Coriolis I

The Coriolis Effect
according to Coriolis

The scientific-mathematical basis for these lectures

Quarterly Journal of the Royal Meteorological Society *Q. J. R. Meteorol. Soc.* 141: 1957–1967, July 2015 A DOI:10.1002/qj.2477

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Notes and Correspondence
Is the Coriolis effect an 'optical illusion'?

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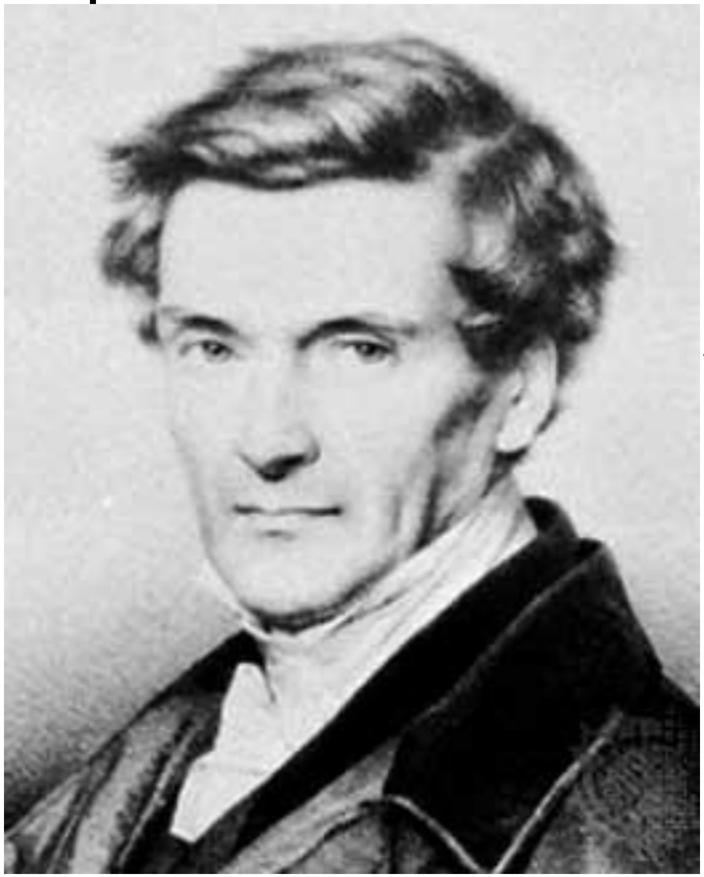
The difference between the derivations of the Coriolis effect on a rotating turntable and on the rotating Earth is discussed. In the latter case a real force, the component of the earth's gravitational attraction, non-parallel to the local vertical, plays a central role by balancing the centrifugal force. That a real force is involved leaves open, not only the question on the inertial nature of the 'inertial oscillations', but also the way we tend to physically conceptualize the terrestrial Coriolis effect.

When reading a lot of literature dealing with dynamic meteorology I saw an aside comment that the Coriolis effect had been derived by its discoverer in a quite different way compared to all our modern textbooks

Gaspard Gustave Coriolis 1784-1843



Further, I could read, Coriolis was interested, neither in the atmosphere nor in the oceans – ***but in machines***

MÉMOIRE

Sur les équations du mouvement relatif des systèmes de corps;

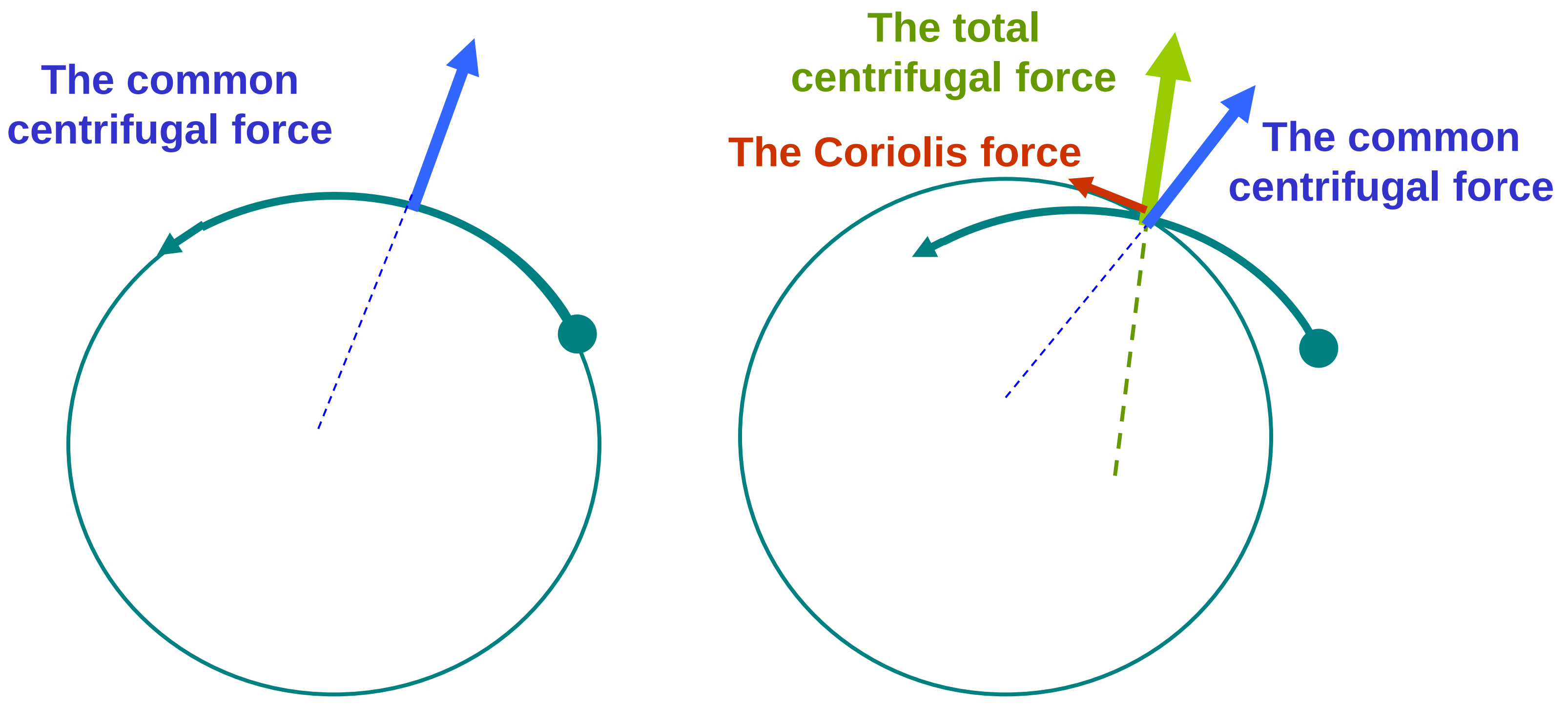
PAR G. CORIOLIS.

