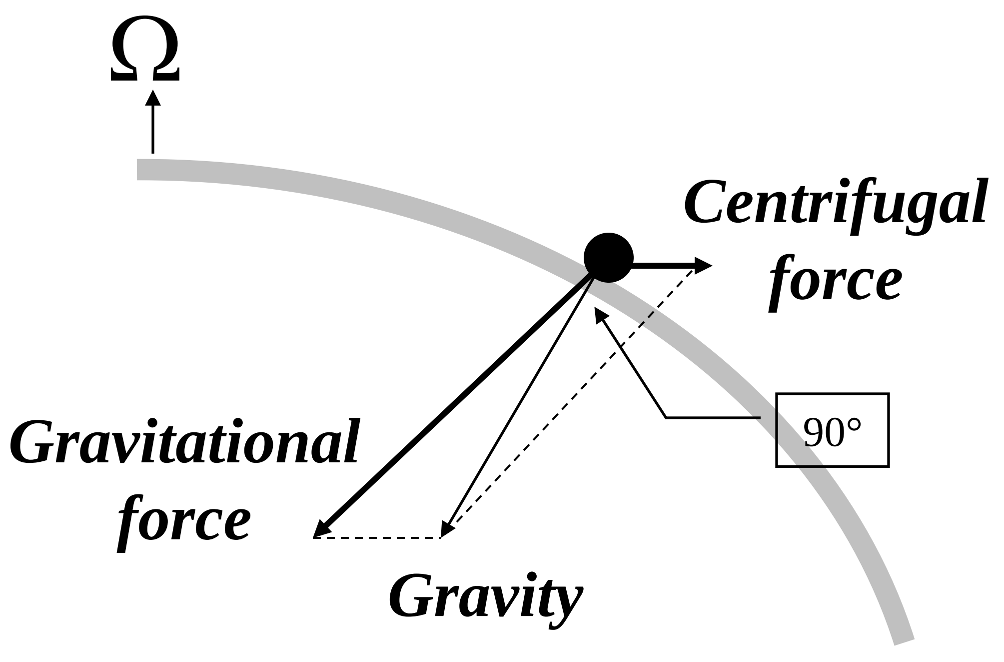
The small body is perturbed



The same seen from "inside" the rotating parabola

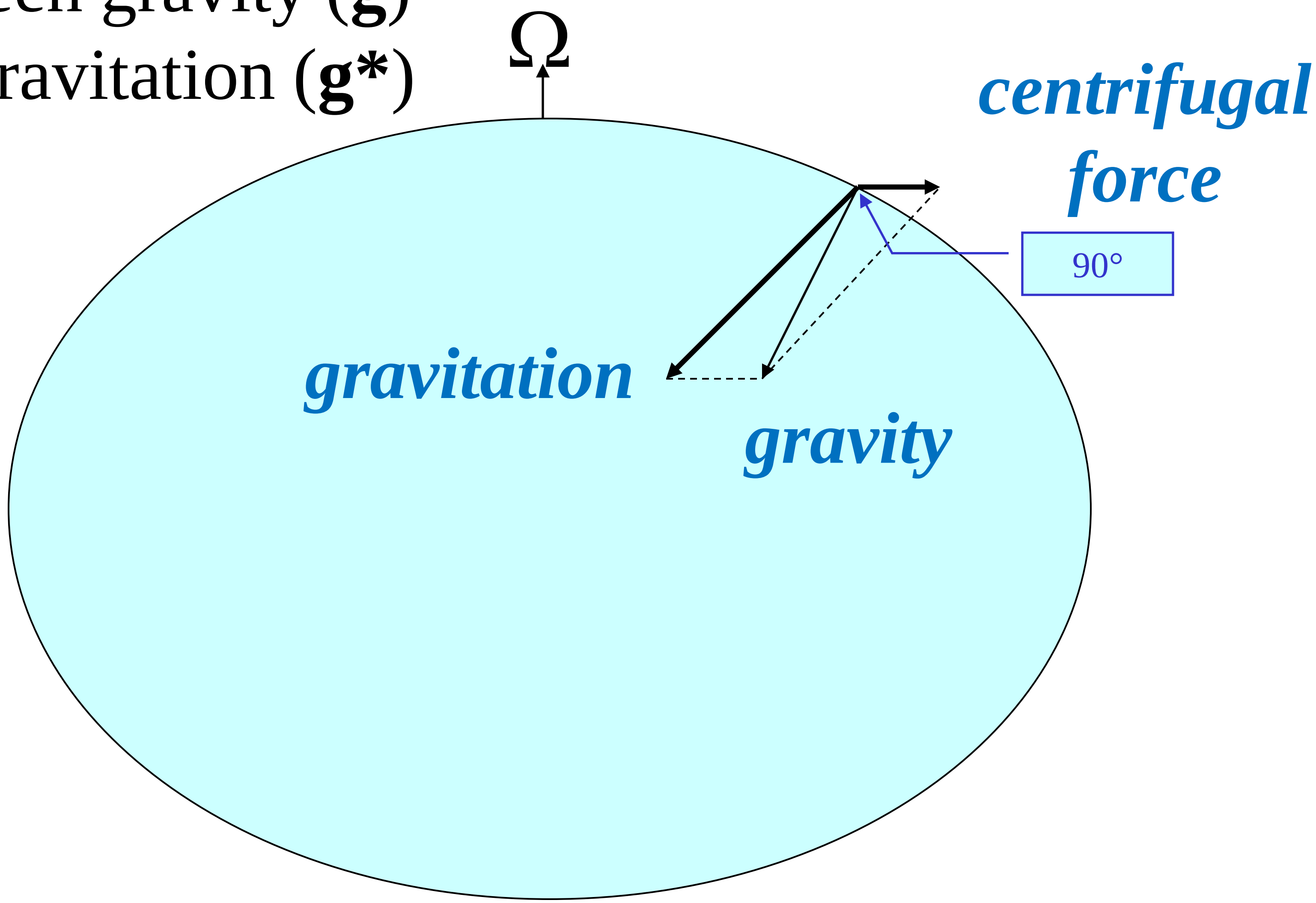
So now we know how to "get rid of" the centrifugal force on a carousel, but what about the earth?





