Part II

Four types of errors:

Model errors

Systematic errors

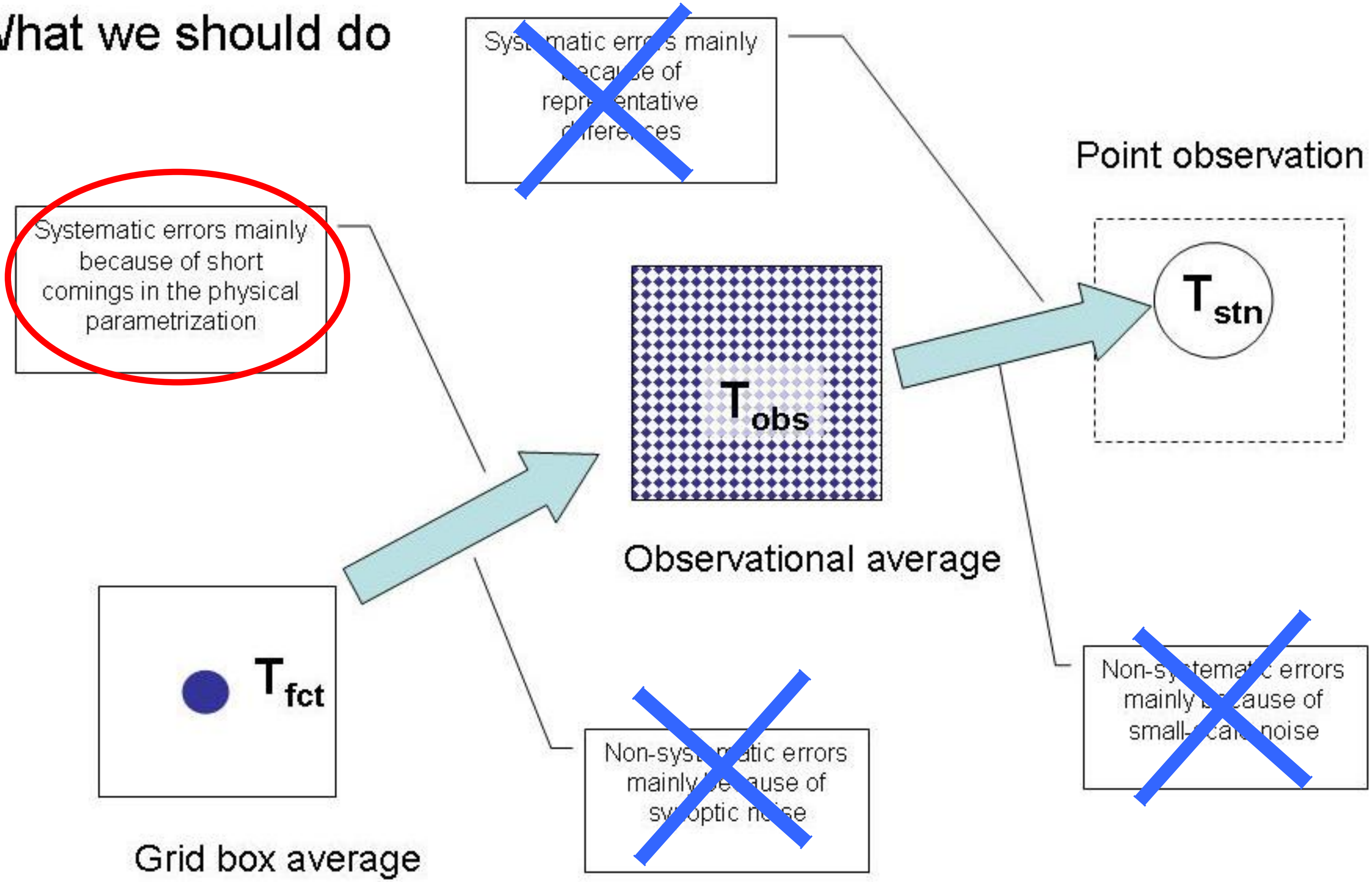
Representativeness

Synoptic errors

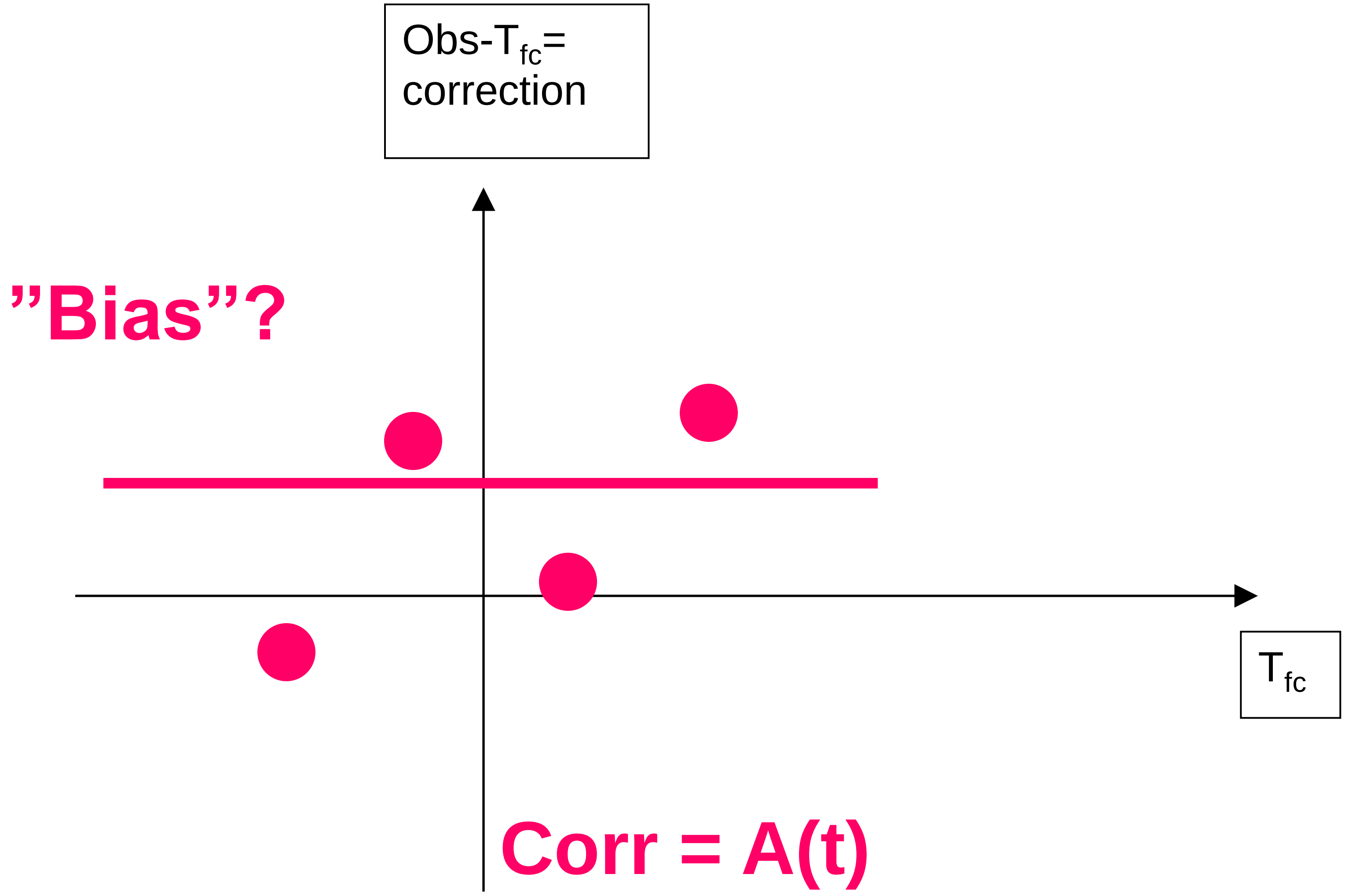
Non-systematic errors

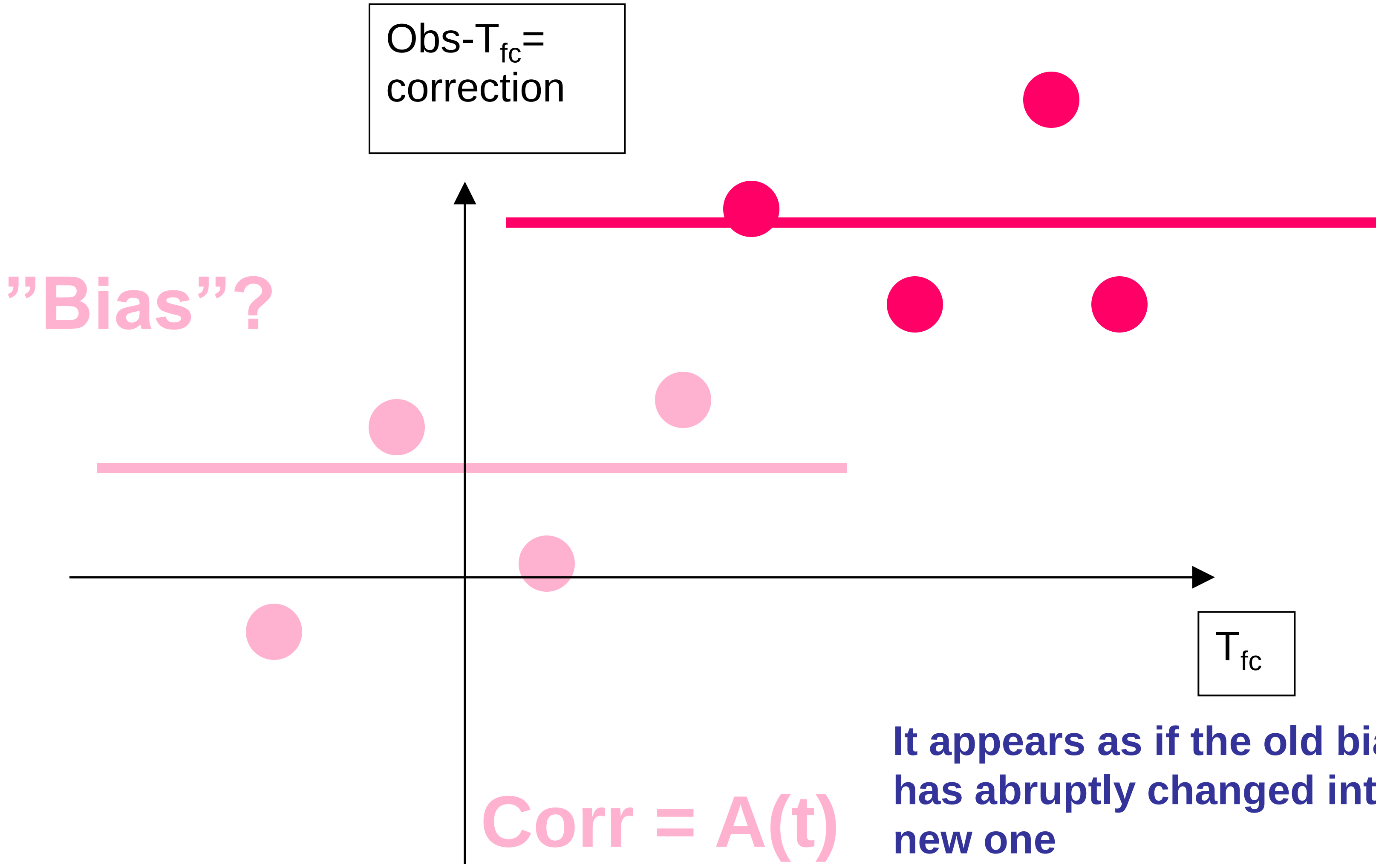
Small scale "noise"

What we should do

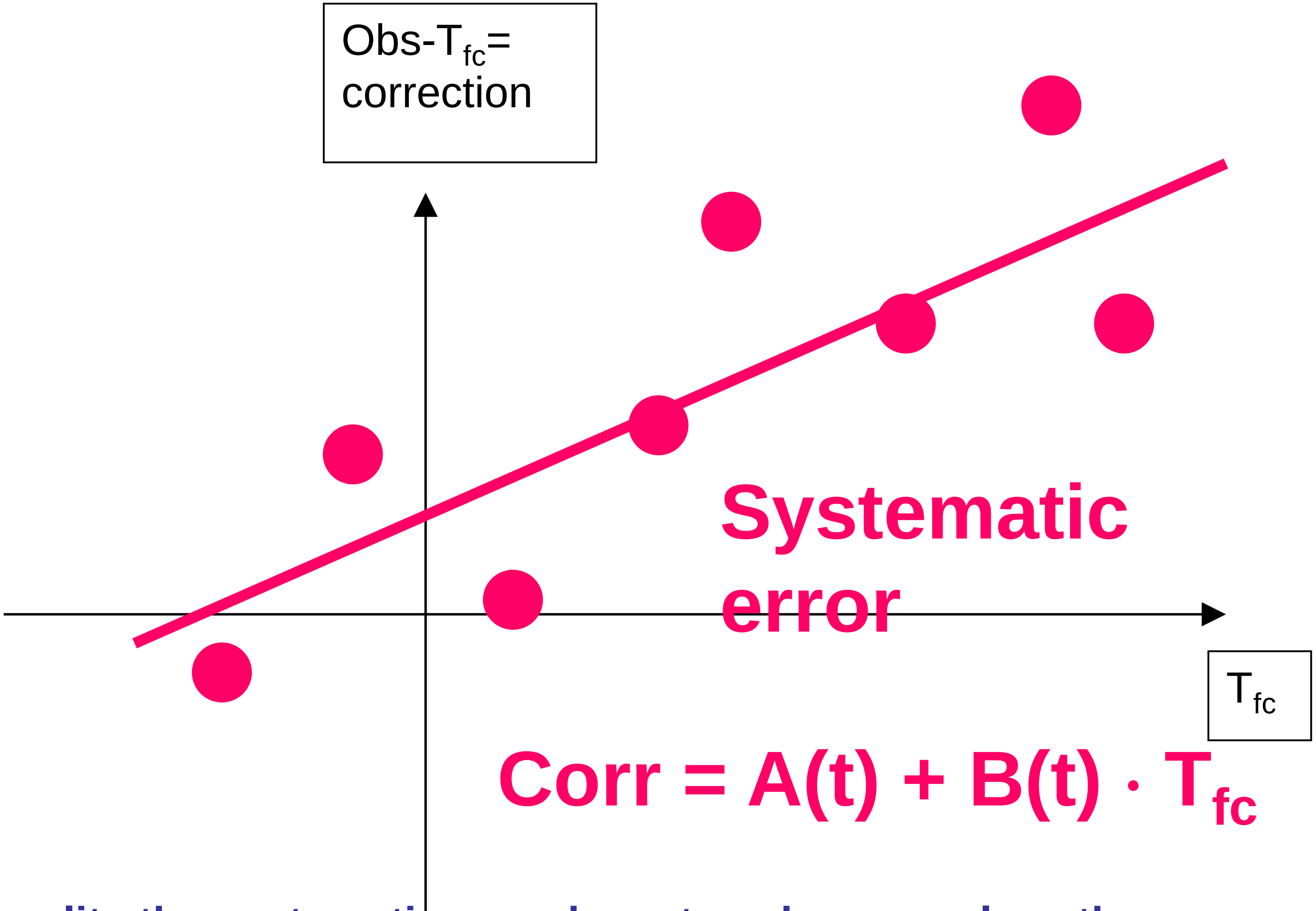


The systematic errors we want to correct for are not only 1-dimensional “flat biases” . . .


Obs-Tfc=
correction
"Bias"?
Tfc
Corr = A(t)



It appears as if the old bias has abruptly changed into a new one

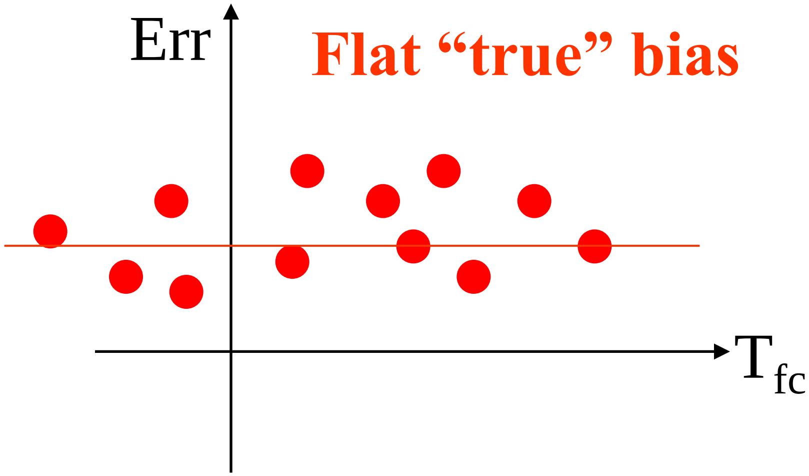


$$Corr = A(t) + B(t) \cdot T_{fc}$$

In reality the systematic error has stayed more or less the same, but defined by *<u>two</u>* coefficients, A and B

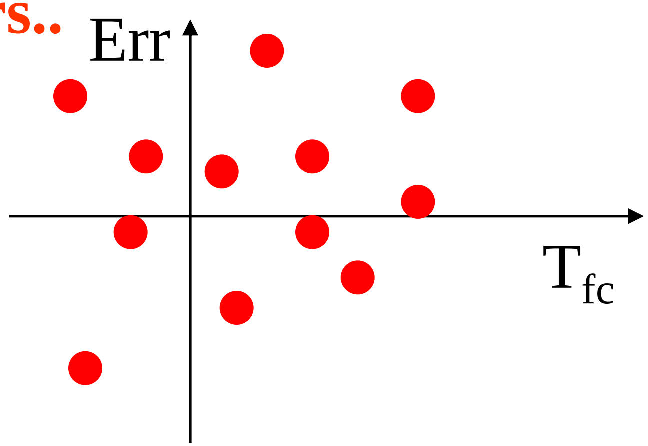
Mean errors are not the only "systematic" errors

a) A mean error which is much smaller than the variance does not point to any "systematic" error
b) In the opposite case we are dealing with a plain bias. But that is not the only type of "systematic" error
c) Errors are systematic if they can be described/explained by a (linear) equation