1835

Dans un Mémoire qui fait partie du XXI^{e} Cahier du *Journal de l'École Polytechnique*, j'ai montré que pour appliquer le principe des forces vives aux mouvemens relatifs des systèmes entraînés avec des plans coordonnés ayant un mouvement quelconque dans l'espace, il suffisait d'ajouter aux forces données d'autres forces opposées à celles qui sont capables de forcer les points matériels à rester invariablement liés aux plans mobiles auxquels on rapporte les mouvemens relatifs.

J'ai fait remarquer dans ce Mémoire que la proposition qui en est l'objet, ne peut s'appliquer en général à d'autres équations du mouvement que celles des forces vives; mais je n'avais pas examiné alors s'il y a des circonstances où la marche qu'elle fournit peut s'appliquer à certaines équations du mouvement; et si, dans le sens où elle ne s'applique pas, on peut donner une expression simple des nouveaux termes

Coriolis was interested in how the centrifugal effect acted on moving parts in rotating machines



A stationary object in the rotating system

An object moving (inwards) in the rotating system

The Coriolis force was the extra force that had to be added to the common centrifugal force for an relatively moving object

$$\frac{d\mathbf{A}}{dt} = \left(\frac{d\mathbf{A}}{dt}\right)_r + \mathbf{\Omega} \times \mathbf{A}$$

which applied on **R** and **V** yields

$$\frac{d\mathbf{R}}{dt} = \left(\frac{d\mathbf{R}}{dt}\right)_r + \mathbf{\Omega} \times \mathbf{R}$$

$$\mathbf{V} = \mathbf{V}_r + \mathbf{\Omega} \times \mathbf{R}$$

$$\frac{d\mathbf{V}}{dt} = \left(\frac{d\mathbf{V}}{dt}\right)_r + \mathbf{\Omega} \times \mathbf{V}$$

$$\frac{d\mathbf{V}}{dt} = \left(\frac{d\mathbf{V}}{dt}\right)_r + \mathbf{\Omega} \times \mathbf{V} = \left(\frac{d\mathbf{V}_r}{dt}\right)_r + \left(\frac{d(\mathbf{\Omega} \times \mathbf{R})}{dt}\right)_r + \mathbf{\Omega} \times \mathbf{V}_r + \mathbf{\Omega} \times (\mathbf{\Omega} \times \mathbf{R})$$

$$\left(\frac{d\,\mathbf{V}}{dt}\right) = \left(\frac{d\,\mathbf{V}_r}{dt}\right)_r + \left(\frac{d\,(\boldsymbol{\Omega}\times\mathbf{R})}{dt}\right)_r + \boldsymbol{\Omega}\times\mathbf{V}_r + \boldsymbol{\Omega}\times(\boldsymbol{\Omega}\times\mathbf{R})$$

The centripetal acceleration

simplifies into

The Coriolis acceleration!!

$$\frac{d\,\mathbf{V}}{dt} = \left(\frac{d\,\mathbf{V}_r}{dt}\right)_r + 2\boldsymbol{\Omega}\times\mathbf{V}_r + \boldsymbol{\Omega}\times(\boldsymbol{\Omega}\times\mathbf{r})$$

The centrifugal acceleration

and then

The Coriolis force (per unit mass)

$$\left(\frac{d\,\mathbf{V}_r}{dt}\right)_r = \frac{d\,\mathbf{V}}{dt} - 2\boldsymbol{\Omega}\times\mathbf{V}_r - \boldsymbol{\Omega}\times(\boldsymbol{\Omega}\times\mathbf{r})$$

From the Coriolis force to the Coriolis acceleration

The Coriolis force $\mathbf{-2m\Omega \times V}_r$:

1. Fictitious force
2. To the right of anti-cl. motion
3. Non-inertial system

The Coriolis acceleration: $\mathbf{+2\Omega \times V}_r$:

1. Acceleration caused by a real force
2. Pointing to the left of anti-cl. motion
3. Inertial, fixed, system

The Coriolis acceleration is caused by **the real force** we have to apply to **prevent** the Coriolis Effect from deflecting the object

Also note that the Coriolis acceleration (force) was derived in conjunction with the centripetal (centrifugal) force. That the Coriolis acceleration (force) can **not be derived separately** was conjectured in my QJRMS 2015 article.