Exactly in the same way, with the shape of the earth having an important role

The crucial difference between gravity (**g**) and gravitation (**g***)



The general equation for relative motion in a rotating system

The Coriolis force (per unit mass)

The centrifugal acceleration

$$\left(\frac{d\mathbf{V}_r}{dt}\right)_r = \frac{d\mathbf{V}}{dt} - 2\mathbf{\Omega}\times\mathbf{V}_r - \mathbf{\Omega}\times(\mathbf{\Omega}\times\mathbf{r})$$

. . . applied on the rotating earth at distance **R** from axis

$$\left(\frac{d\mathbf{V}_r}{dt}\right)_r = \mathbf{g}^* - 2\mathbf{\Omega}\times\mathbf{V}_r - \mathbf{\Omega}\times(\mathbf{\Omega}\times\mathbf{R})$$

The earth's gravitational attraction

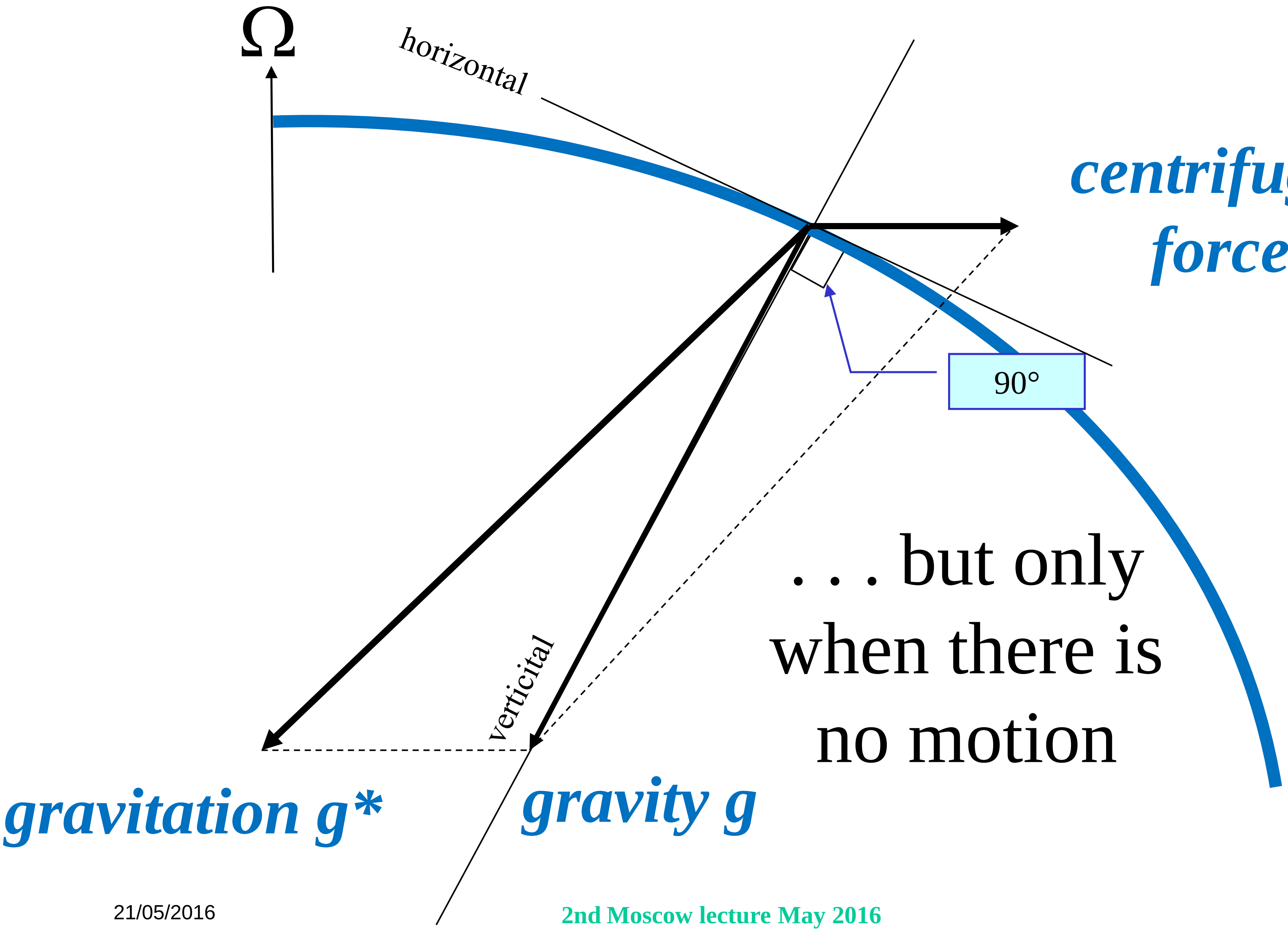
R = distance to the earth's rotational axis

We combine the gravitational attraction (**g***) with the centrifugal force into gravity (**g**)

$$\left(\frac{d\mathbf{V}_r}{dt}\right)_r = \mathbf{g}^* - 2\mathbf{\Omega}\times\mathbf{V}_r - \mathbf{\Omega}\times(\mathbf{\Omega}\times\mathbf{R})$$