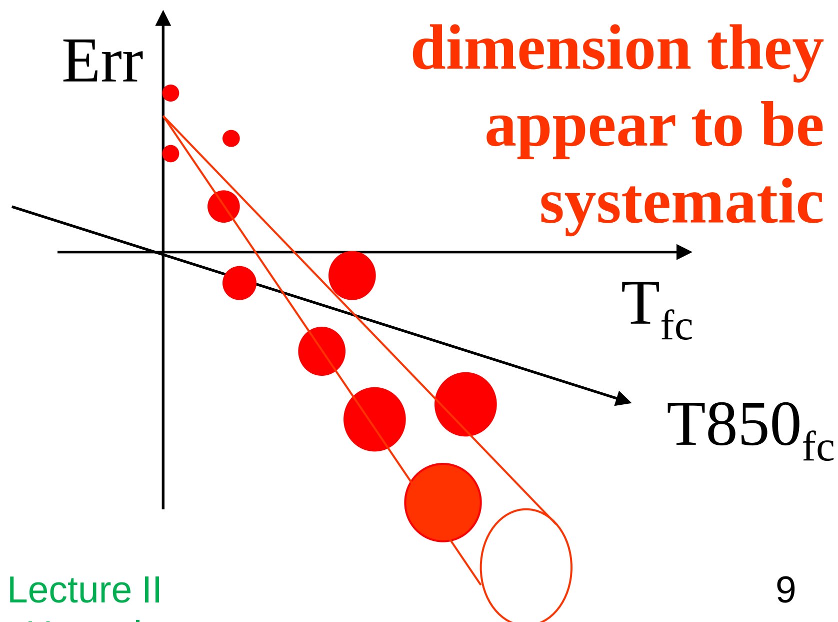


Systematic errors can be more complex than a "flat bias"

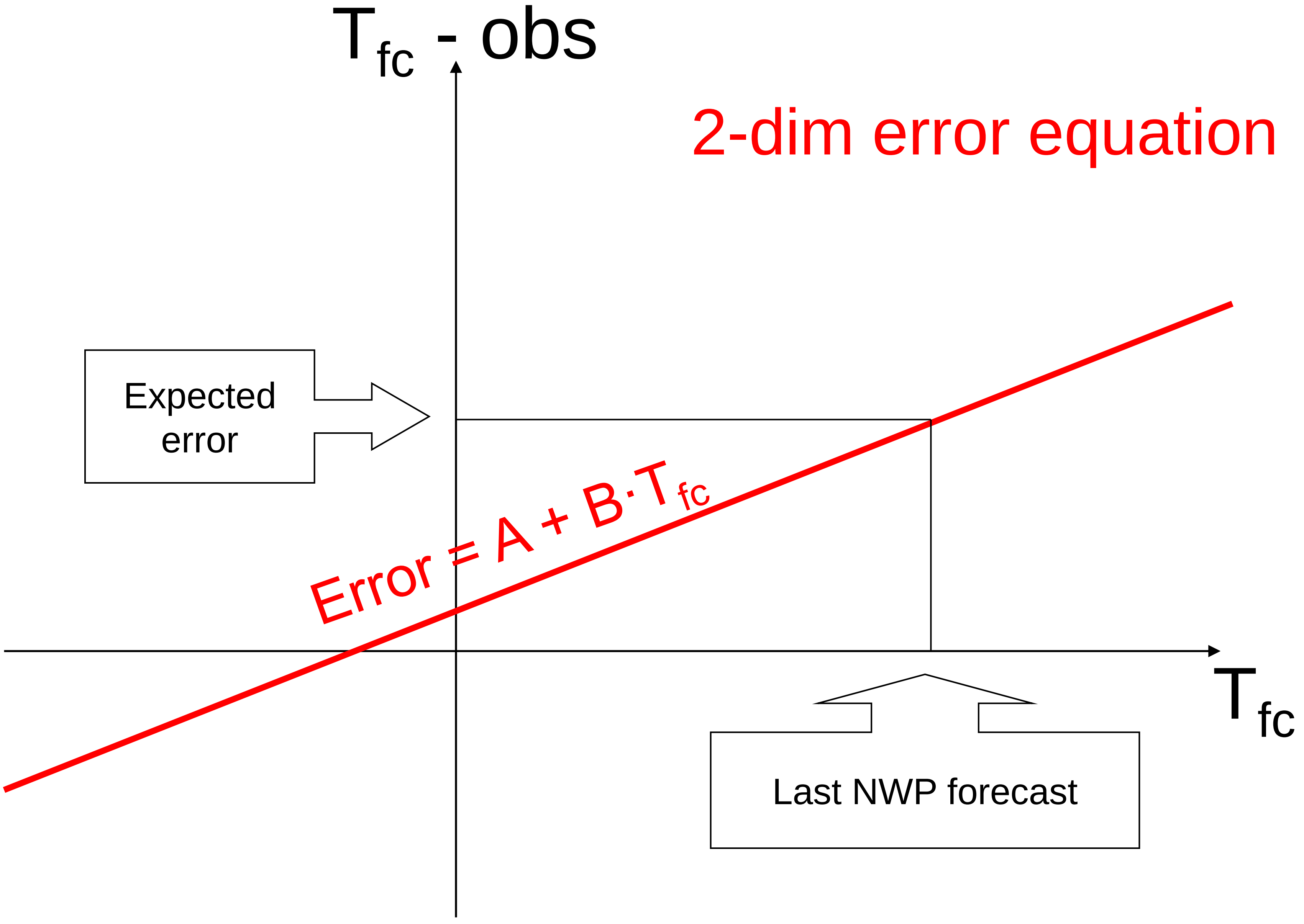
Apparent non-systematic errors..



…but when projected into an additional dimension they appear to be systematic

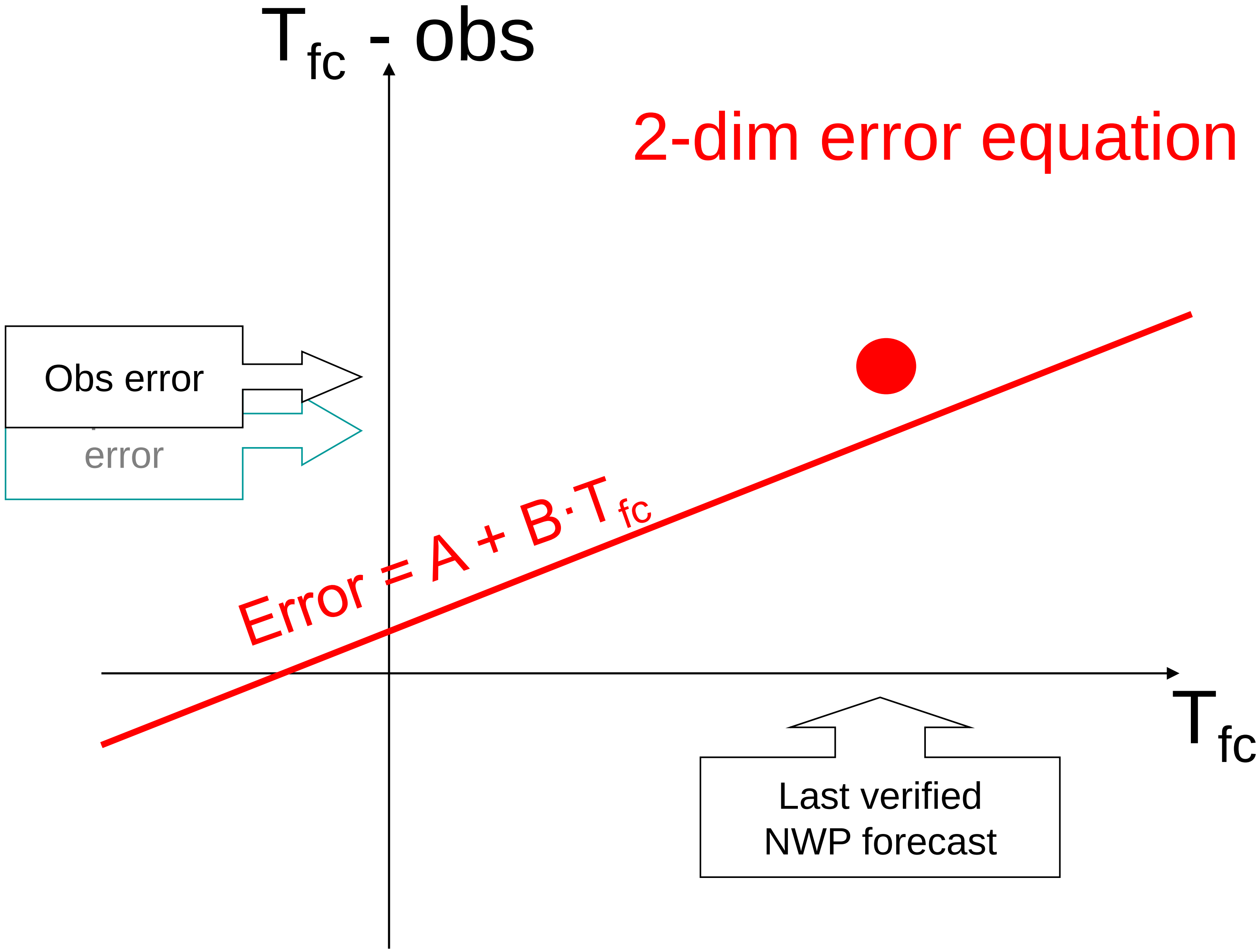


A very, very brief introduction to the adaptive Kalman filtering procedure

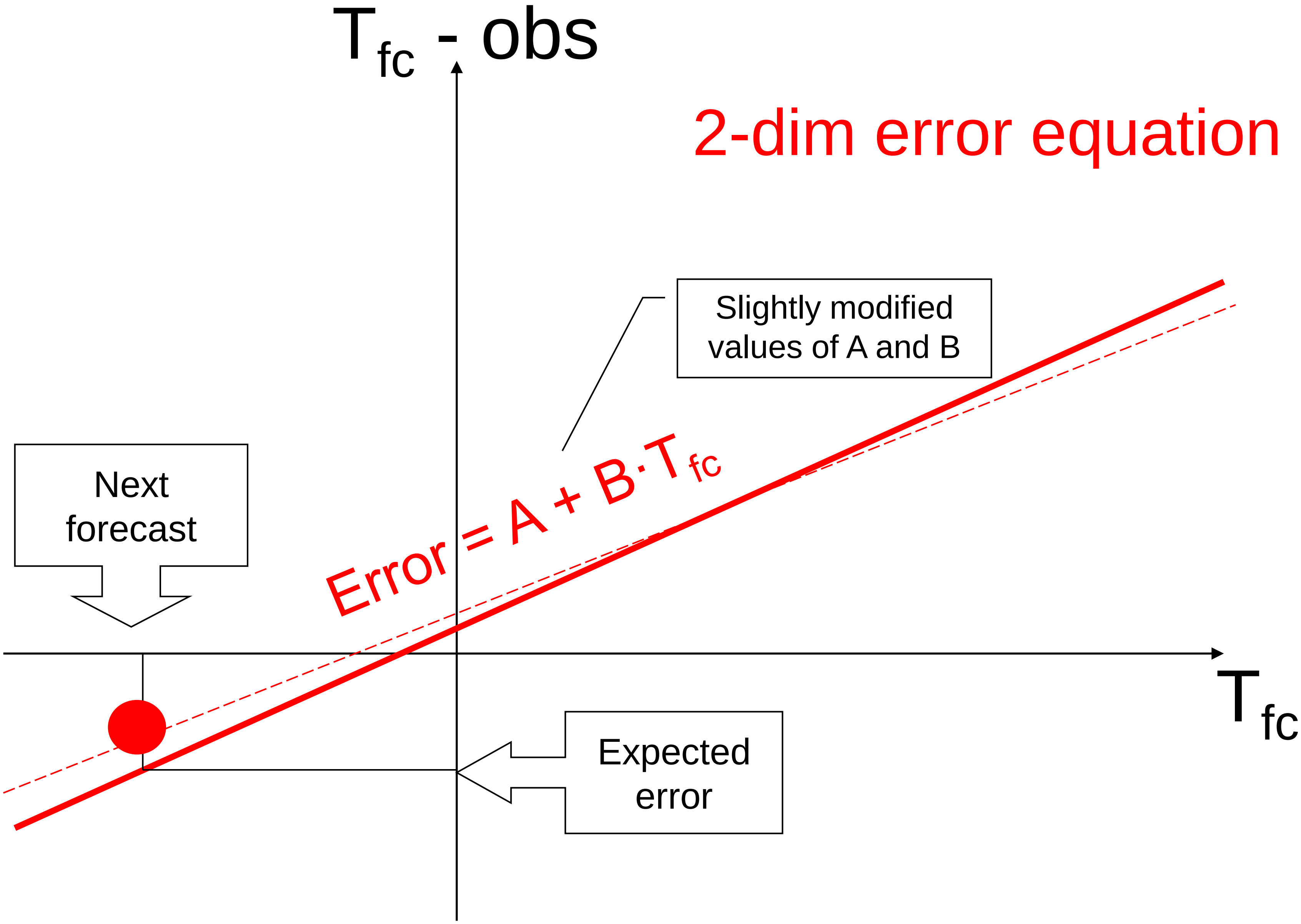


T fc - obs
2-dim error equation
Expected error
Error = A + B·T fc
T fc
Last NWP forecast