The absolute frame of reference was used by Euler when he in 1759 derived the Coriolis acceleration

Otons l'une de ces équations de l'autre, & nous aurons:

$$(2\,dr\,d\Phi + r\,dd\Phi)\,(\mathrm{tang}\,\Phi + \cot\Phi) = 0$$

ou bien $2\,dr\,d\Phi + r\,dd\Phi = 0$

Multiplions la premiere par cot Φ & la ſeconde par tang Φ, & nous aurons en les ajoutant enſemble:

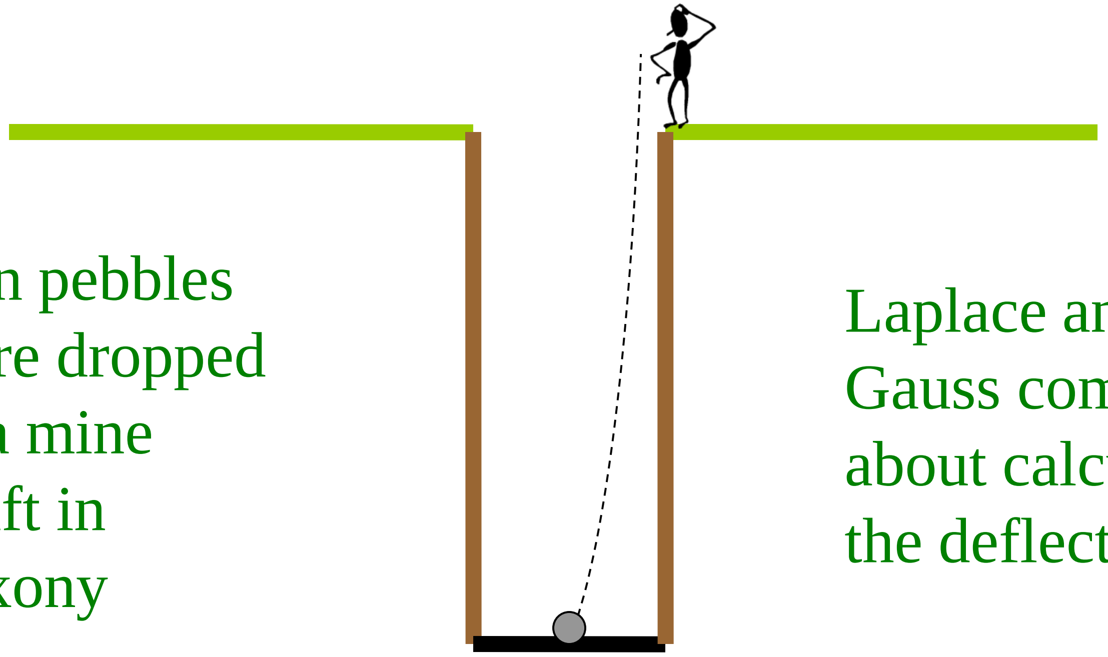
$$(ddr - r\,d\Phi^2)\,(\cot\Phi + \mathrm{tang}\,\Phi) = -\tfrac{1}{2}\,V\,dt^2\,(\cot\Phi + \mathrm{tang}\,\Phi)$$

ou bien $ddr - r\,d\Phi^2 = -\tfrac{1}{2}\,V\,dt^2.$

The next step of progress was at the end of the 18th century when Laplace derived his "tidal equations"

But neither he nor Euler really understood physically what they had mathematically derived

It was the 1803 experiment in the Schlebusch mines in Saxony that for the first time confirmed agreement with theory



Iron pebbles were dropped in a mine shaft in Saxony

Laplace and Gauss competed about calculating the deflection

From Simeon de Laplace's paper 1803

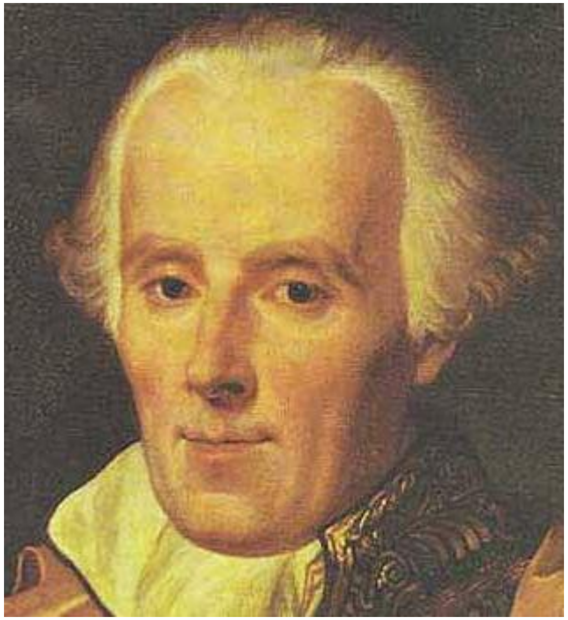
Si l'on égale à zéro les coefficients des trois variations δr, $\delta \theta$ et $\delta \omega$, et si l'on observe que $-\left(\frac{dQ}{dr}\right)$ représente la pesanteur que nous désignerons par g (¹), on aura, en prenant pour l'unité le rayon r, ce qu'on peut faire ici sans erreur sensible, les trois équations suivantes :

$$0 = \alpha \frac{d^2 s}{dt^2} + 2\alpha n \frac{dv}{dt} \sin^2\theta + \alpha K \frac{ds}{dt} - g,$$

$$0 = \alpha \frac{d^2 u}{dt^2} - 2\alpha n \frac{dv}{dt} \sin\theta \cos\theta + \alpha K \frac{du}{dt} - g\left(\frac{dy}{d\theta}\right),$$

$$0 = \alpha \frac{d^2 v}{dt^2} \sin\theta + 2\alpha n \frac{du}{dt} \cos\theta - 2\alpha n \frac{ds}{dt} \sin\theta + \alpha K \frac{dv}{dt} \sin\theta - \frac{g}{\sin\theta}\left(\frac{dy}{d\omega}\right).$$