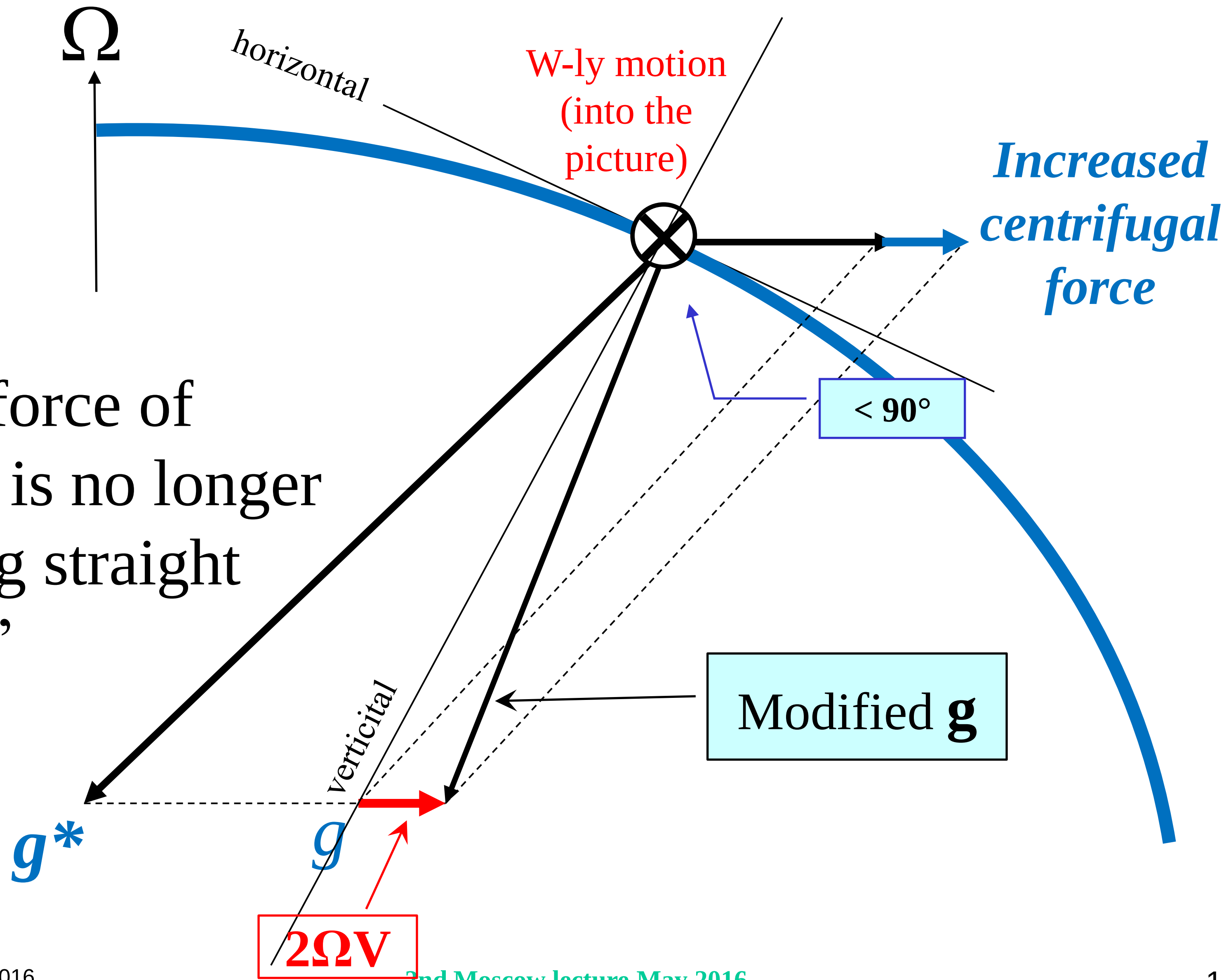
$$\left(\frac{d\mathbf{V}_r}{dt}\right)_r = \mathbf{g} - 2\mathbf{\Omega}\times\mathbf{V}_r$$

The force of gravity is pointing straight "down"