2-dim error equation

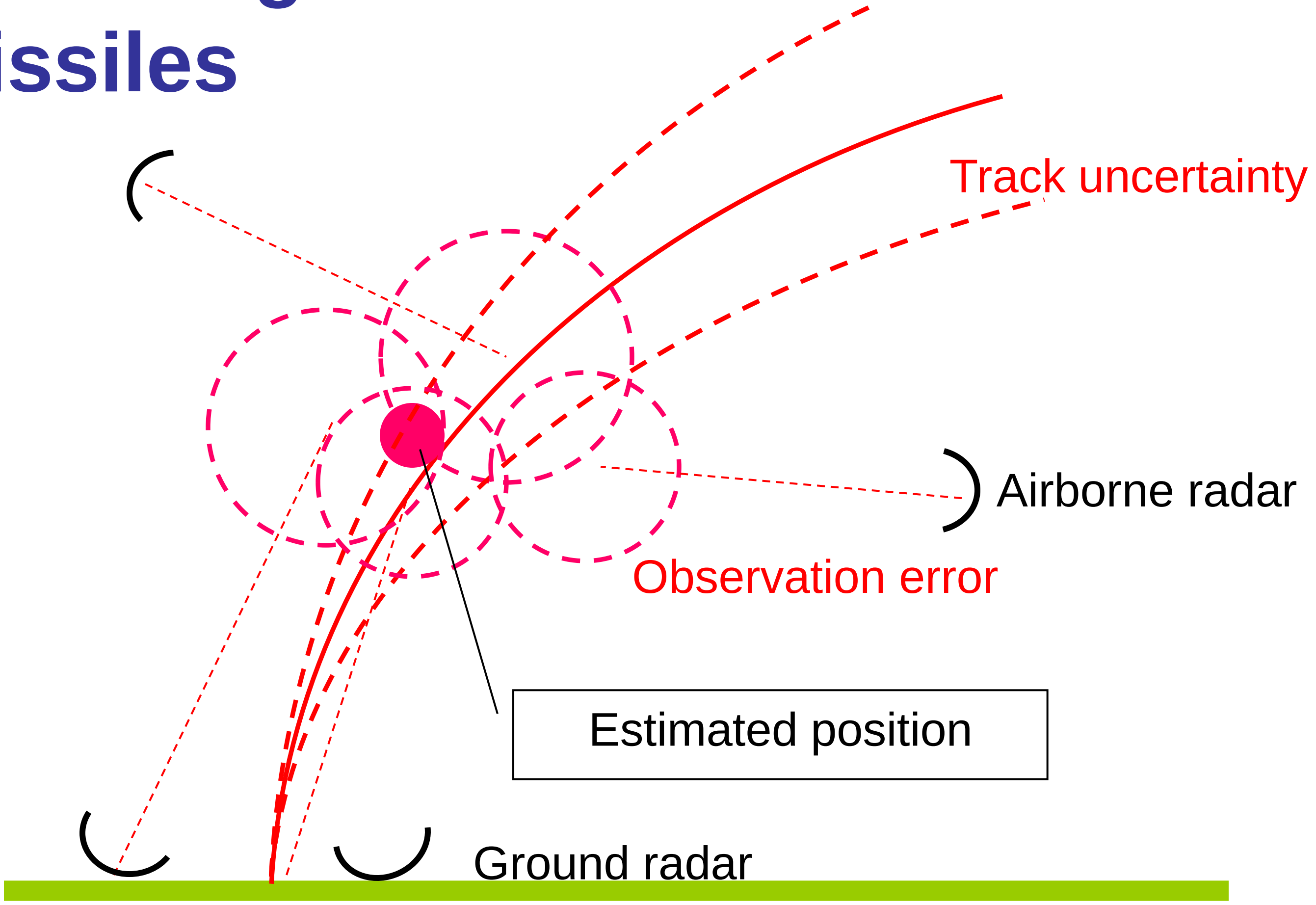


2-dim error equation

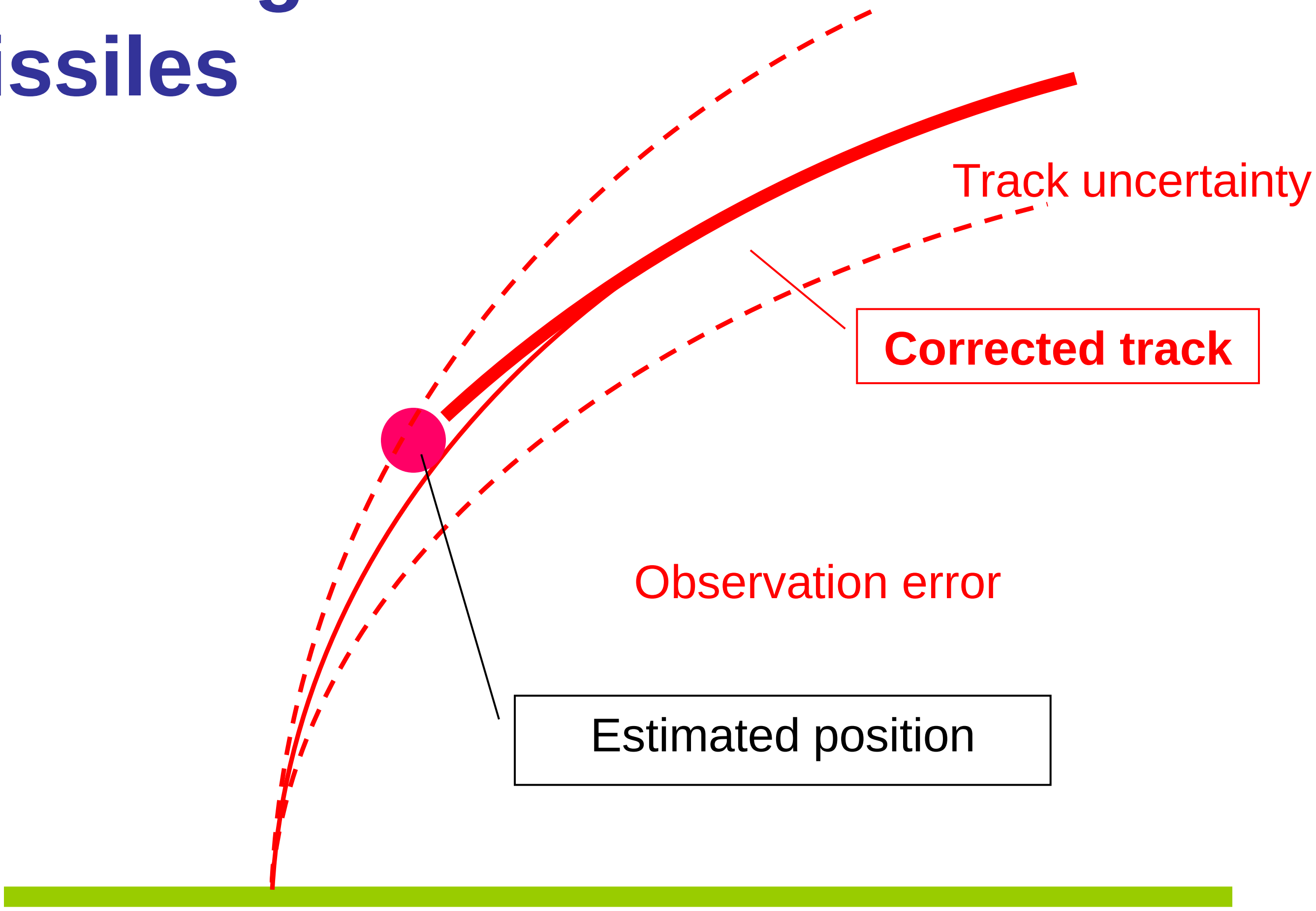


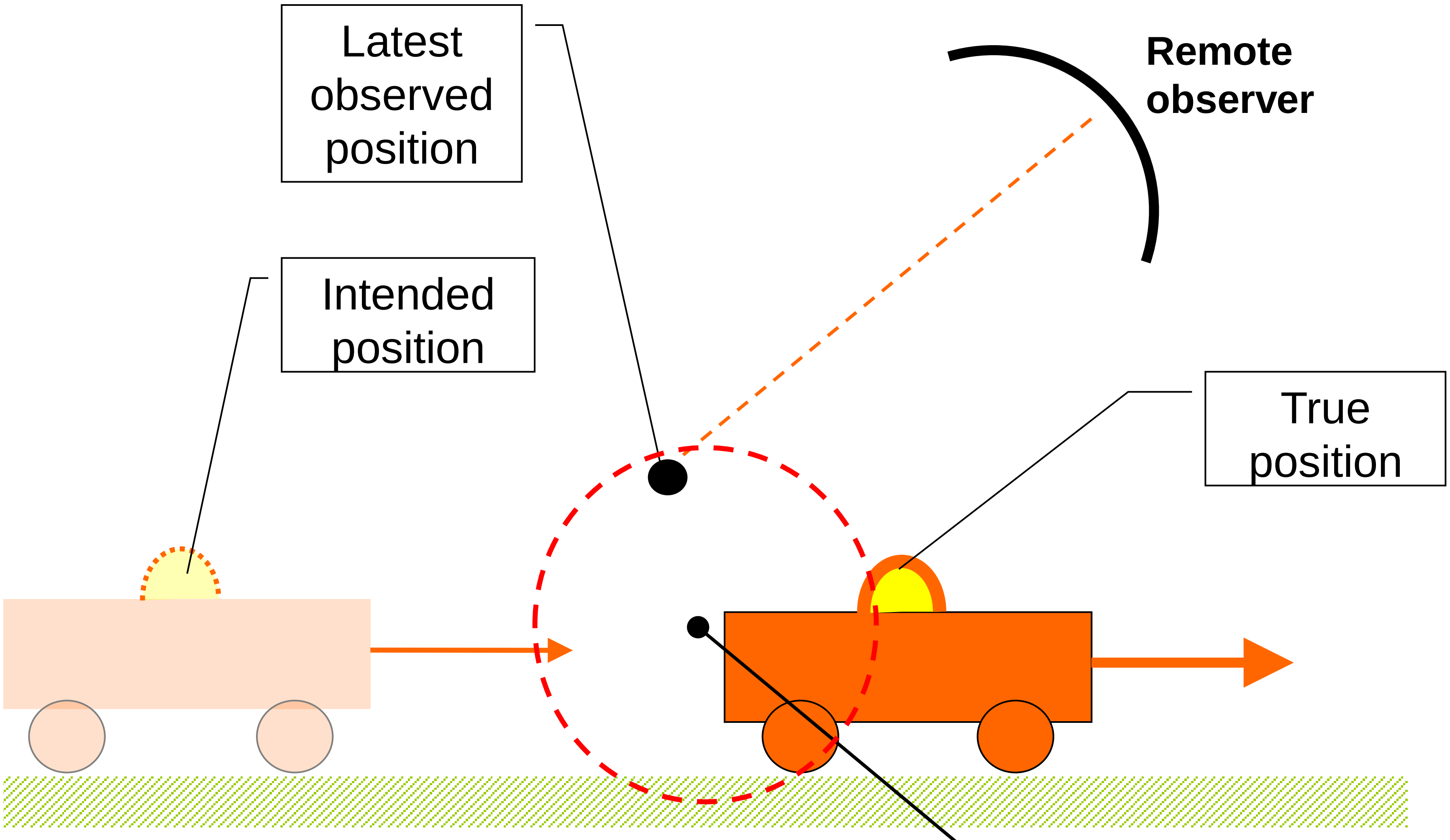
The historical background to Kalman filtering and its classical application

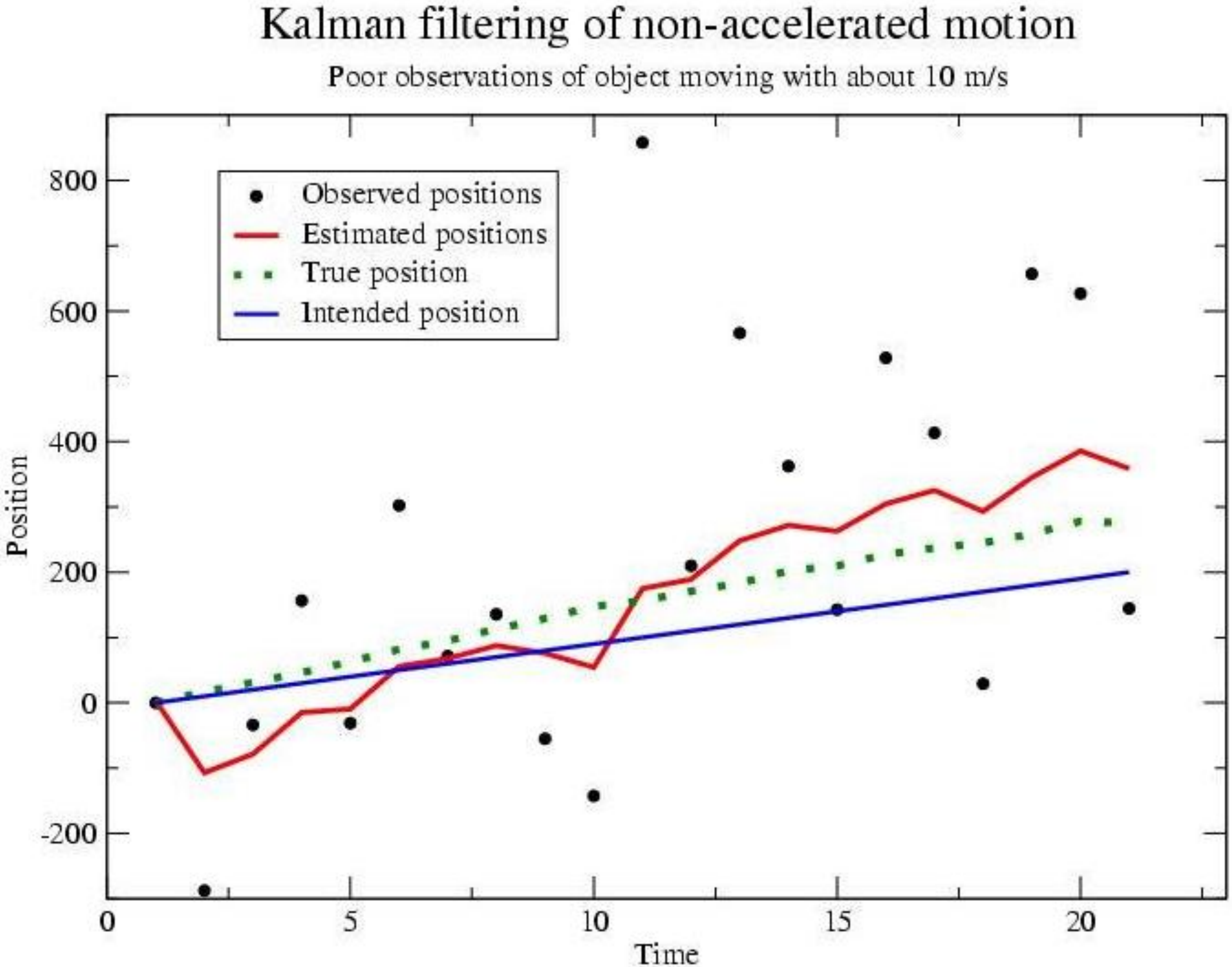
The origin of the Kalman filter 1960 – launching intercontinental ballistic missiles



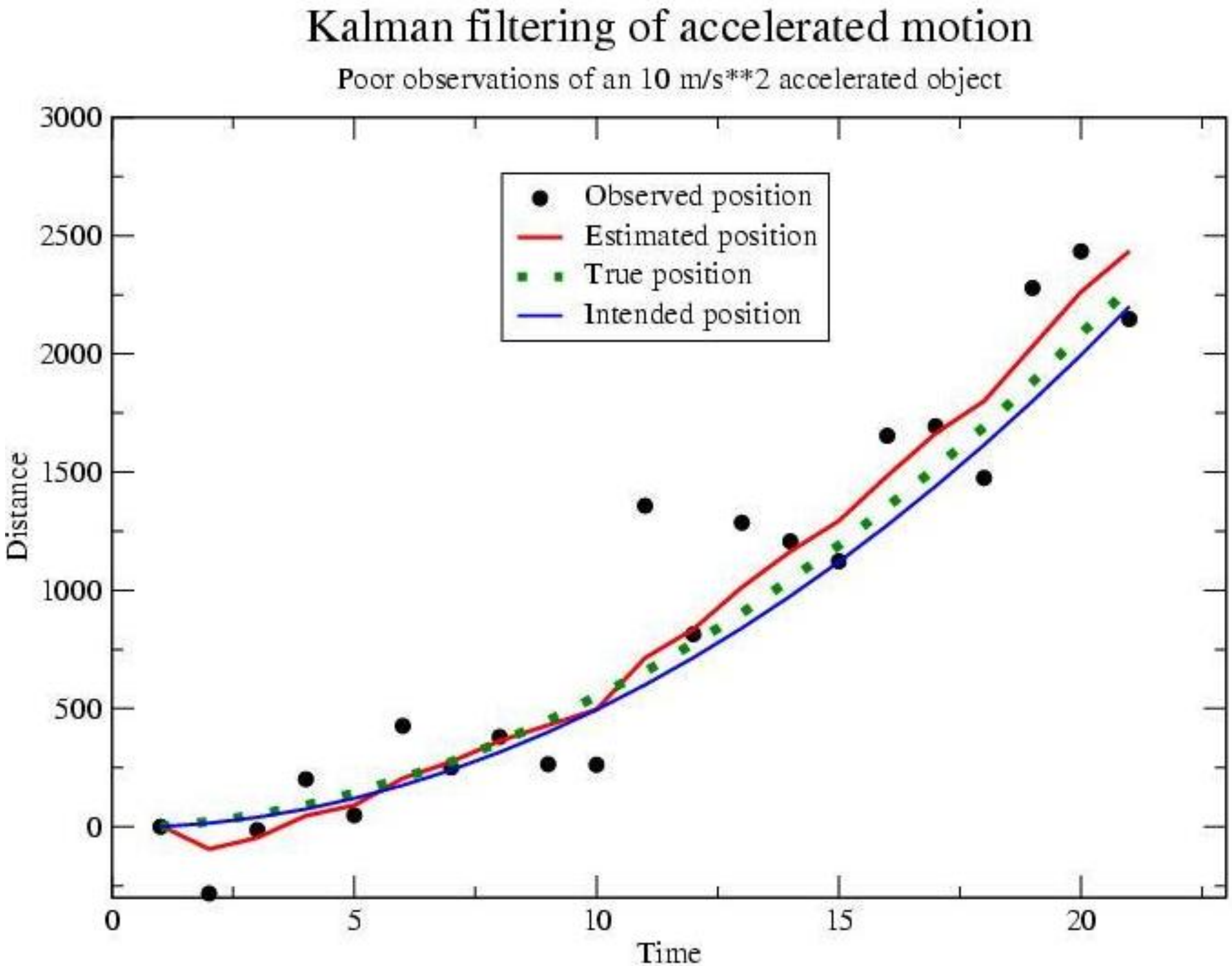
The origin of the Kalman filter 1960 – launching intercontinental ballistic missiles




Latest observed position
Remote observer
Intended position
True position
Estimated position



Kalman filtering of non-accelerated motion
Poor observations of object moving with about 10 m/s
Observed positions
Estimated positions
True position
Intended position
800
600
400
200
0
-200
Position
0
5
10
15
20
Time

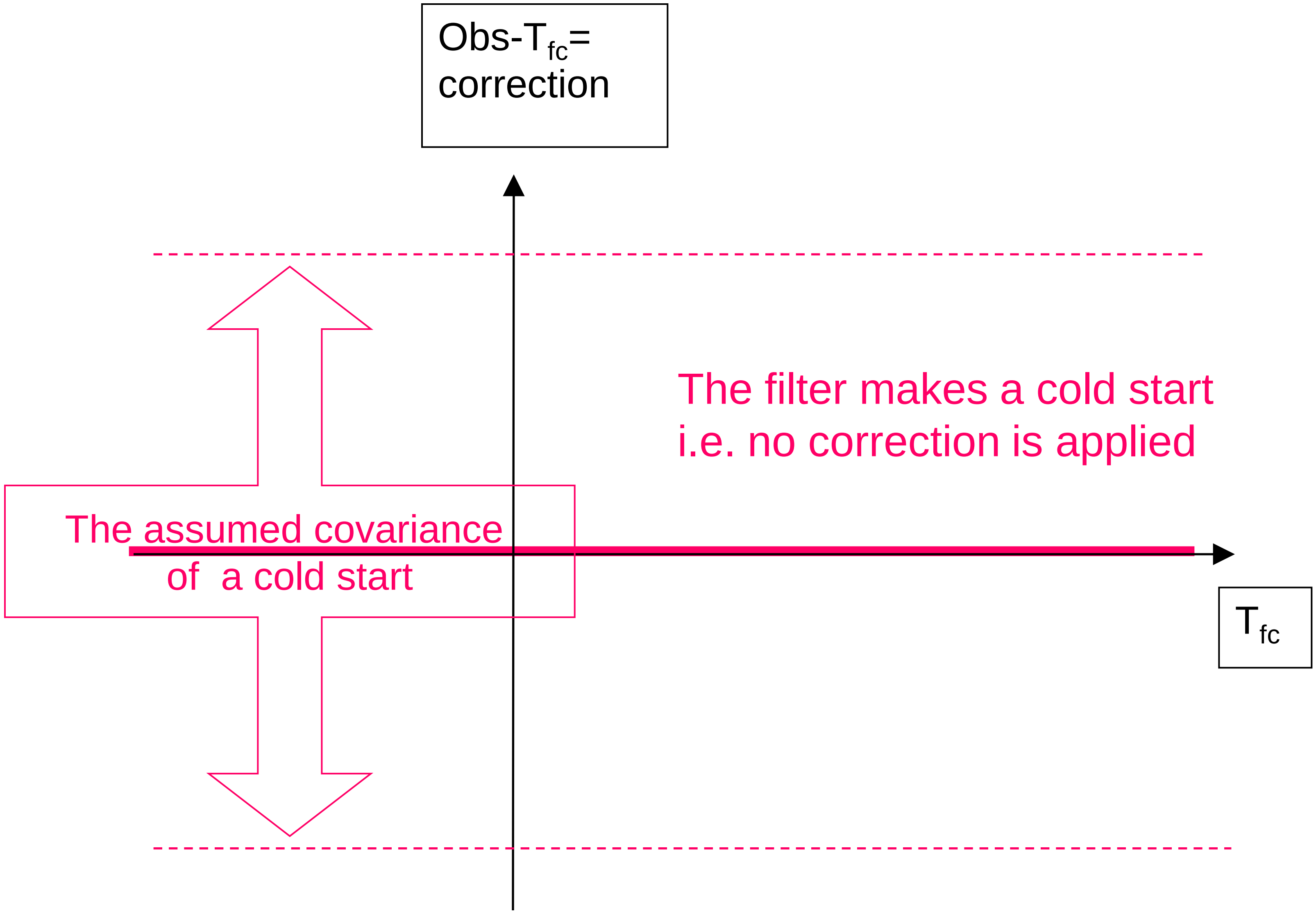

Kalman filtering of accelerated motion
Poor observations of an 10 m/s**2 accelerated object
Observed position
Estimated position
True position
Intended position
Distance
Time
3000
2500
2000
1500
1000
500
0
0
5
10
15
20