Si l'on prend la seconde décimale, ou la cent-millième partie du jour moyen, pour unité de temps, n est le petit angle décrit dans une seconde par la rotation de la Terre. Cet angle est extrêmement petit; et comme αu et αv sont de très petites quantités par rapport à αs, on peut négliger, dans la première de ces trois équations, le terme $2\alpha n \frac{dv}{dt} \sin^2\theta$; dans la deuxième, le terme $-2\alpha n \frac{dv}{dt} \sin\theta \cos\theta$ et, dans la troisième, le terme $2\alpha n \frac{du}{dt} \cos\theta$; ce qui réduit ces trois équations



From Friedrich Gauss's paper 1804

FÜR DIE BEWEGUNG SCHWERER KÖRPER AUF DER ROTIRENDEN ERDE. 501

$$(x-a)\left(\frac{p}{r}-nn\right)+\cos\varphi\left(\frac{pq}{r}-nnZ\right)$$
$$y\left(\frac{p}{r}-nn\right)$$
$$(z+c)\left(\frac{p}{r}-nn\right)-\sin\varphi\left(\frac{pq}{r}-nnZ\right)$$

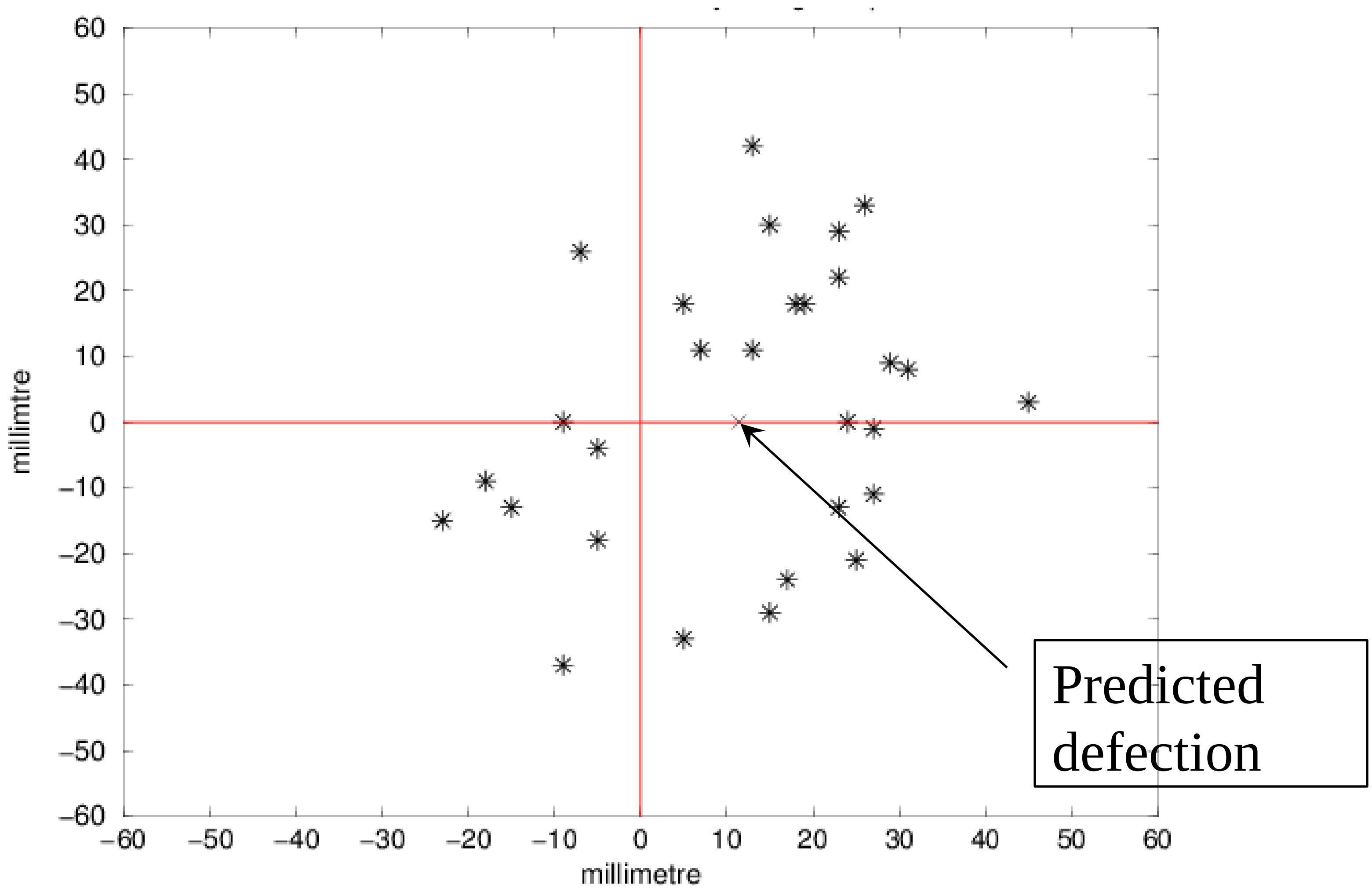
sollicitirt zu werden. Ein schon in Bewegung begriffener Körper hingegen wird anders afficirt. Denn ausser dem Widerstande der Luft, der den Körper nach diesen Richtungen wie Kräfte, deren Maass $Mu\frac{dx}{dt}$, $Mu\frac{dy}{dt}$, $Mu\frac{dz}{dt}$ ist, treibt und folglich auf der rotirenden Erde völlig eben so wirkt, als er auf der ruhenden wirken würde, kommen nach jenen Richtungen noch die drei Kräfte