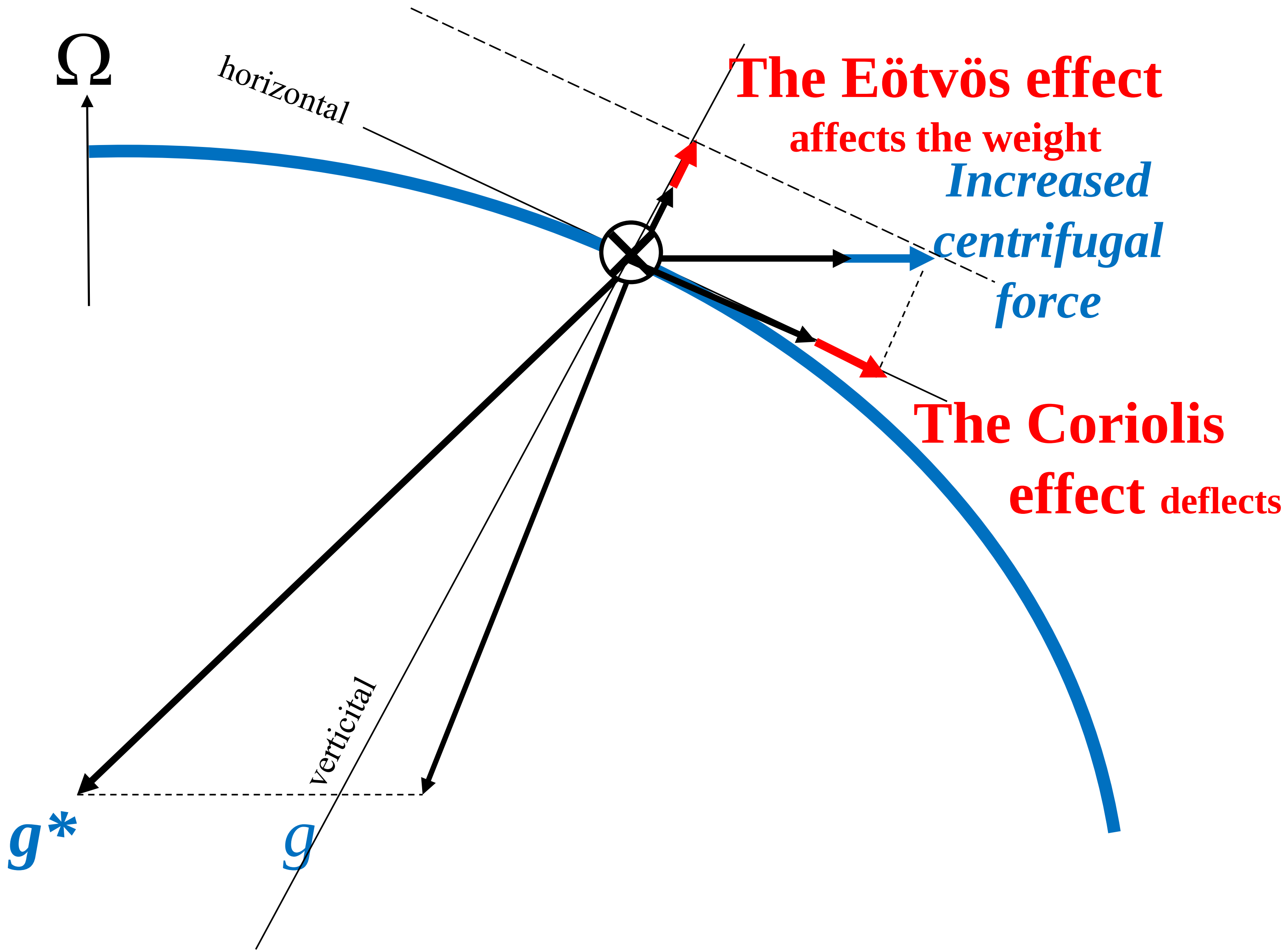


. . . but only when there is no motion

When there is motion . . .



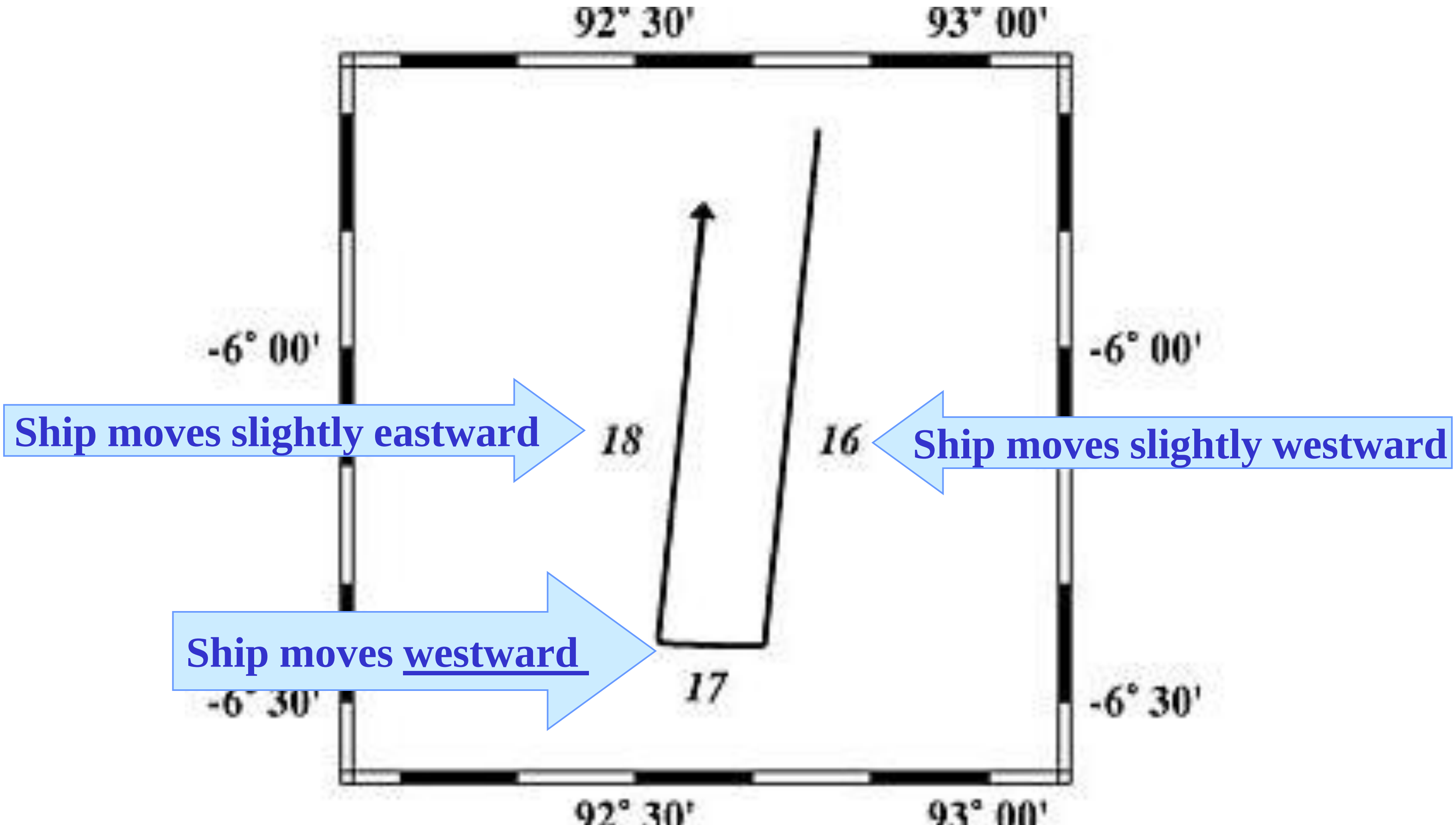
. . .the force of gravity is no longer pointing straight "down"



Ω
horizontal
The Eötvös effect
affects the weight
Increased
centrifugal
force
The Coriolis
effect deflects
verticital
g*
g

Example of the Eötvös effect

The **weight** of a French research vessel sailing in the Indian Ocean



http://www.geologie.ens.fr/~hebert/THESE/CHAP2/FIGURES/fig1.html)