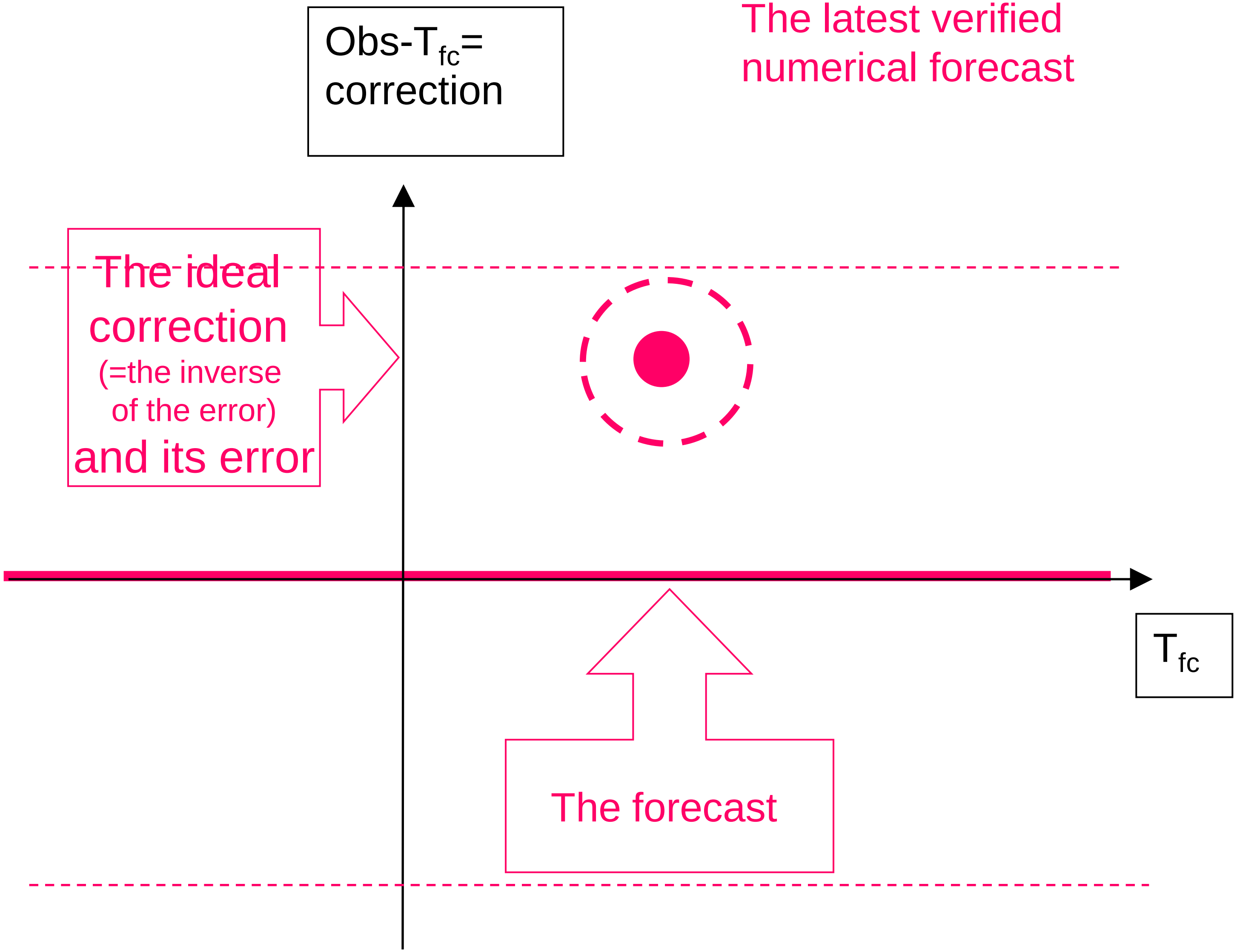
1-D Kalman filtering

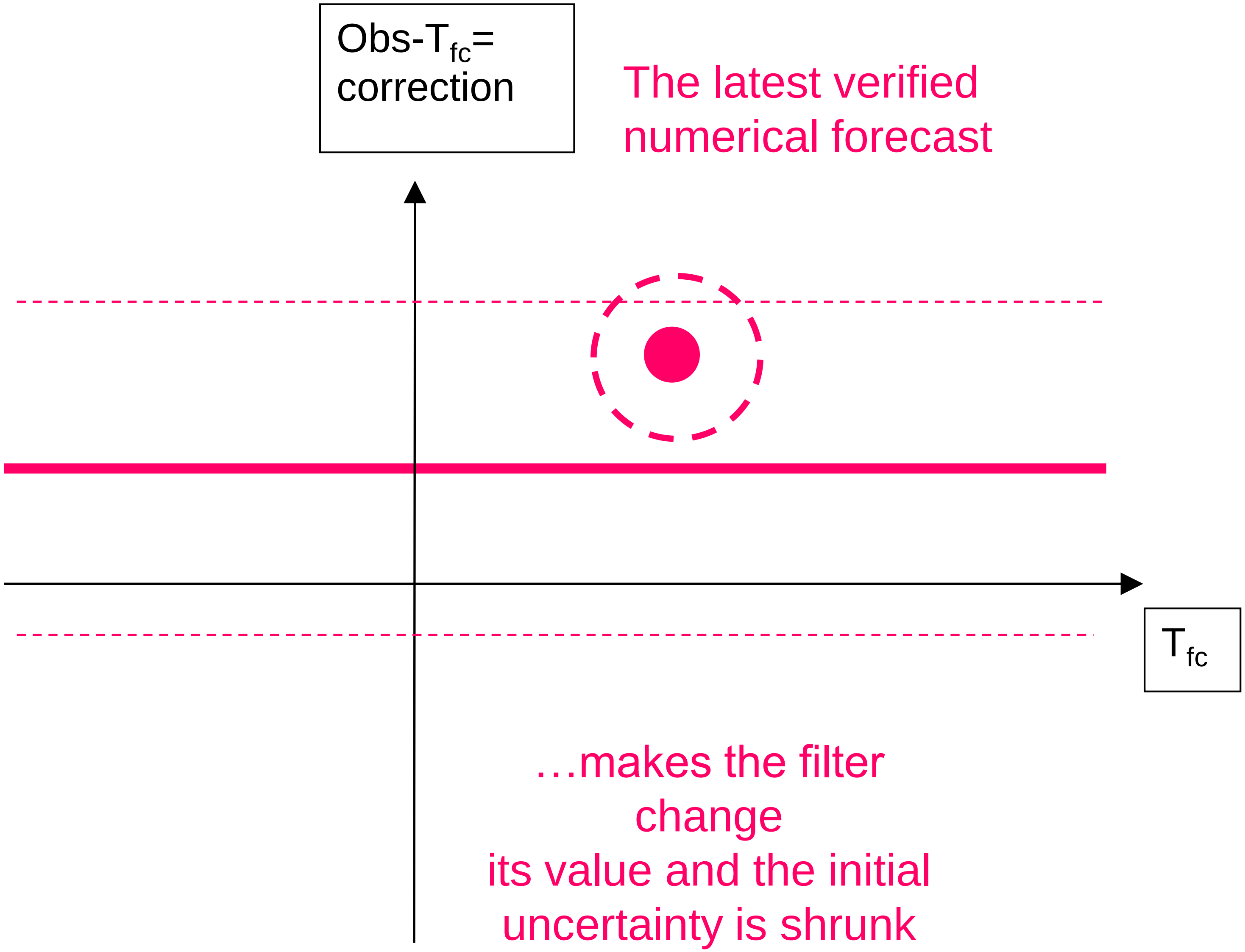
corrects for mean errors (”biases”) but can also illustrate the basic philosophy, here in three ways

1. Pictorial description
2. Mathematical derivation
3. Graphical illustration

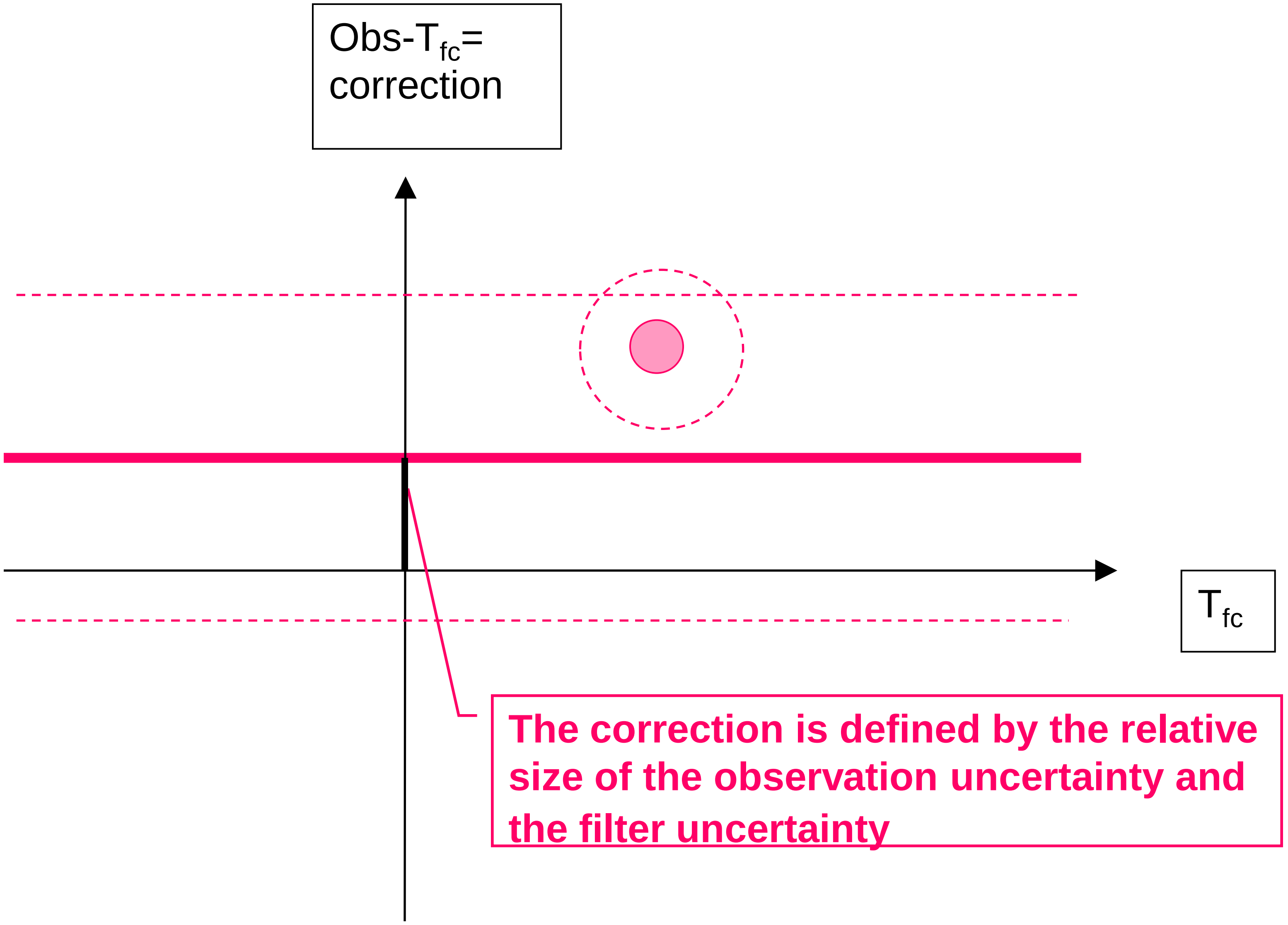
How is it done? The pictorial version

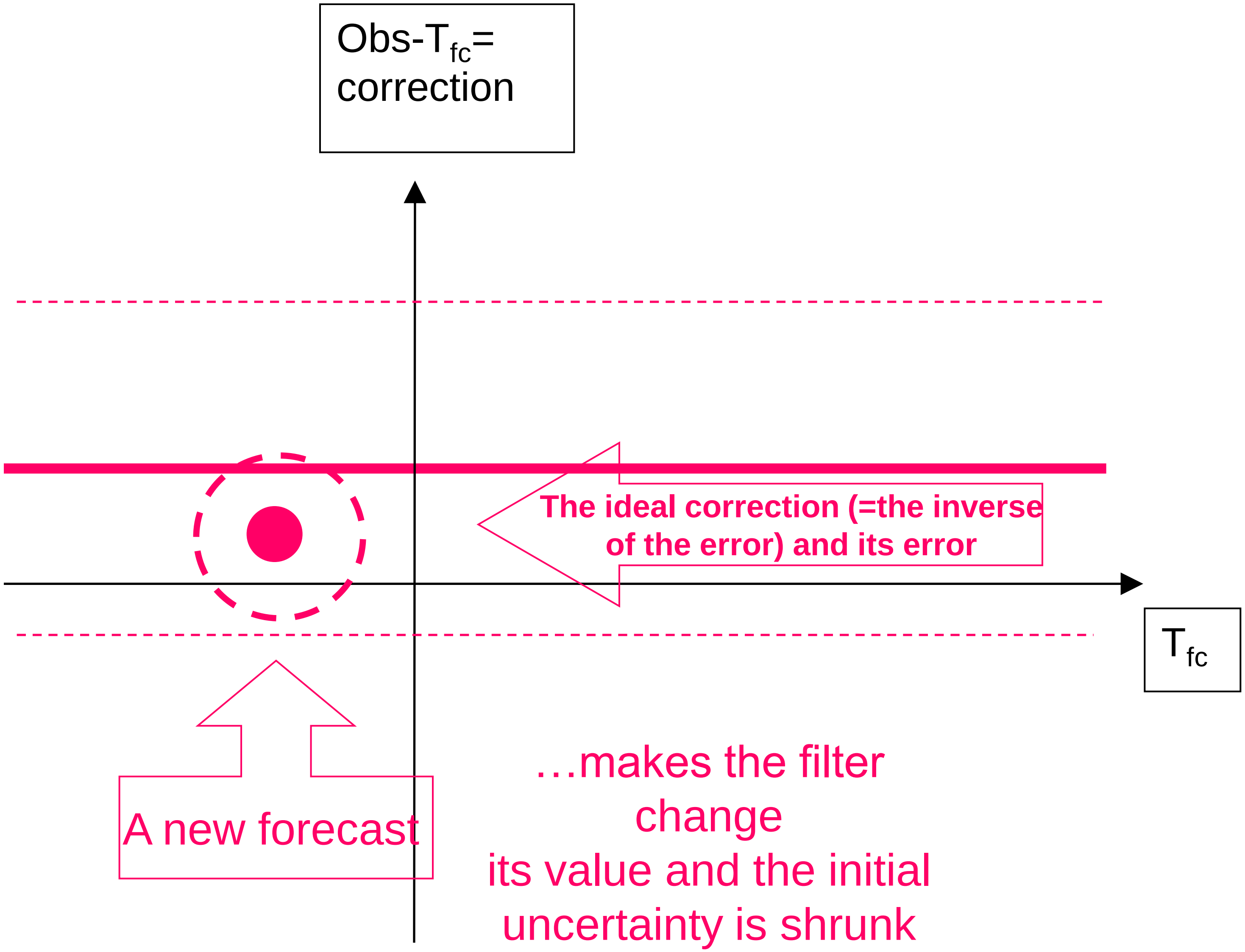

Obs-T$_{fc}$= correction
The filter makes a cold start
i.e. no correction is applied
The assumed covariance
of a cold start
T$_{fc}$


Obs-T fc=
correction
The latest verified
numerical forecast
The ideal
correction
(=the inverse
of the error)
and its error
T fc
The forecast

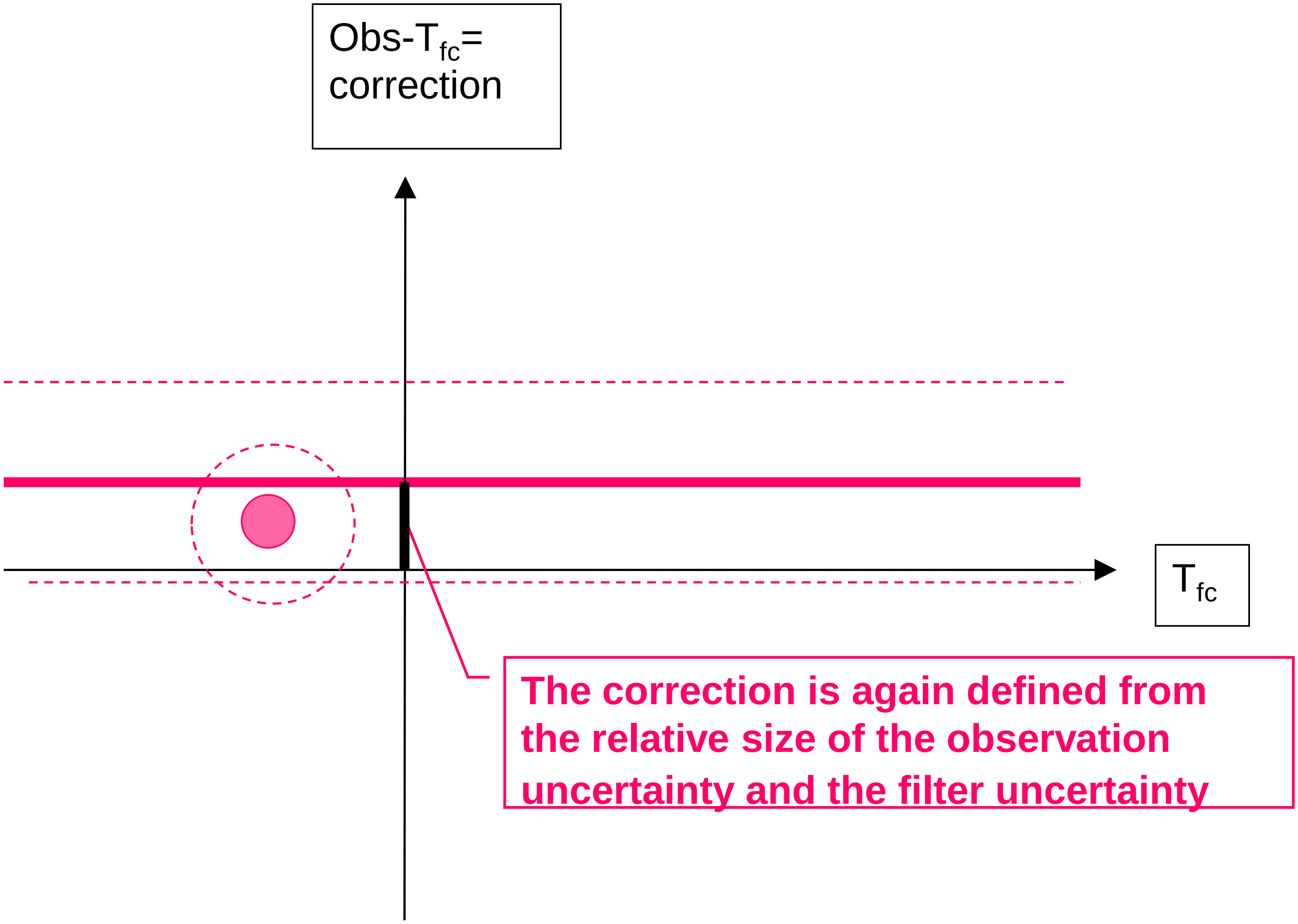


Obs-T fc= correction
The latest verified numerical forecast
T fc
…makes the filter change its value and the initial uncertainty is shrunk