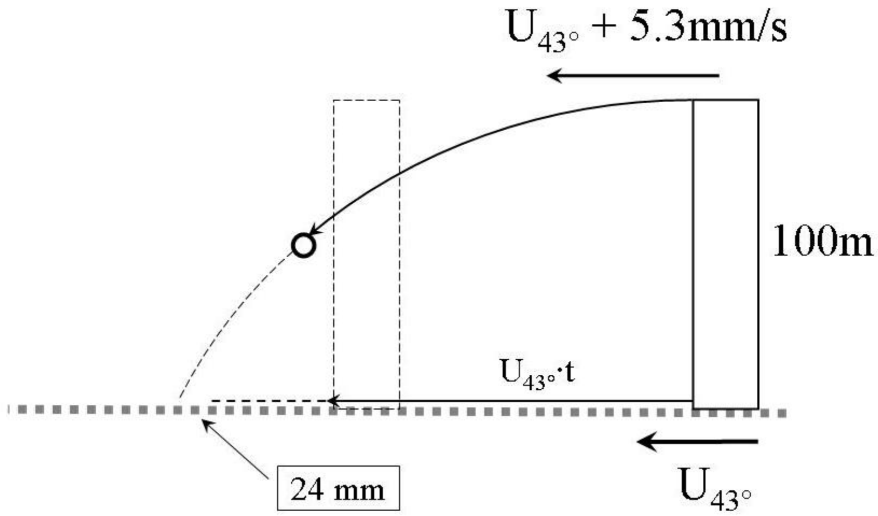
$$-2n\sin\varphi\frac{dy}{dt},\qquad 2n\sin\varphi\frac{dx}{dt}+2n\cos\varphi\frac{dz}{dt},\qquad -2n\cos\varphi\frac{dy}{dt}$$

hinzu, und diese sind es allein, wodurch die Rotation der Erde an fallenden Körpern sichtbar wird. Die bisherigen Schlüsse und Folgerungen sind streng und allgemein richtig.

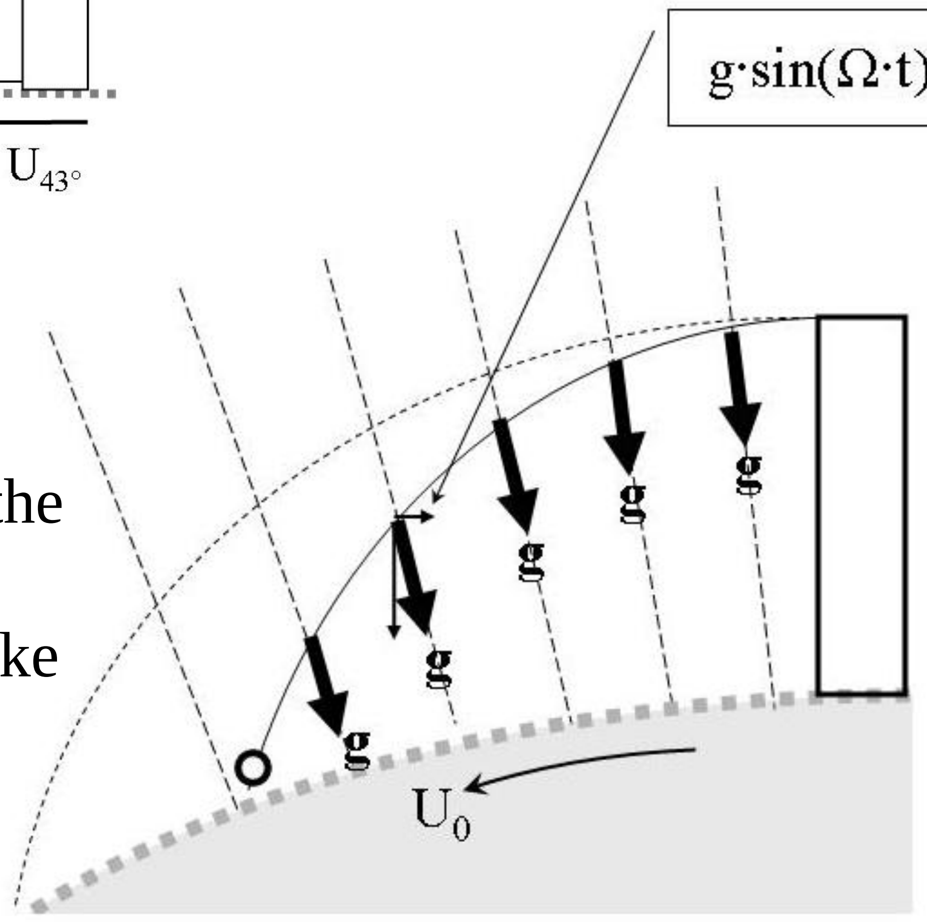
Bei *Versuchen*, die in dieser Hinsicht angestellt werden, geschieht allemal die Bewegung des Körpers in einem so kleinen Raume, dass man die Stärke der auf ruhende Körper wirkenden scheinbaren Schwere innerhalb desselben, als unveränderlich $=g$, und ihre Richtung als immer parallel, also senkrecht auf die Ebne der z annehmen kann. Es wird also ohne Bedenken erlaubt sein, statt der obigen drei Grössen