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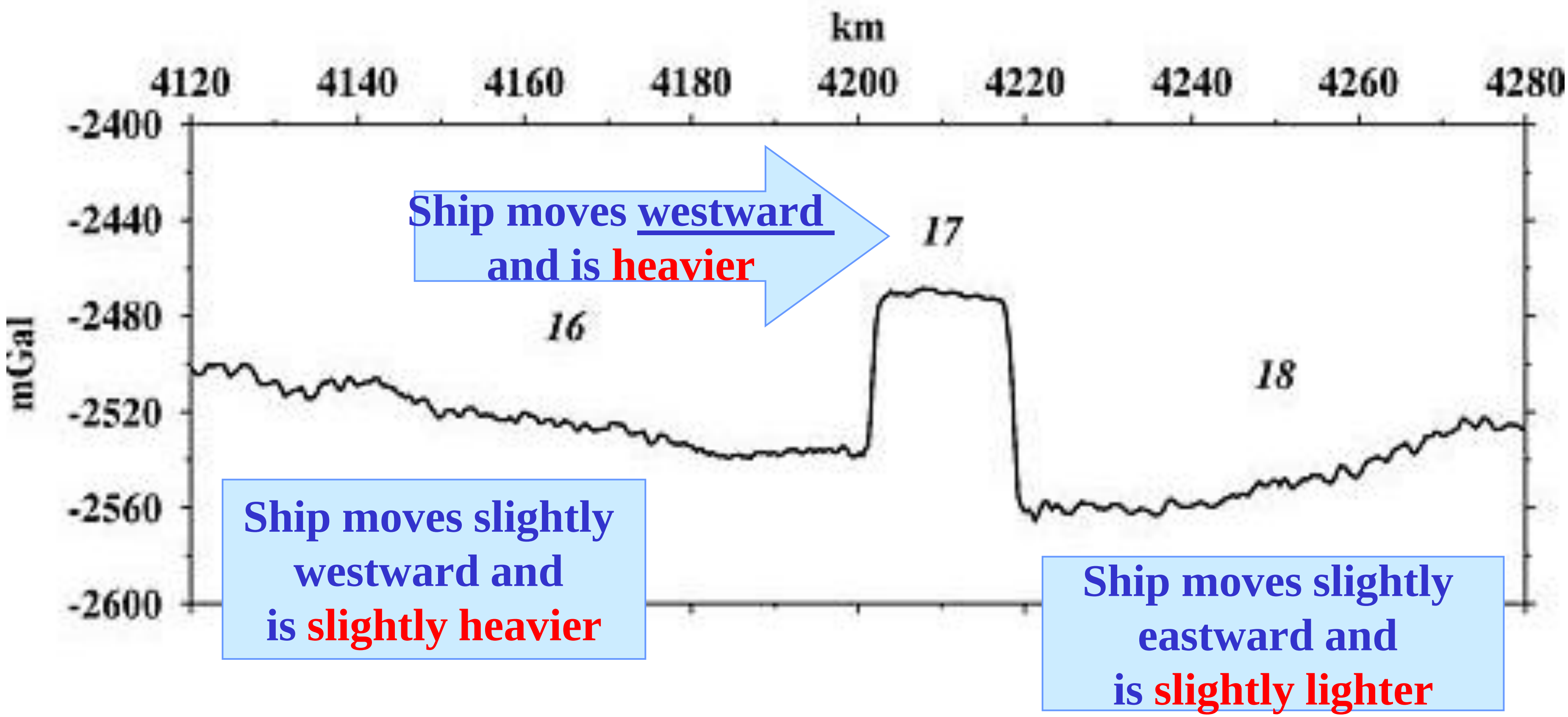
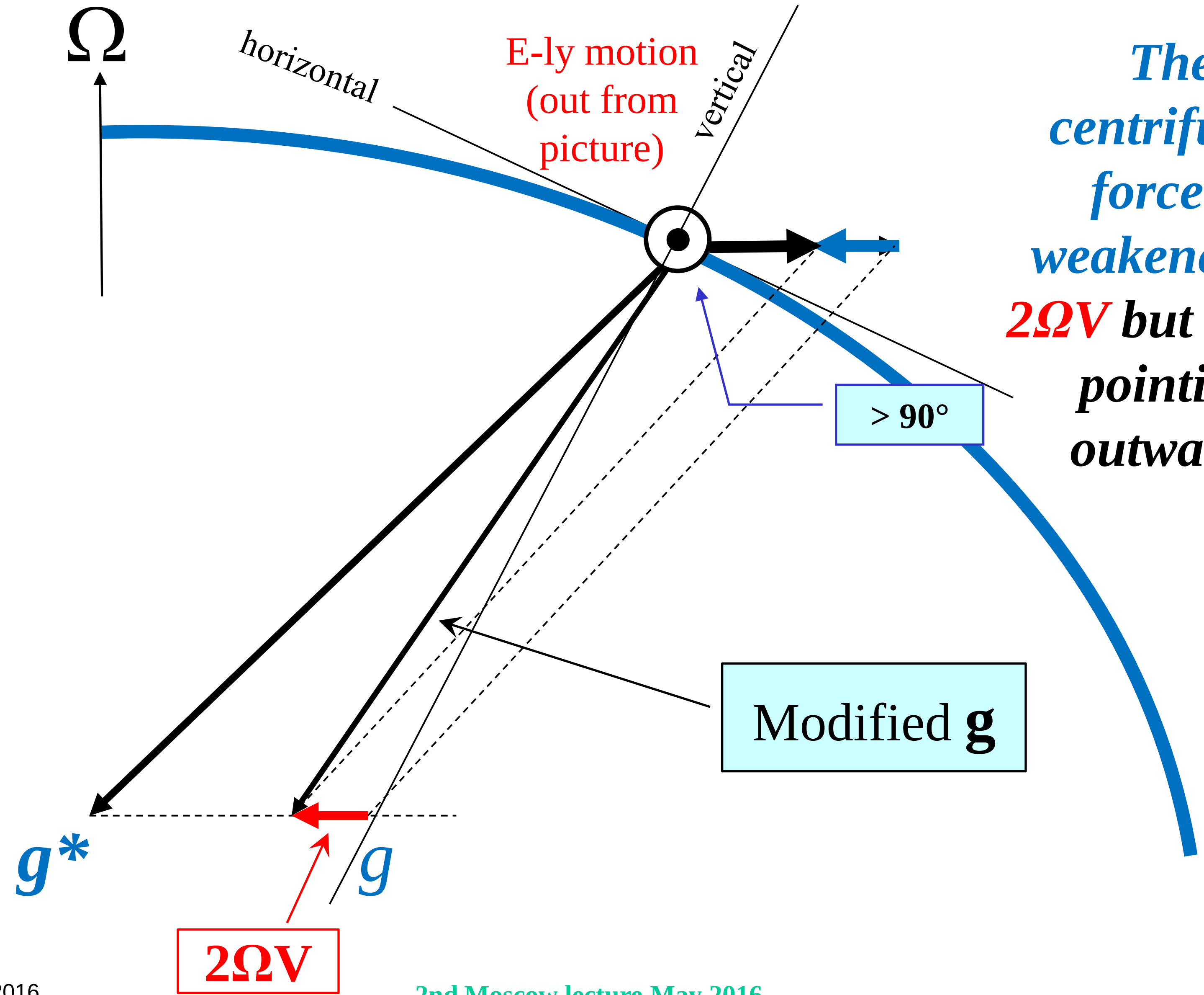