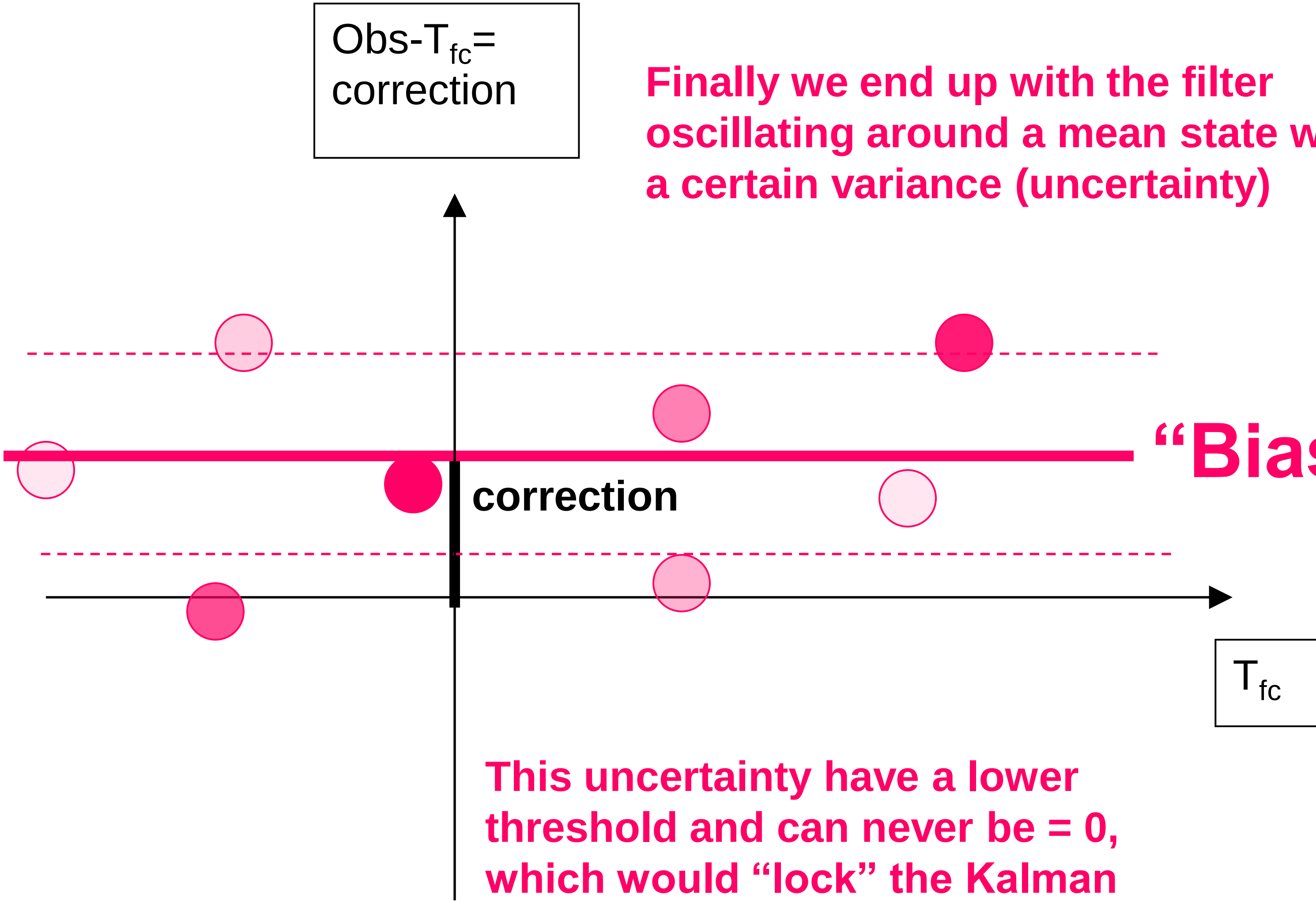

Obs-T fc=
correction
T fc
The correction is defined by the relative size of the observation uncertainty and the filter uncertainty



…makes the filter change
its value and the initial uncertainty is shrunk


Obs-T fc=
correction
T fc
The correction is again defined from the relative size of the observation uncertainty and the filter uncertainty

Finally we end up with the filter oscillating around a mean state with a certain variance (uncertainty)



This uncertainty have a lower threshold and can never be = 0, which would "lock" the Kalman filter

How is it done? The mathematical derivation

Y_τ = the observed forecast error at verification time τ

$$Y_\tau = Tfc_\tau - Tobs_\tau$$

Y_τ is the sum of the ideal correction χ_τ and the noise ν_τ

$$Y_\tau = \chi_\tau + \nu_\tau$$

1

We introduce the "first guess" values

$X_{\tau/\tau-1} = A_\tau \cdot X_{\tau/\tau-1}$

where $A_\tau = 1-F_1$, where $F_1 << 1$ and

$Q_{\tau/\tau-1}$ to be discussed later

The difference between the "first guess" value $X_{\tau/\tau-1}$ and the observed value Y_τ must obviously affect how much we shall modify $X_{\tau/\tau-1}$

2

We now introduce δ_τ ($0 < \delta_\tau < 1$) which indicates how much of the difference between Y_τ and $X_{\tau/\tau-1}$ that shall modify $X_{\tau/\tau-1}$

$$X_{\tau/\tau} = X_{\tau/\tau-1} + \delta_\tau (Y_\tau - X_{\tau/\tau-1})$$

Assume that the error in our estimation of χ_τ is ε_τ

$$\varepsilon_\tau = \chi_\tau - X_{\tau/\tau}$$

which yields

$$\varepsilon_\tau = \chi_\tau - X_{\tau/\tau-1} - \delta_\tau (Y_\tau - X_{\tau/\tau-1})$$

3

And with the noise term $Y_\tau = \chi_\tau + \nu_\tau$ we get

$$\varepsilon_\tau = \chi_\tau - X_{\tau/\tau-1} - \delta_\tau (\chi_\tau + \nu_\tau - X_{\tau/\tau-1})$$

...and after rearrangement of the terms

$$\varepsilon_\tau = (1-\delta_\tau)(\chi_\tau - X_{\tau/\tau-1}) - \delta_\tau \nu_\tau$$

4

The (co) variance term which indicates the (un) certainty of our estimation

The uncertainty of

$$Y_\tau = \chi_\tau + \nu_\tau$$

depends on sub-grid turbulence, non-systematic synoptic errors or measurement errors, what we choose to call the observation error D_τ

$$\mathrm{cov}(\nu_\tau) = D_\tau$$

5

$$\mathrm{cov}(\varepsilon_\tau) = \mathrm{cov}(\chi_\tau - X_{\tau/\tau}) = Q_{\tau/\tau}$$

and $\mathrm{cov}(\chi_\tau - X_{\tau/\tau-1}) = Q_{\tau/\tau-1}$

and $\mathrm{cov}(\nu_\tau) = D_\tau$

...yields:

$$Q_{\tau/\tau} = (1-\delta_\tau)^2 Q_{\tau/\tau-1} + \delta_\tau^2 D_\tau$$

6

We differentiate

$$Q_{\tau/\tau} = (1-\delta_\tau)^2 Q_{\tau/\tau-1} + \delta_\tau^2 D_\tau$$

...with respect to δ_τ

$$\frac{dQ_{\tau/\tau}}{d\delta_\tau} = -2(1-\delta_\tau) Q_{\tau/\tau-1} + 2\delta_\tau D_\tau$$

7

from

$$\frac{dQ_{\tau/\tau}}{d\delta_\tau} = -2(1-\delta_\tau) Q_{\tau/\tau-1} + 2 D_\tau \delta_\tau$$

we get

$$\delta_{\tau_{\min}} D_\tau - (1-\delta_\tau) Q_{\tau/\tau-1} = 0$$

$$\delta_{\tau_{\min}} = \frac{Q_{\tau/\tau-1}}{D_\tau + Q_{\tau/\tau-1}}$$

Which is the final result

8

with

$$\delta_{\tau_{\min}} = \frac{Q_{\tau/\tau-1}}{D_\tau + Q_{\tau/\tau-1}}$$

and

$$Q_{\tau/\tau} = Q_{\tau/\tau-1}(1-\delta_\tau)^2 + D_\tau \delta_\tau^2$$

the updated forward (co)variances become

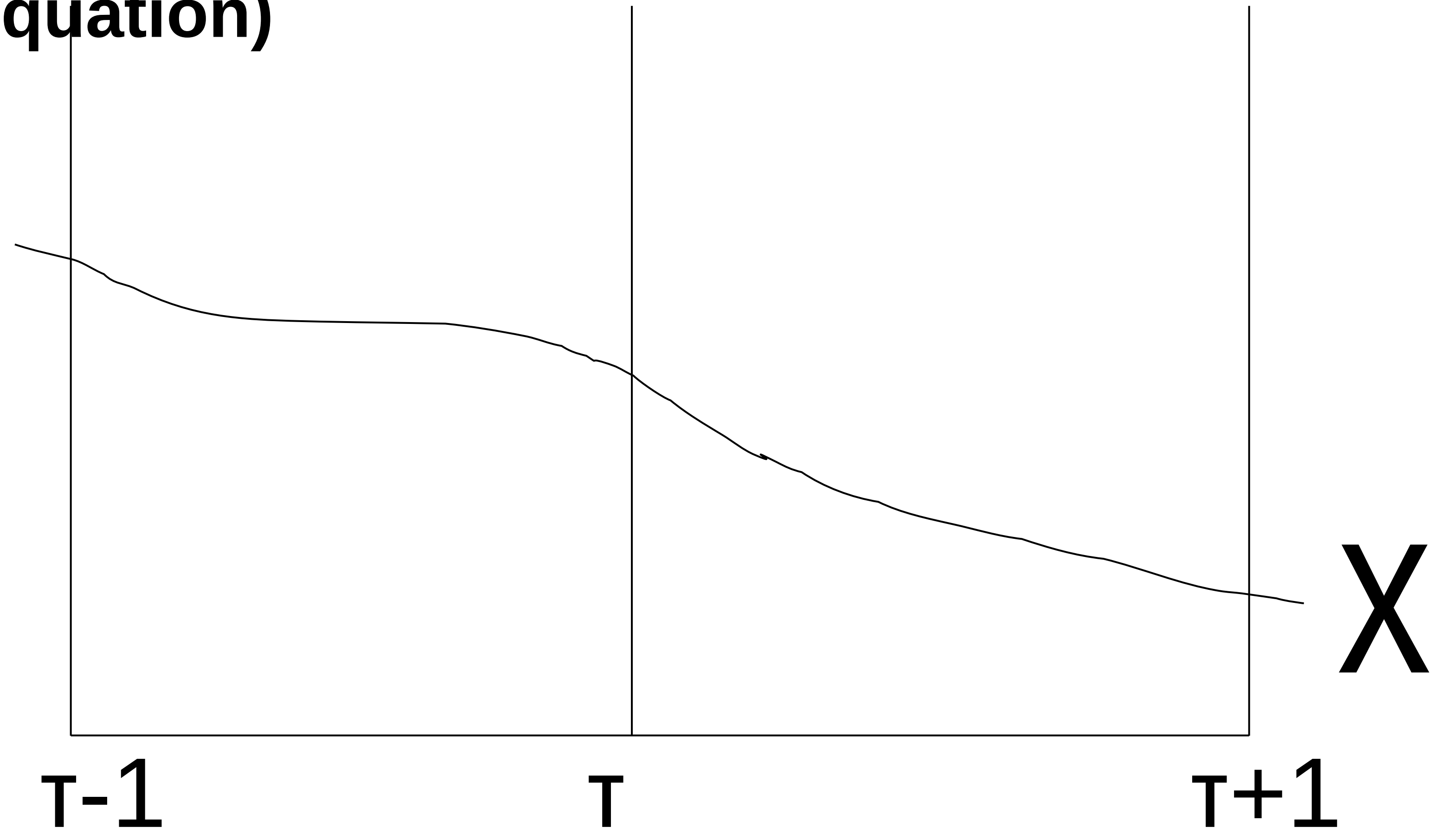
$$Q_{\tau+1/\tau} = Q_{\tau/\tau}(1-\delta_\tau^2) + D_\tau \delta_\tau^2$$

9

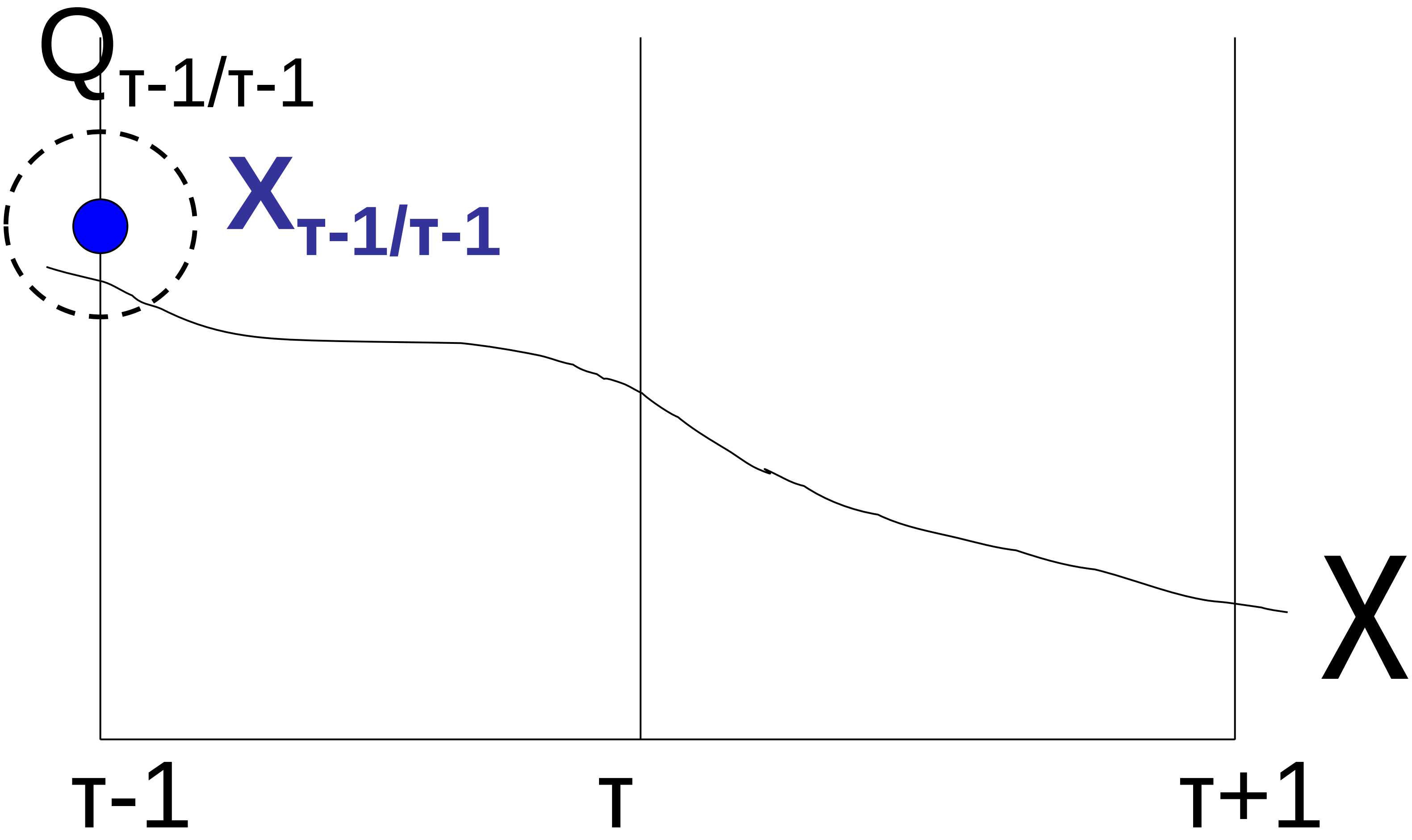
How is it done?

The graphical illustration

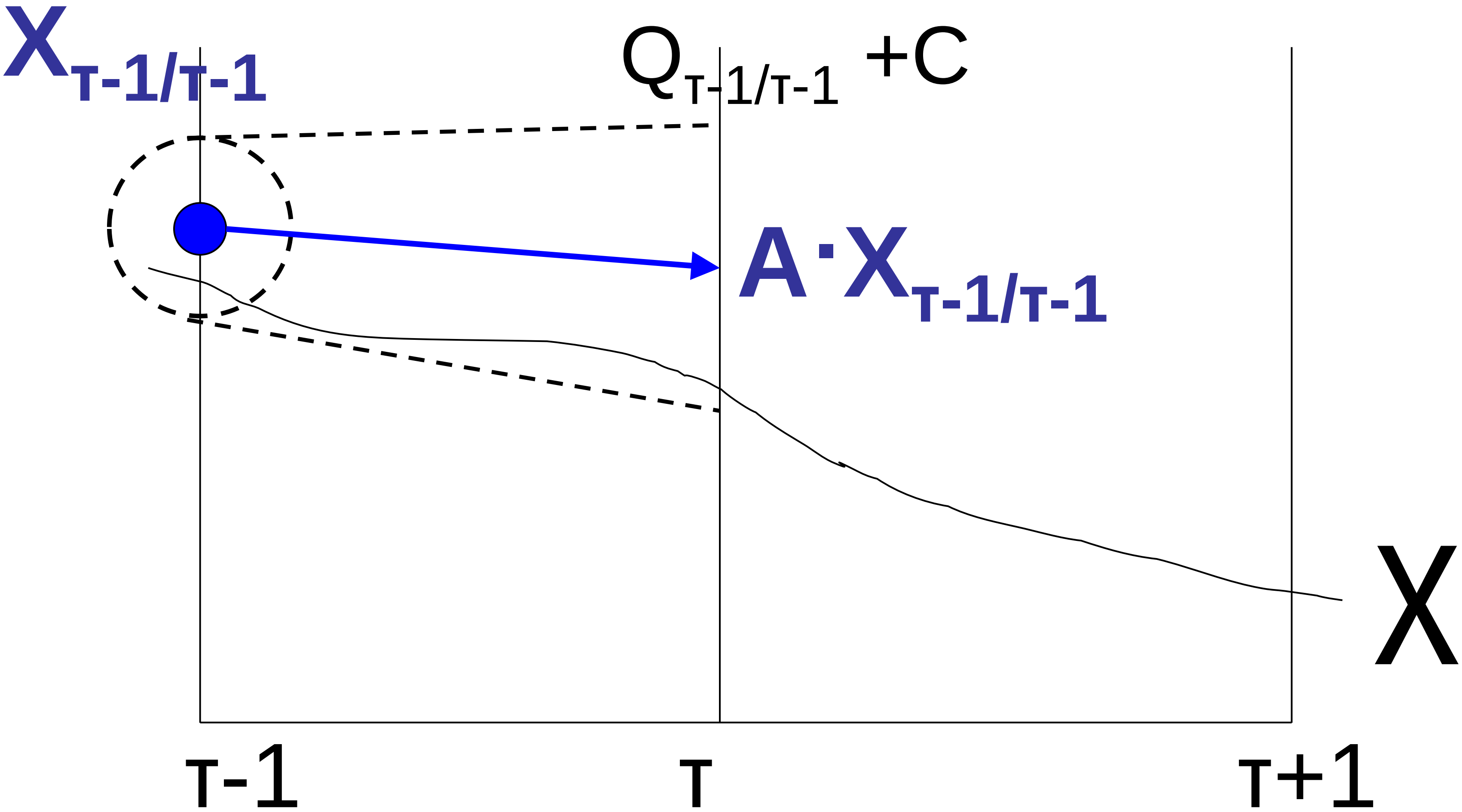
Assume an unknown process χ which can be 1-dim (the mean error or "bias" of NWP) or N-dim (the N coefficients in an error correction equation)