Scatter of the hits in the Schlebusch mine shaft



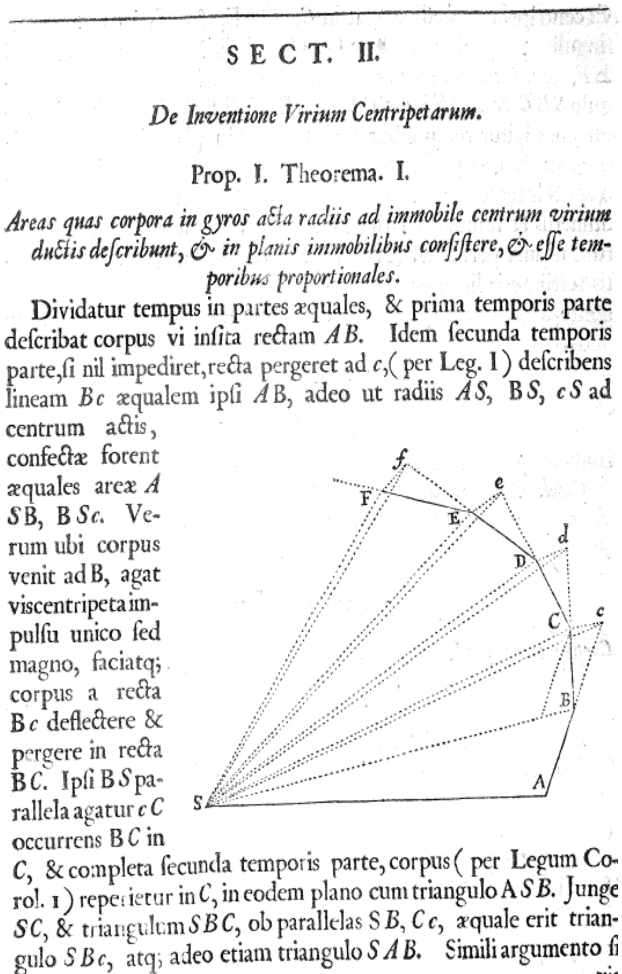


Galileo tried to estimate the deflection of falling objects, but got the maths wrong



Isaac Newton would have the mathematical (and Robert Hooke the technical) to make the 1804 experiment 120 years earlier

One of the first things Newton did in "Principia" was to derive an expression for the centripetal acceleration



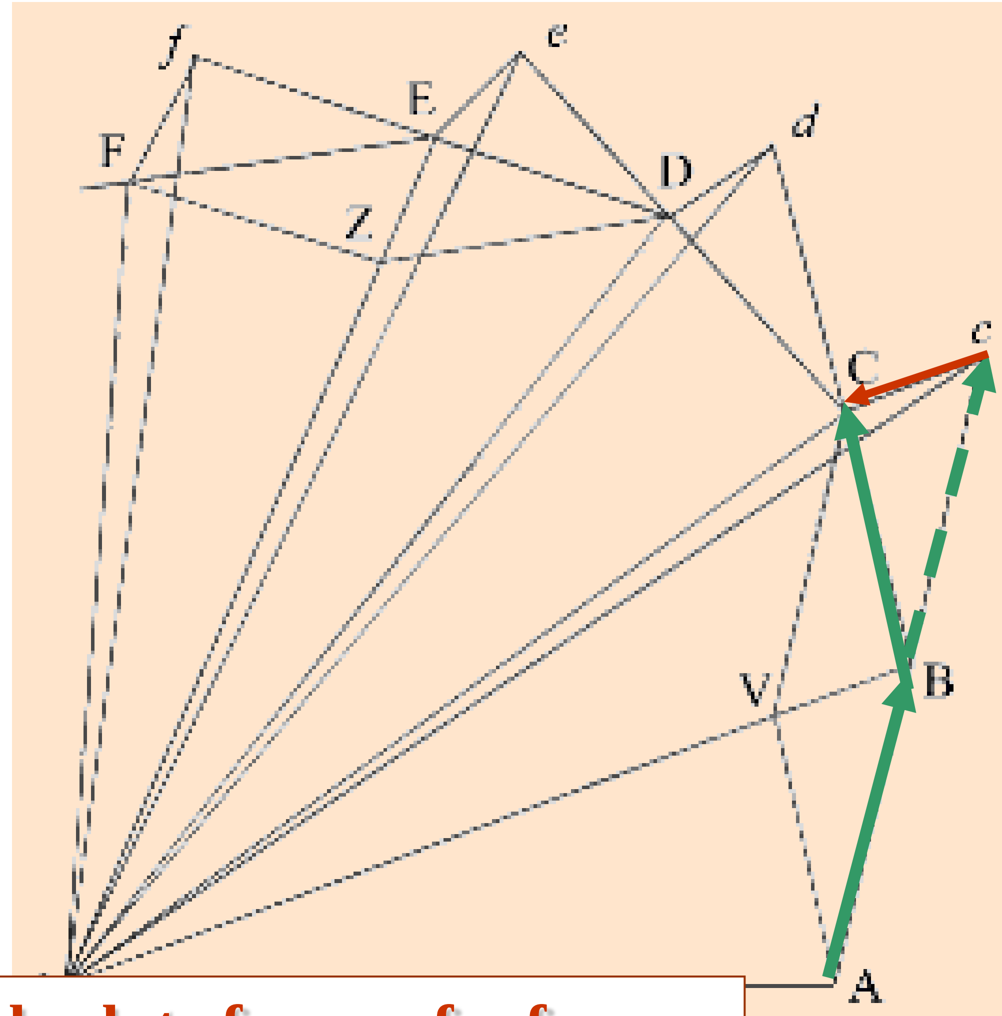
SECT. II.