The Eötvös effect only affects west-east motions

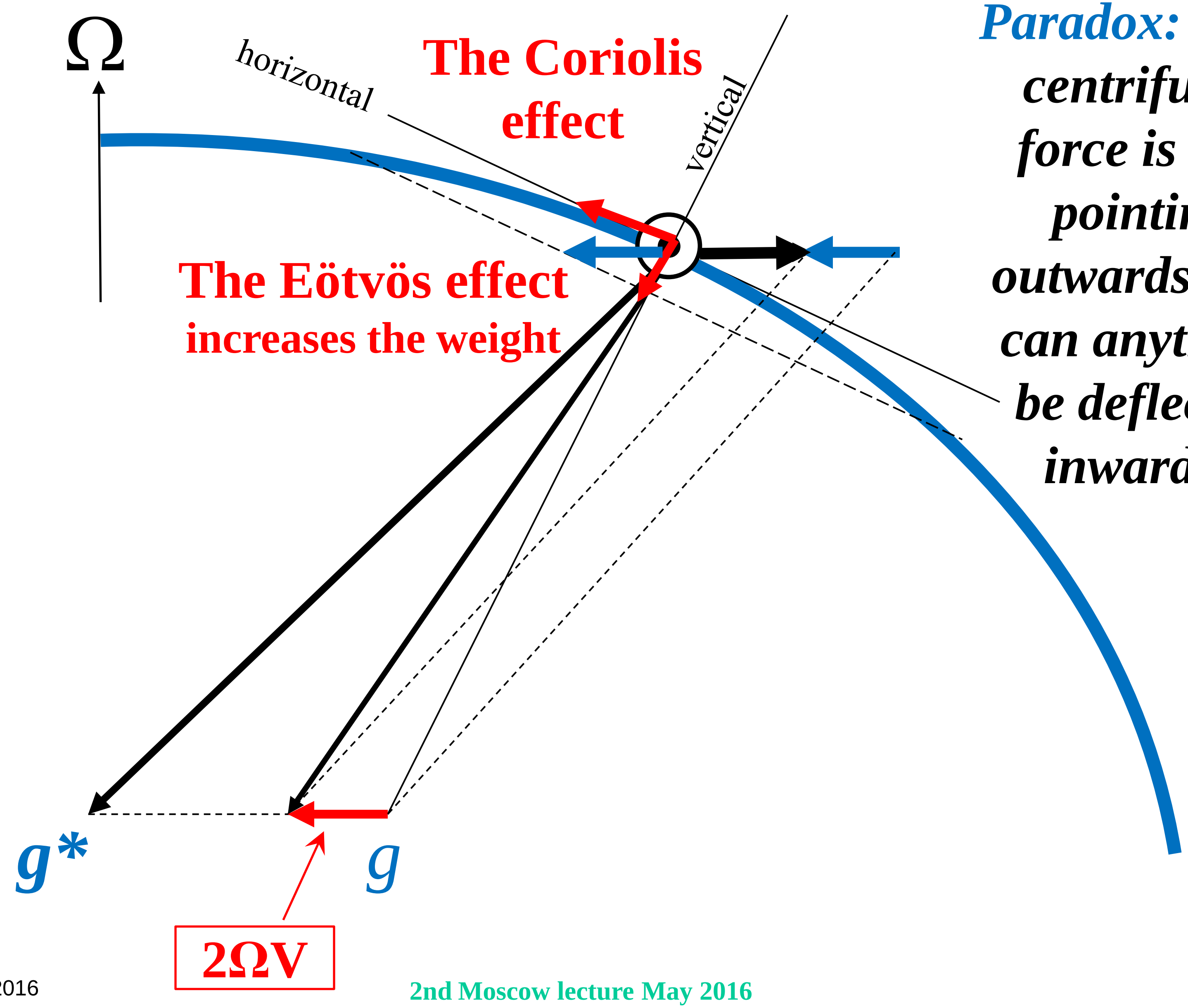
http://www.geologie.ens.fr/~hebert/THESE/CHAP2/FIGURES/fig1.html)

For westward motion the centrifugal force weakens



The centrifugal force is weakened by 2ΩV but is still pointing outwards

The **gravitational force** pulls the motion inwards



***Paradox:* if the centrifugal force is still pointing outwards how can anything be deflected inwards?**

Although the horizontal trajectory of a mass element can be **kinematically** described by