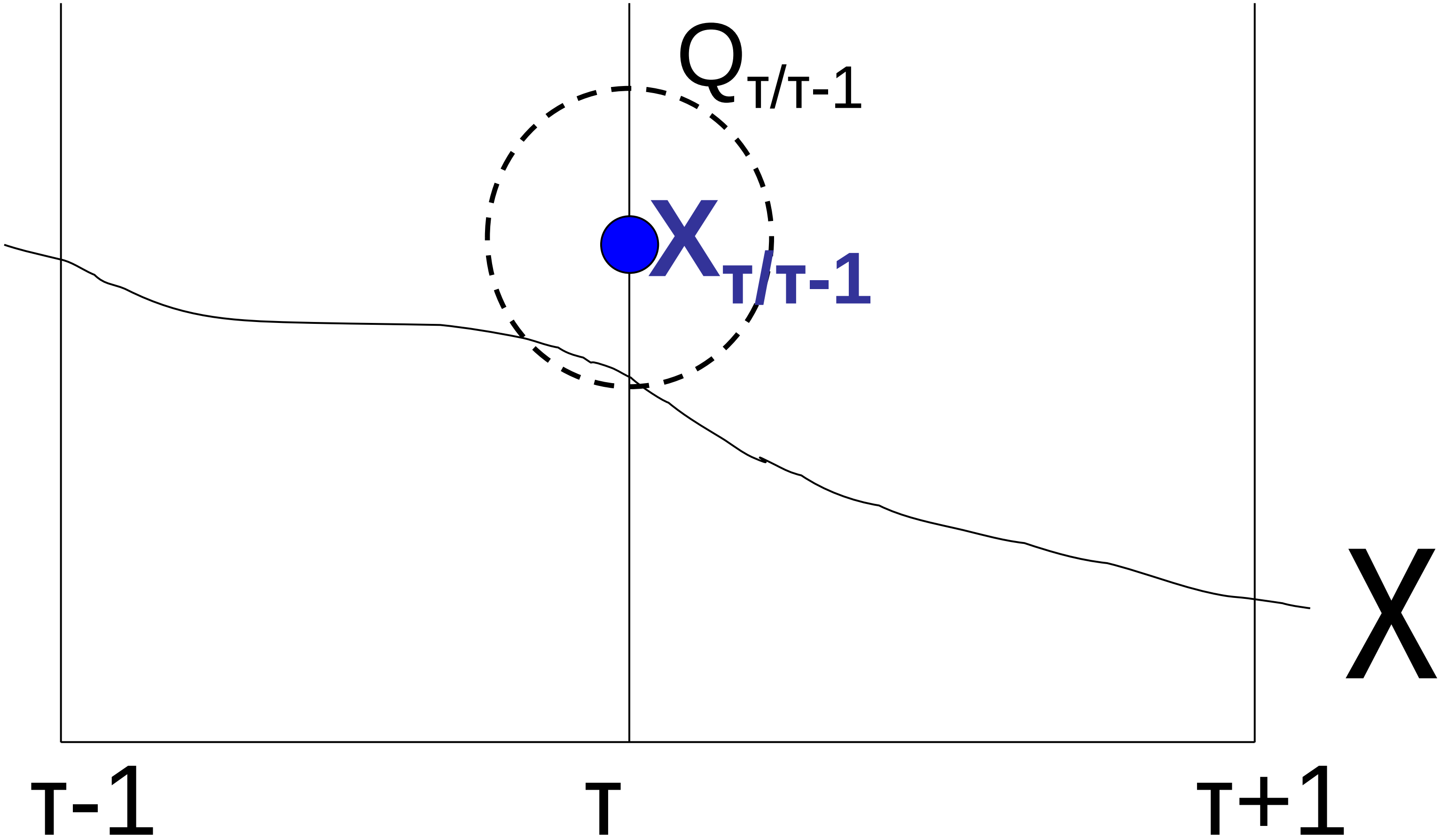
We have at τ-1 an estimated value $X_{\tau-1/\tau-1}$ of the unknown process χ with variance $Q_{\tau-1/\tau-1}$



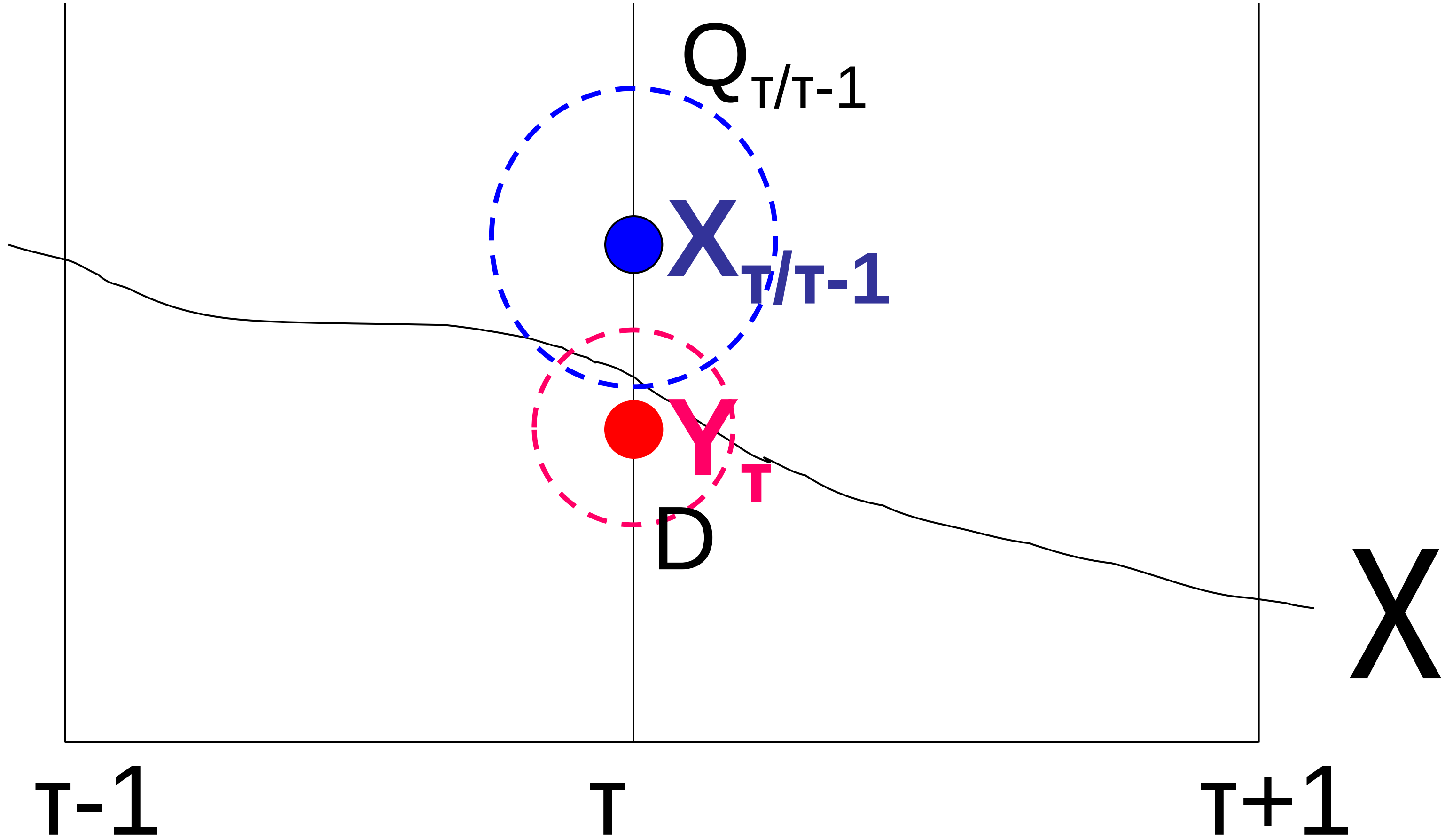
We carry X forward in time by a linear model A, assuming that the variance increases slightly



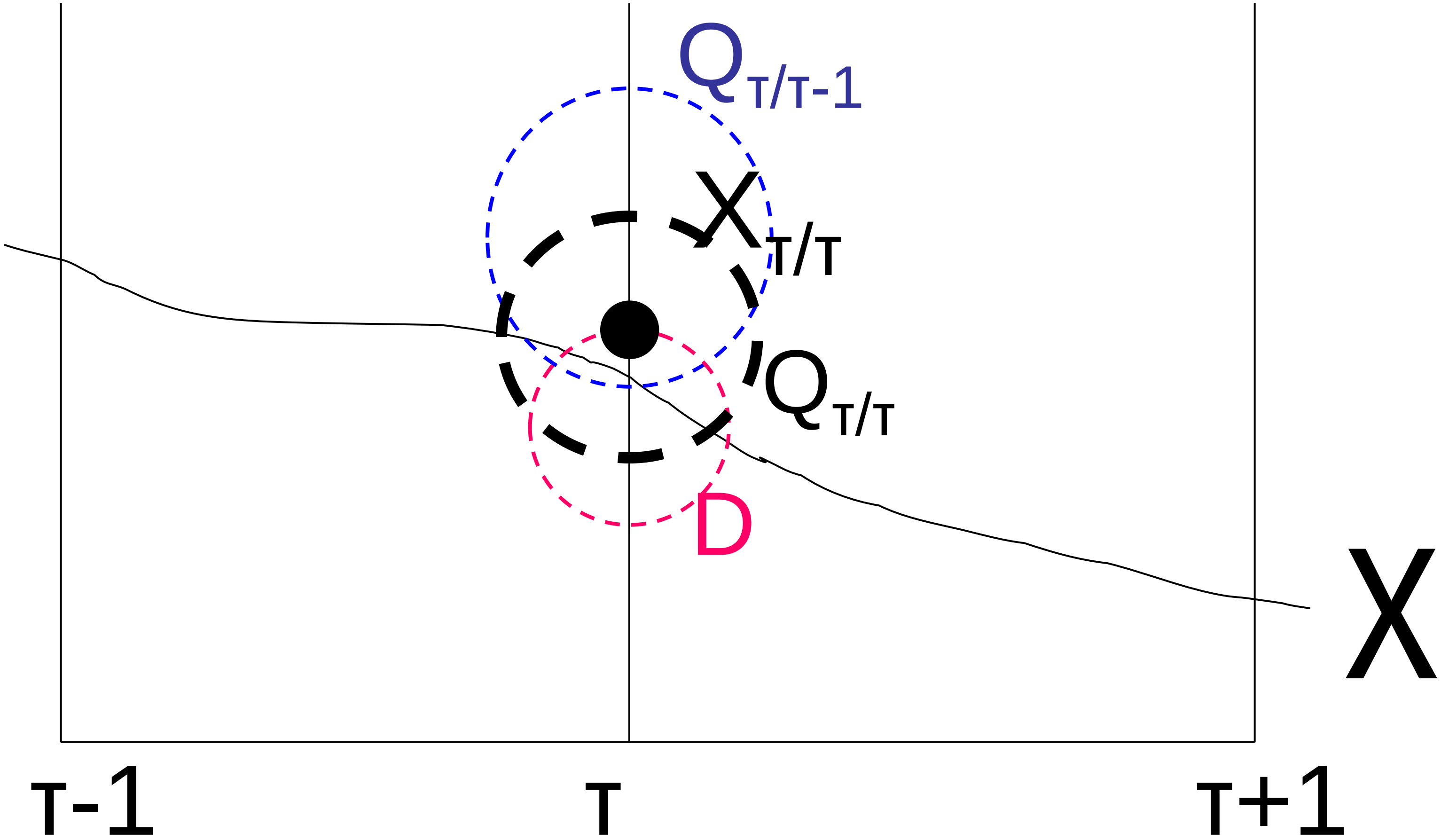
We have a predicted estimate, $X_{\tau/\tau-1}$ and $Q_{\tau/\tau-1}$ similar to the "first guess" in numerical weather prediction



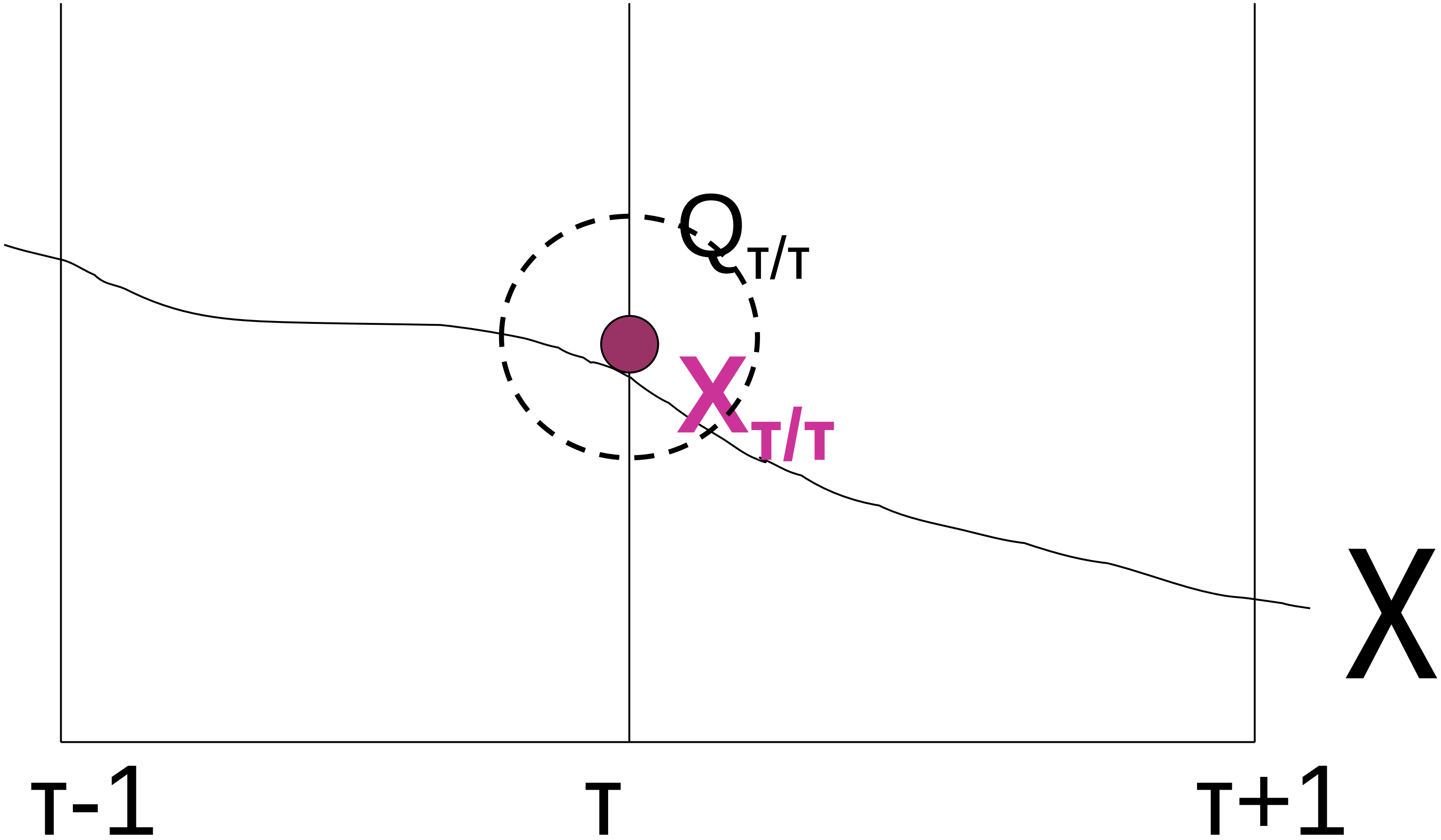
The observation Y_τ , with variance D, of the unknown process χ will modify the "first guess" value $X_{\tau/\tau-1}$



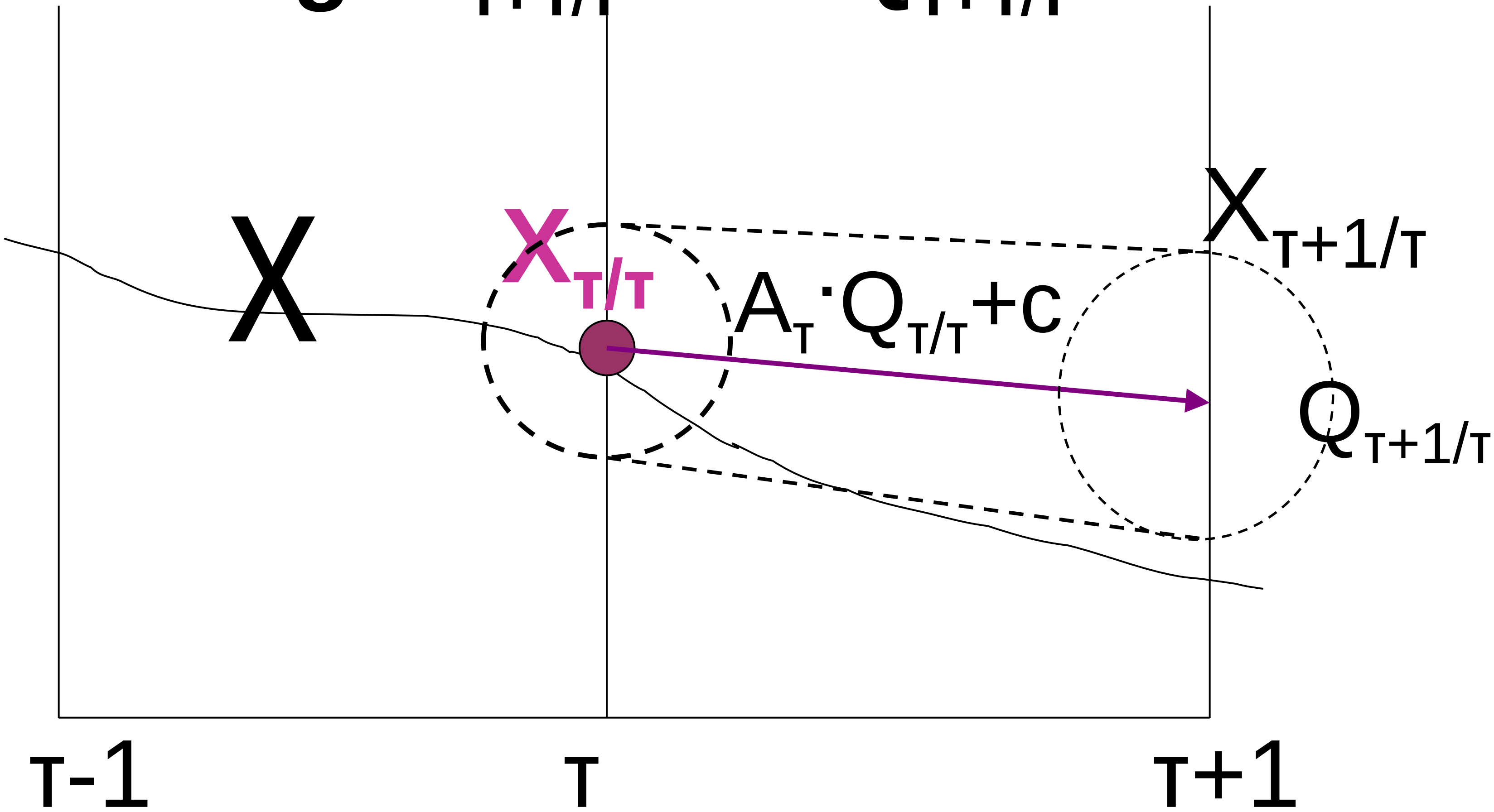
The weighting together of D and $Q_{T/T-1}$ yields a variance of the new estimation $X_{T/T}$ and $Q_{T/T}$