De Inventione Virium Centripetarum.

Prop. I. Theorema. I.

Areas quas corpora in gyros acta radiis ad immobile centrum virium ductis deſcribunt, & in planis immobilibus conſiſtere, & eſſe temporibus proportionales.

Dividatur tempus in partes æquales, & prima temporis parte deſcribat corpus vi inſita rectam *AB*. Idem ſecunda temporis parte, ſi nil impediret, recta pergeret ad *c*, (per Leg. I) deſcribens lineam *Bc* æqualem ipſi *A*B, adeo ut radiis *AS*, B*S*, *cS* ad centrum actis, confectæ forent æquales areæ *A* *S*B, B*Sc*. Verum ubi corpus venit ad B, agat vis centripeta impulſu unico ſed magno, faciatq; corpus a recta B*c* deflectere & pergere in recta B*C*. Ipſi B*S* parallela agatur *c C* occurrens B*C* in *C*, & completa ſecunda temporis parte, corpus (per Legum Corol. 1) reperietur in *C*, in eodem plano cum triangulo A*S*B. Junge *SC*, & triangulum *SBC*, ob parallelas S*B*, *Cc*, æquale erit triangulo *SBc*, atq; adeo etiam triangulo *S* *A* B. Simili argumento ſi



Note: everything in absolute frame of reference

In my 2015 **QJRMS article** I showed how Newton, with his mathematical technique could have derived the Coriolis acceleration

Radial relative motion

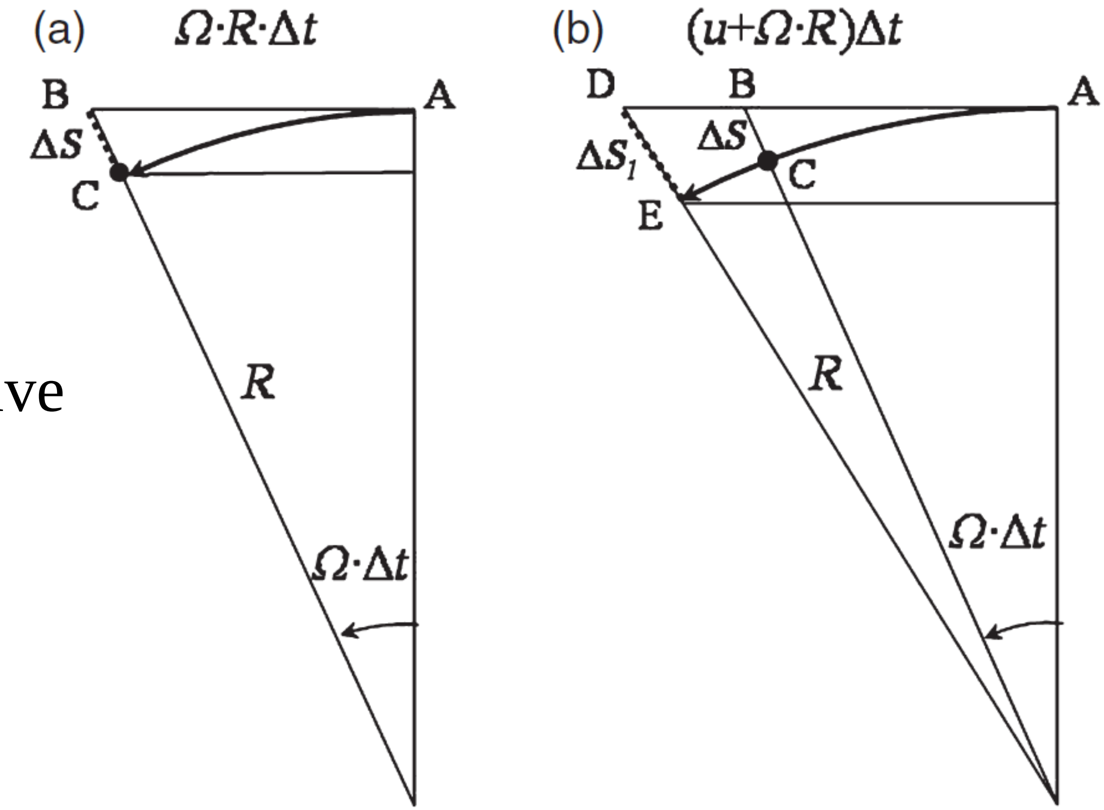


Figure A1. (a) The derivation of the centripetal acceleration for a body fixed in the rotating system. (b) The derivation of the centripetal and Coriolis accelerations for a body moving tangentially relative to the rotating system. See text for further details.