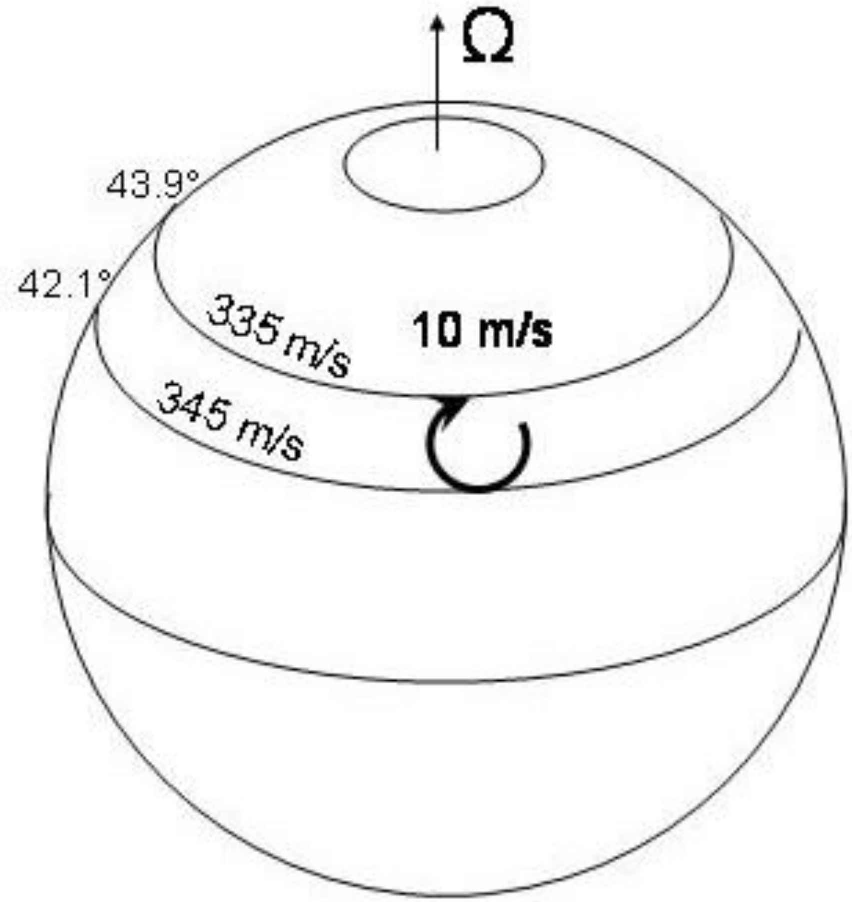
$$\left(\frac{d\,\mathbf{V}_r}{dt}\right)_h = \left(-2\boldsymbol{\Omega}\times\mathbf{V}_r\right)_h$$

...it can **dynamically** only be understood by considering all the three physical forces involved, of which one is a real force

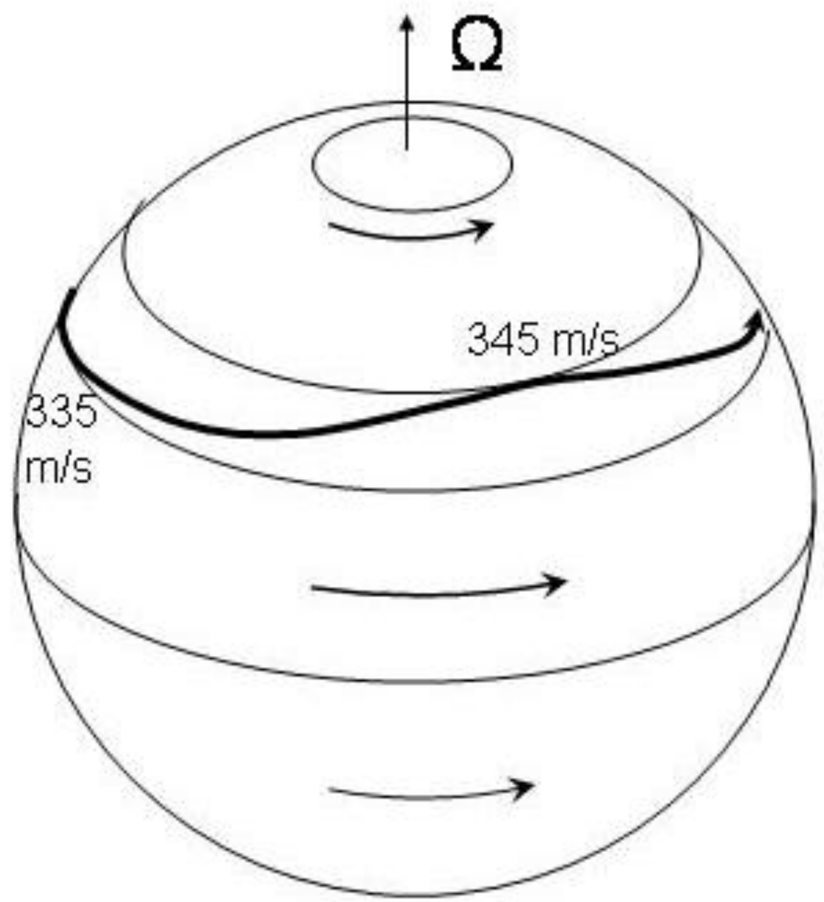
$$\left(\frac{d\,\mathbf{V}_r}{dt}\right)_r = \mathbf{g}^* - 2\boldsymbol{\Omega}\times\mathbf{V}_r - \boldsymbol{\Omega}\times(\boldsymbol{\Omega}\times\mathbf{R})$$