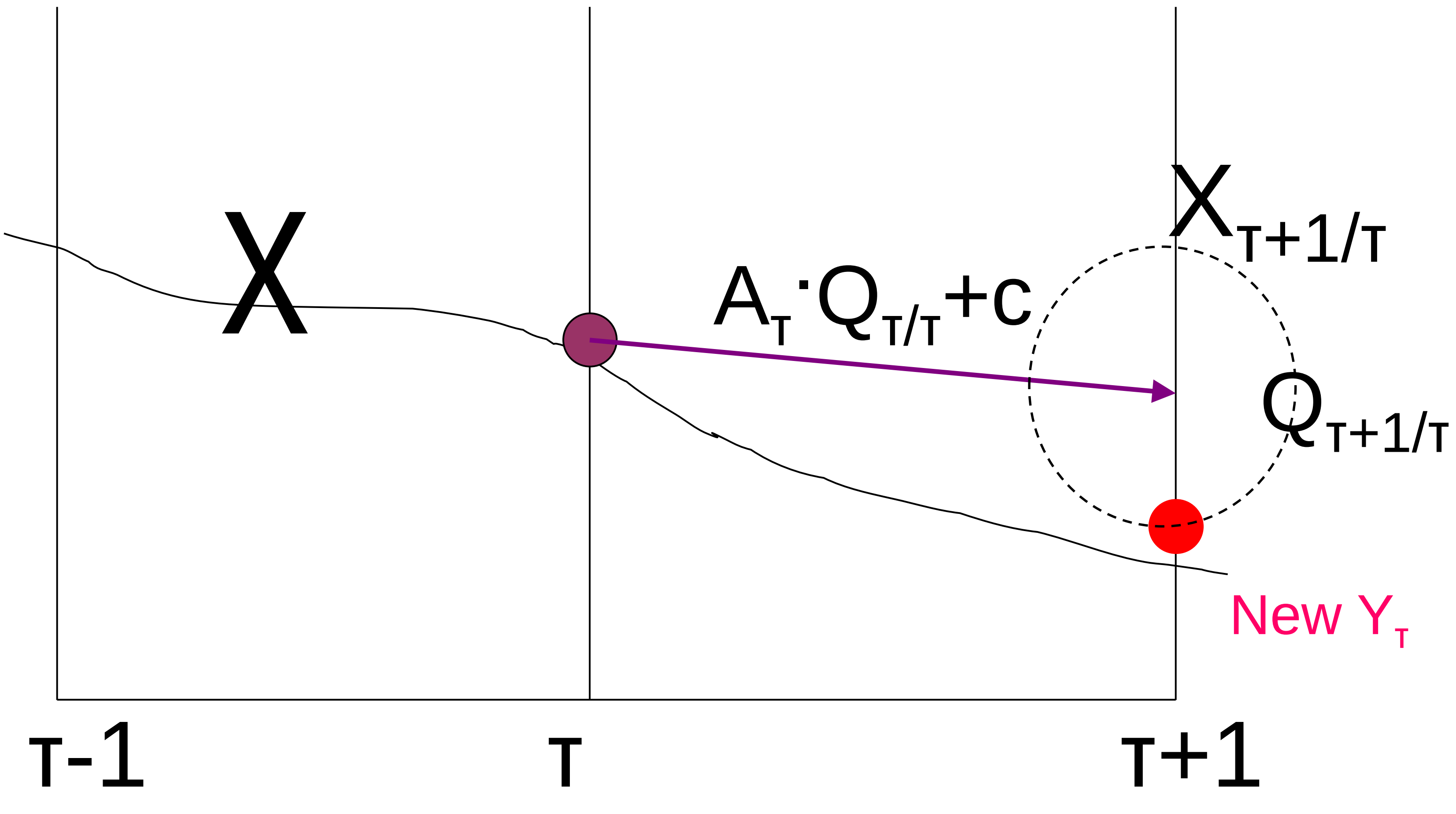
The new estimation of $X_{T/T}$ and $Q_{T/T}$



The new estimation starts with predicting $X_{\tau+1/\tau}$ and $Q_{\tau+1/\tau}$



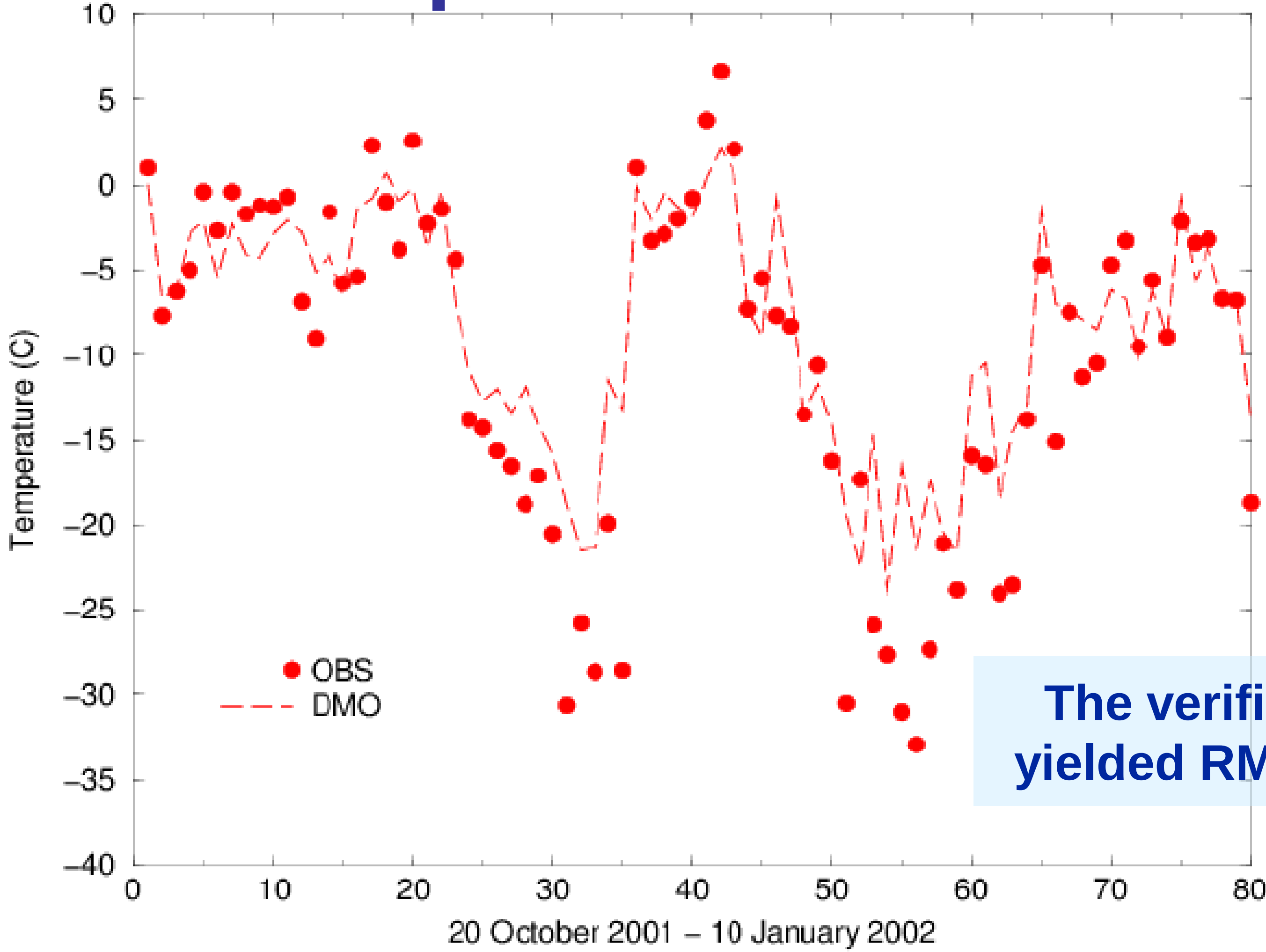
...and a new observation arrives



1-D versus 2-D Kalman filtering

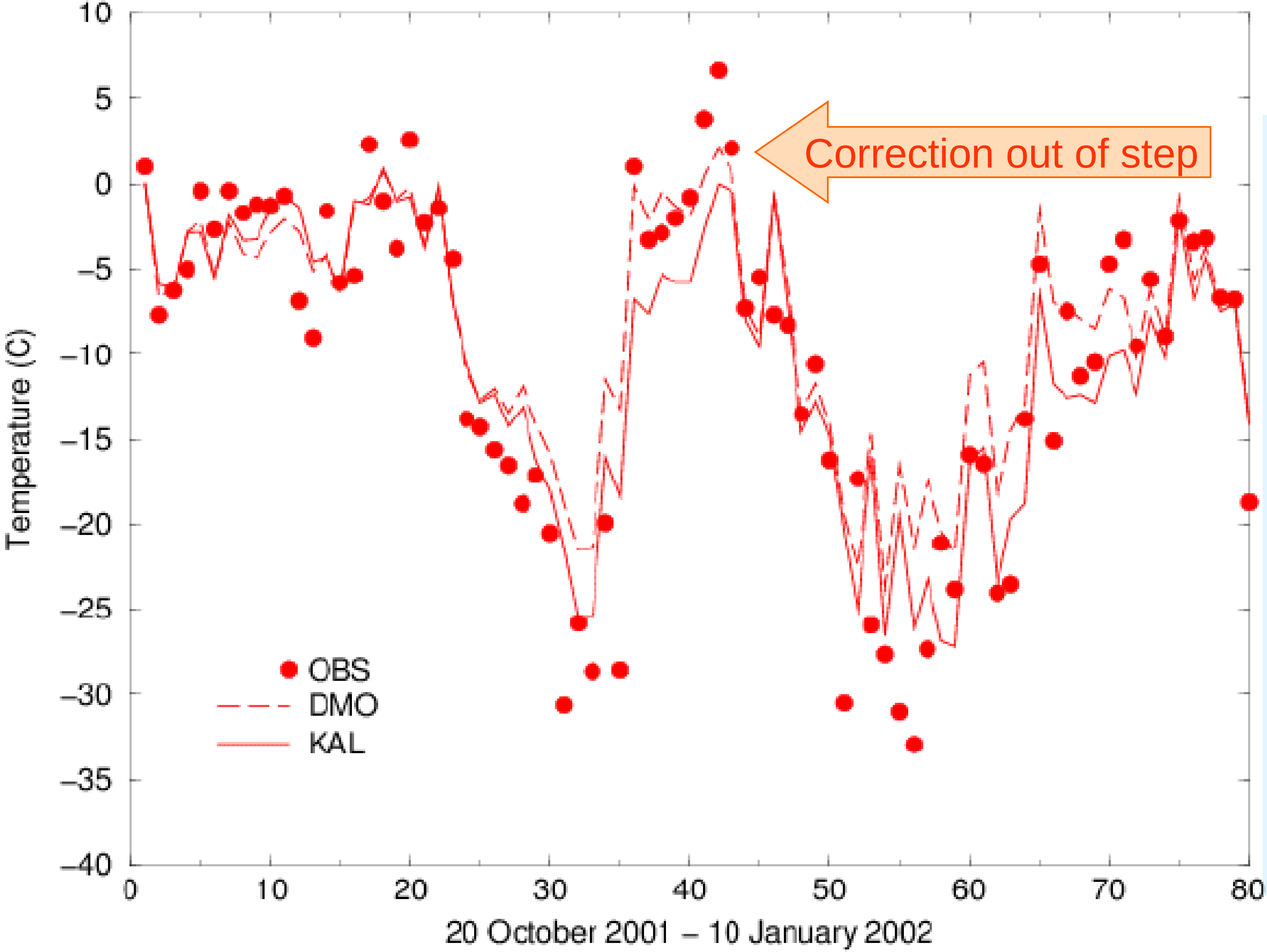
There are fundamental differences between 1-dimensional filtering and multi-dimensional

24 hour 2 m temperature forecast for Kiruna in Lapland winter 2001-2002



The verification yielded RMSE=5.0

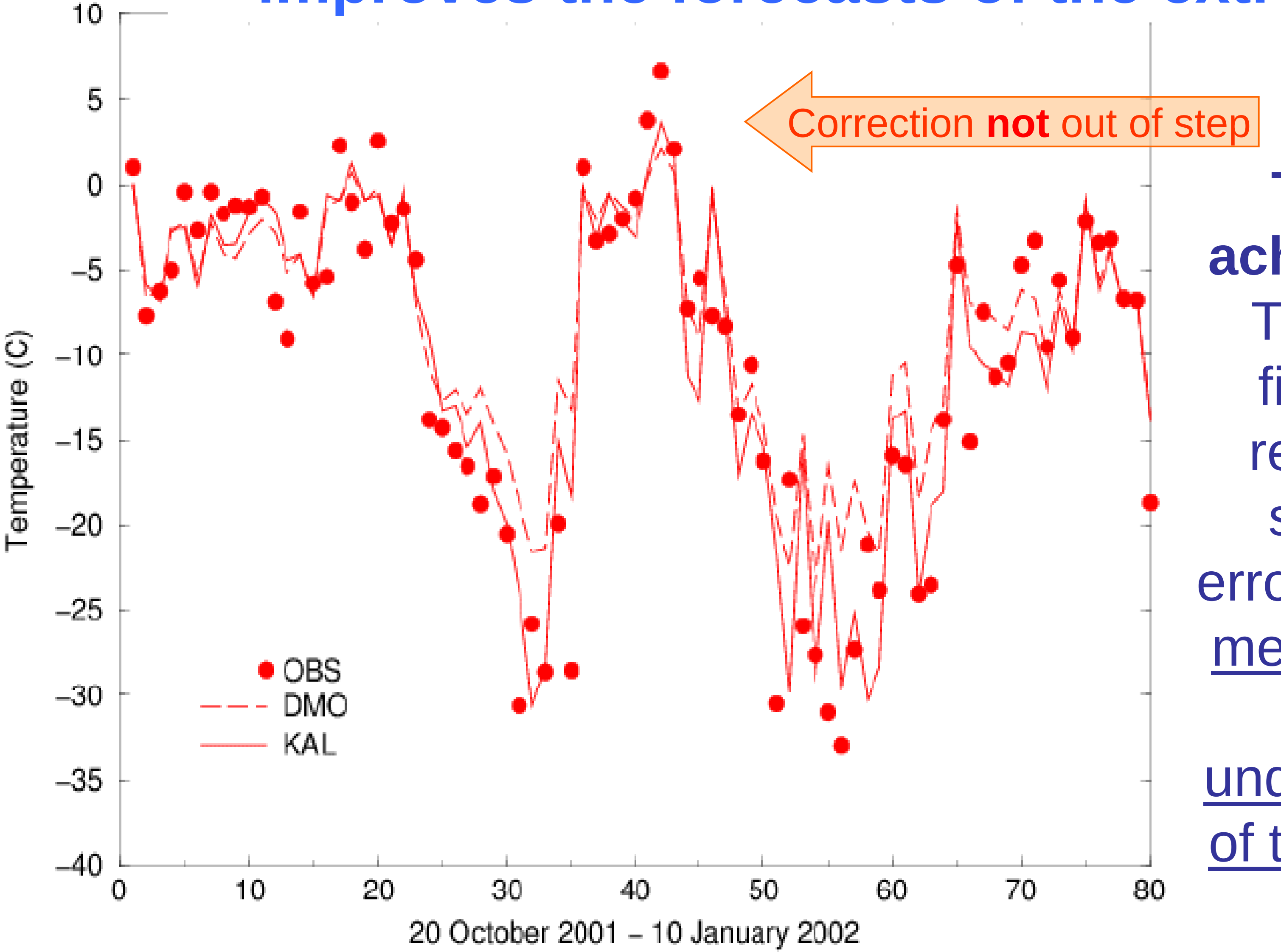
A 1-dimensional Kalman filter reduces an overall bias



The corrections yielded a reduction of the mean error from 2.6 to 0.3 and RMSE from

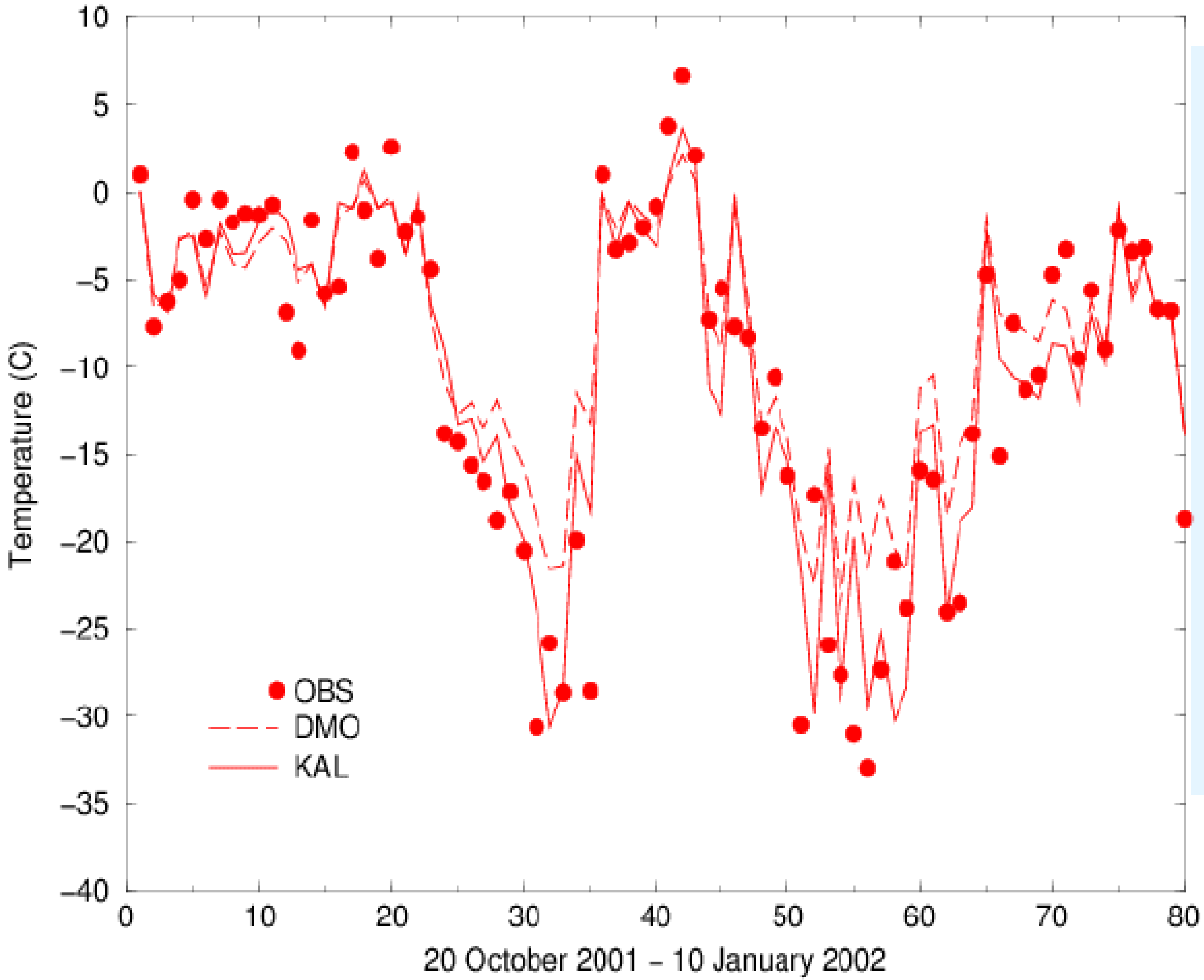
5.0 to **4.2**

A 2-dimensional Kalman filter system also improves the forecasts of the extremes



Two good achievements: The Kalman filtering has reduced two systematic errors: a positive mean error and an underestimation of the variability

Why does the improvement not show up in the verification?



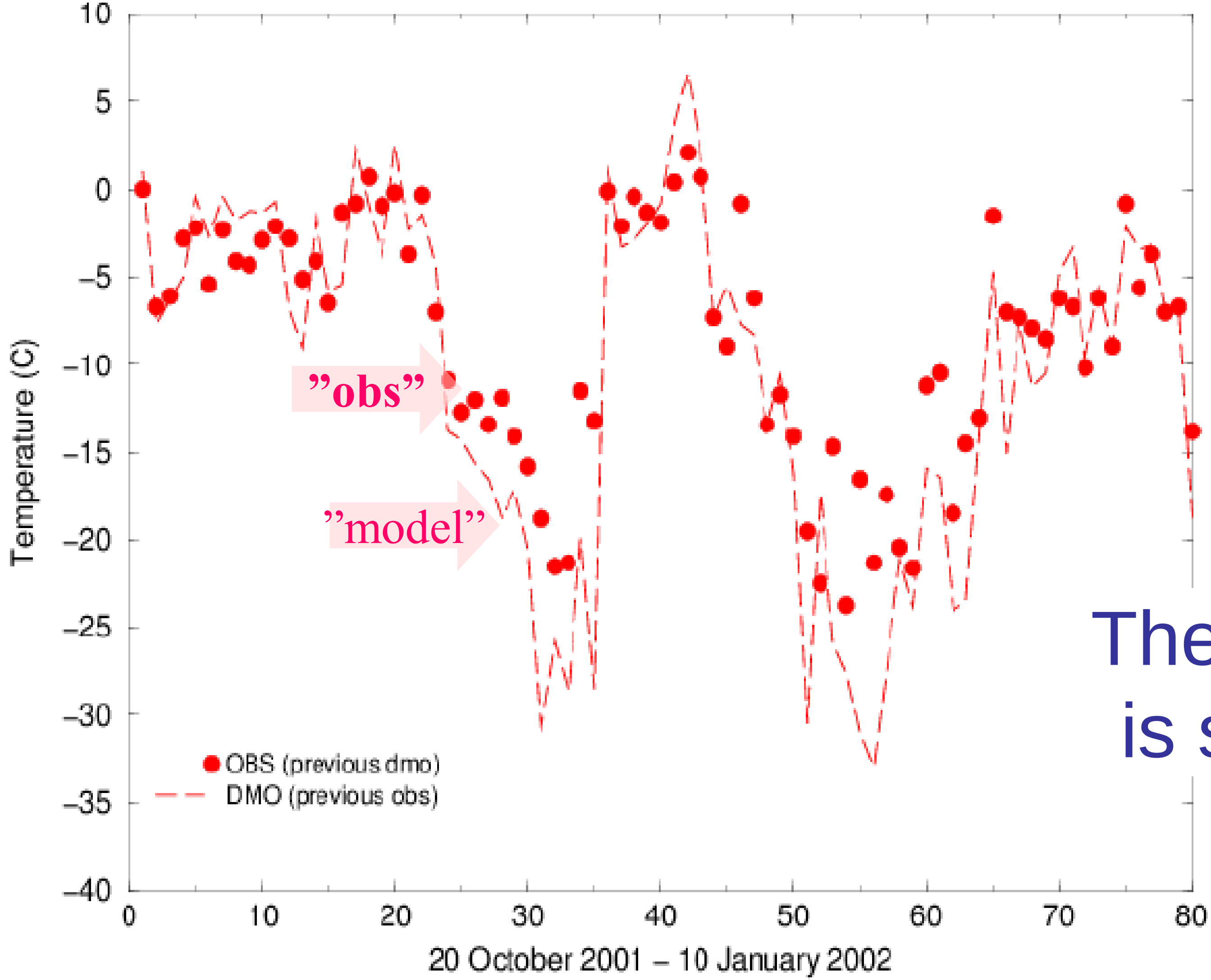
The corrections still yielded a reduction of the mean error from 2.6 to 0.3 but the RMSE from 5.0 only to **4.6** and not to 4.2 as with the 1-D

Is the 2-D worse than the 1-D??

Now we will swap the observations and forecasts. This will not change the size of the errors

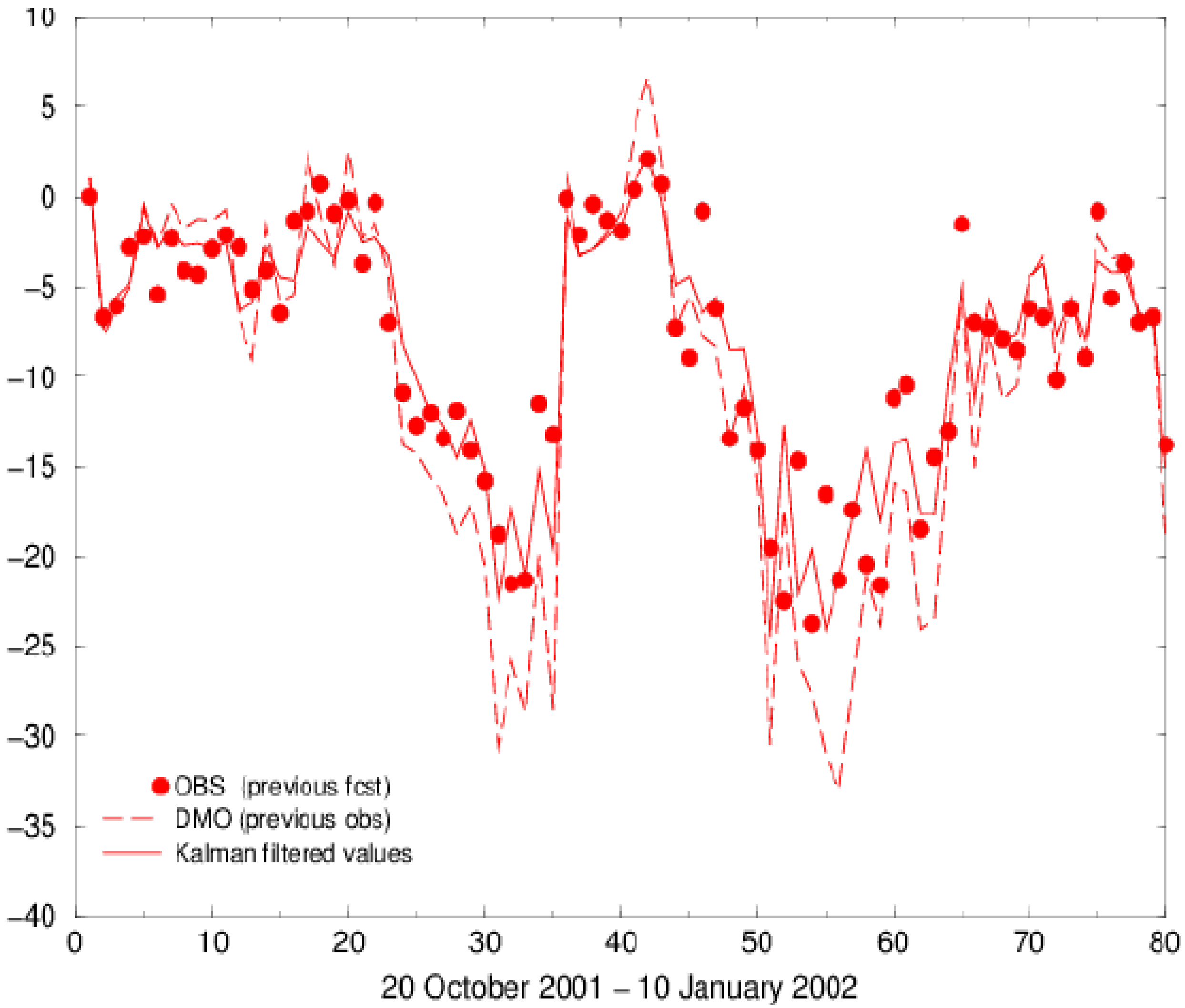
But the forecasts become over-variable compared to the observations

Observations and forecasts swapped so that the forecasts become over-variable



The RMSE is still **5.0**

After 2-D Kalman filtering the RMSE is now reduced from **5.0** to **2.9**

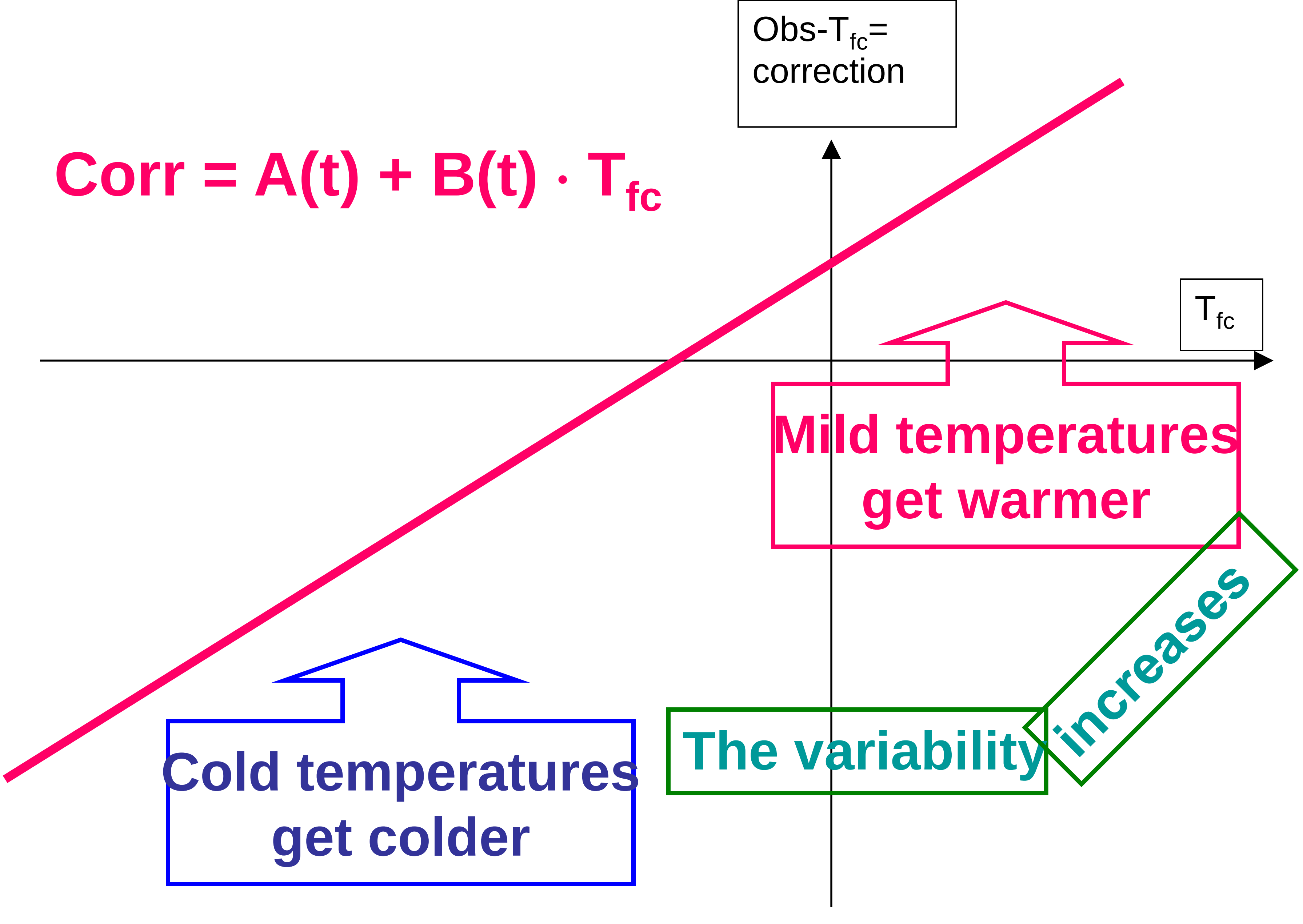


That is much more compared to when it was reduced to only **4.6**

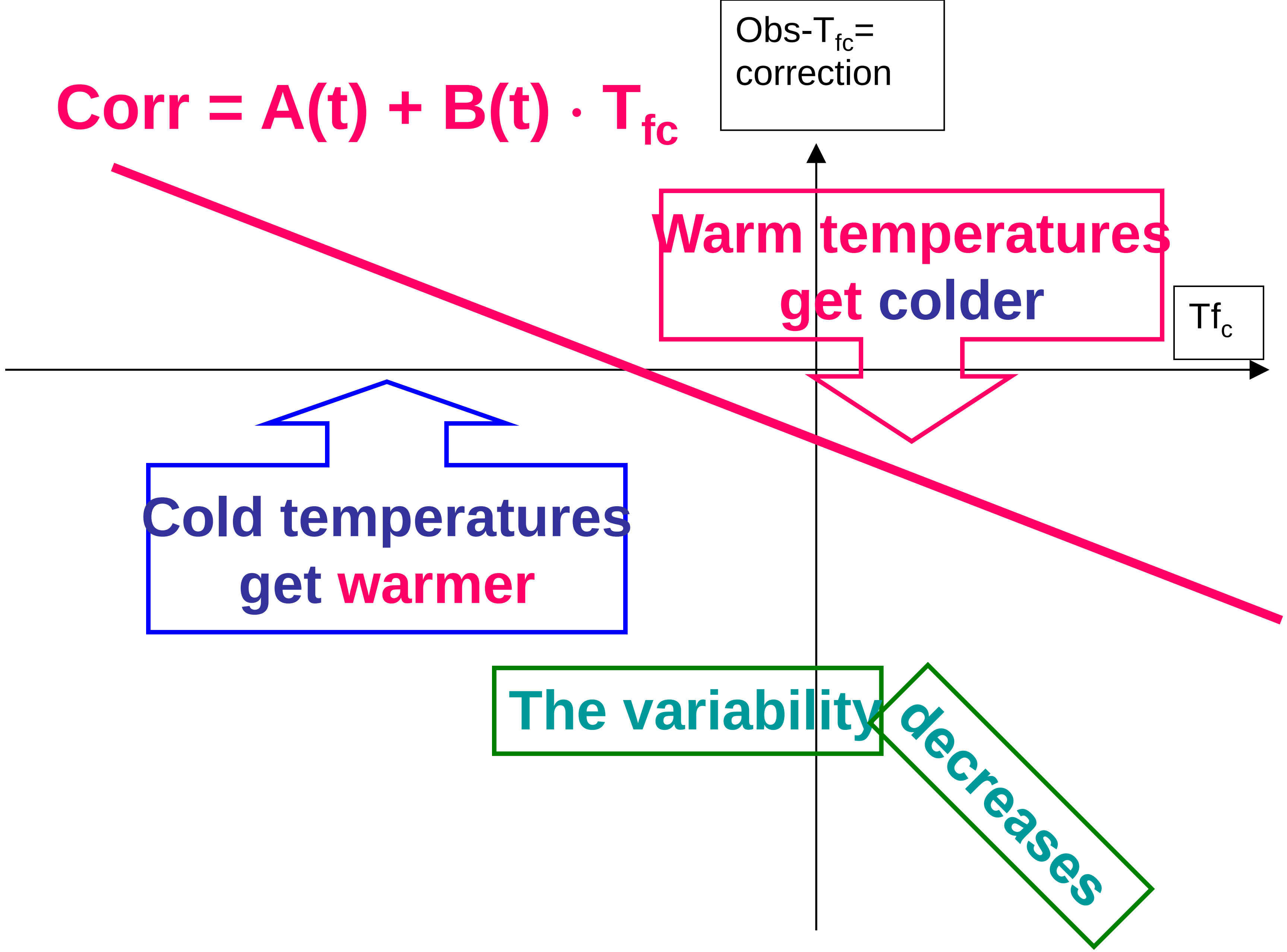
Not only “biases”

A 2- or N-dimensional filter does not only correct mean errors (”biases”) but also *__systematic__* over- and under variability

$$Corr = A(t) + B(t) \cdot T_{fc}$$



$$Corr = A(t) + B(t) \cdot T_{fc}$$



Break