Q: ***Wouldn't a real force accelerate the absolute motion?***

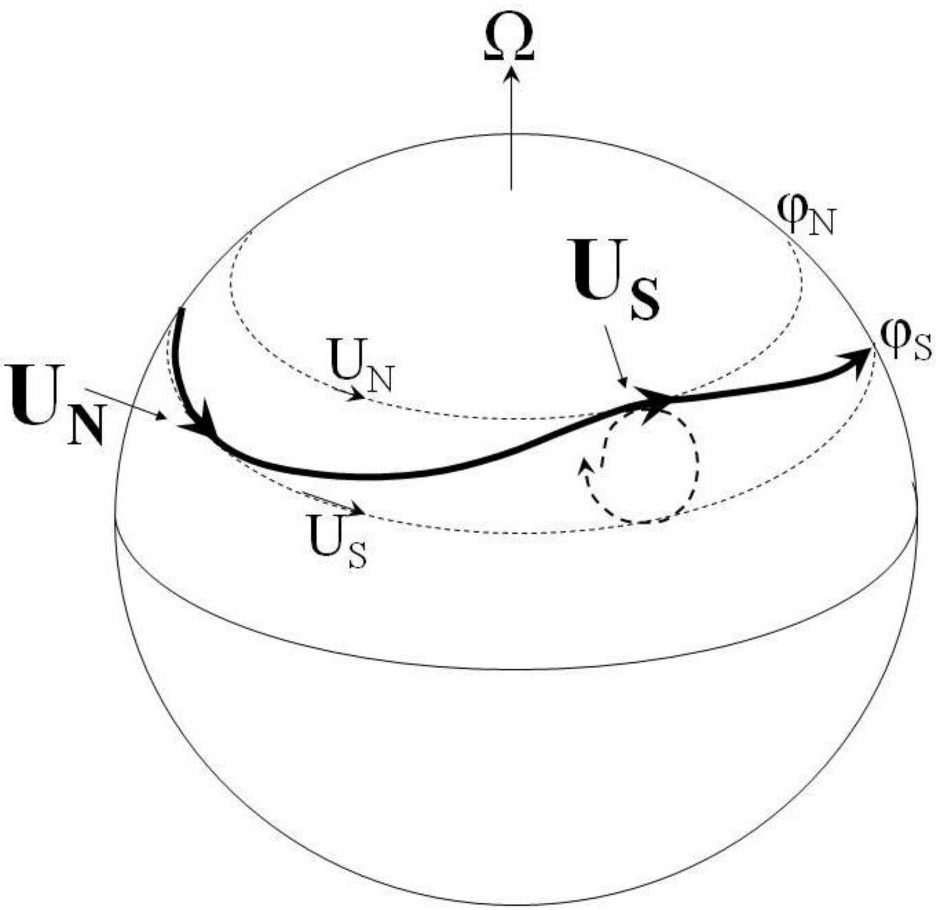
An inertia circle motion has constant relative velocity



The same motion in an absolute frame of reference



The absolute velocities $\mathbf{U_n}$ and $\mathbf{U_s}$ of an object performing an inertia oscillation (seen in an earth bound frame of reference) vary between the absolute velocities $\mathbf{U_n}$ and $\mathbf{U_s}$ of the opposite bounding latitudes



If the absolute velocities vary then also a **real force** is present, not only the **fictitious Coriolis force**

A derivation along the same "centrifugal force" lines is presented at the end of the 2015 QJRMS paper

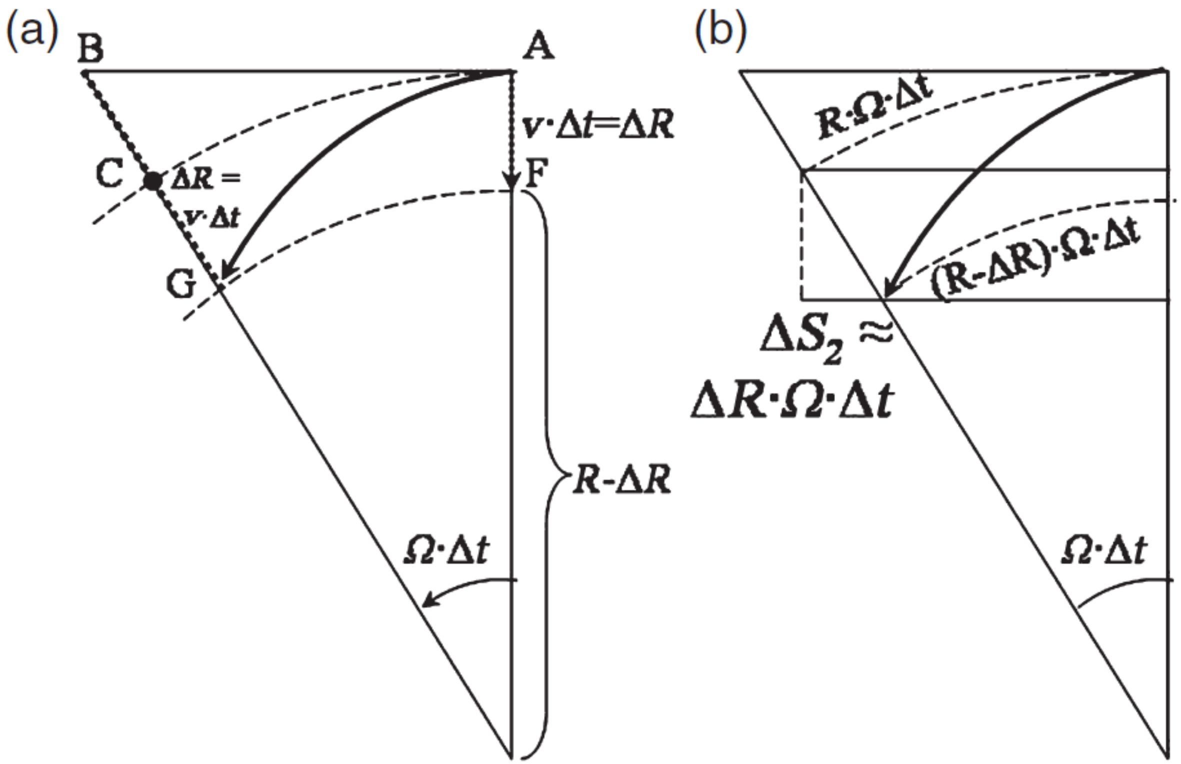


Figure A2. The derivation of the Coriolis acceleration for a radially inward moving object. (a) Highlights the outline of the motion and (b) provides additional mathematical details. See text for further details of the derivation.

END