The same for tangential relative motion

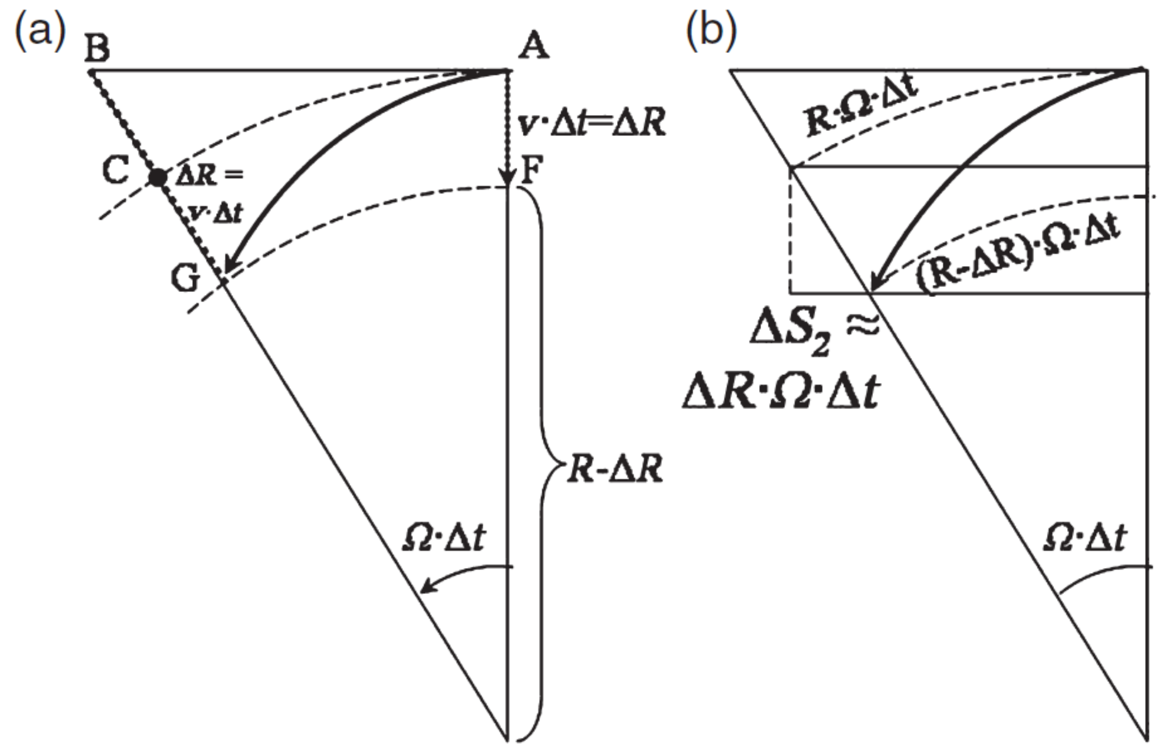
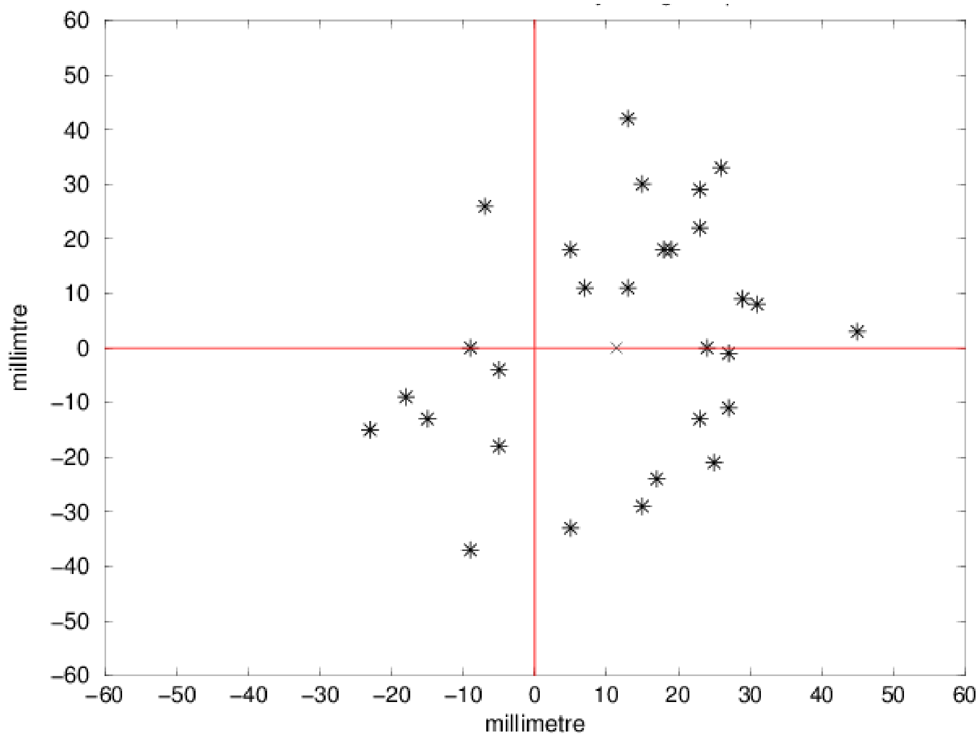


Figure A2. The derivation of the Coriolis acceleration for a radially inward moving object. (a) Highlights the outline of the motion and (b) provides additional mathematical details. See text for further details of the derivation.

So why didn't Newton do it?

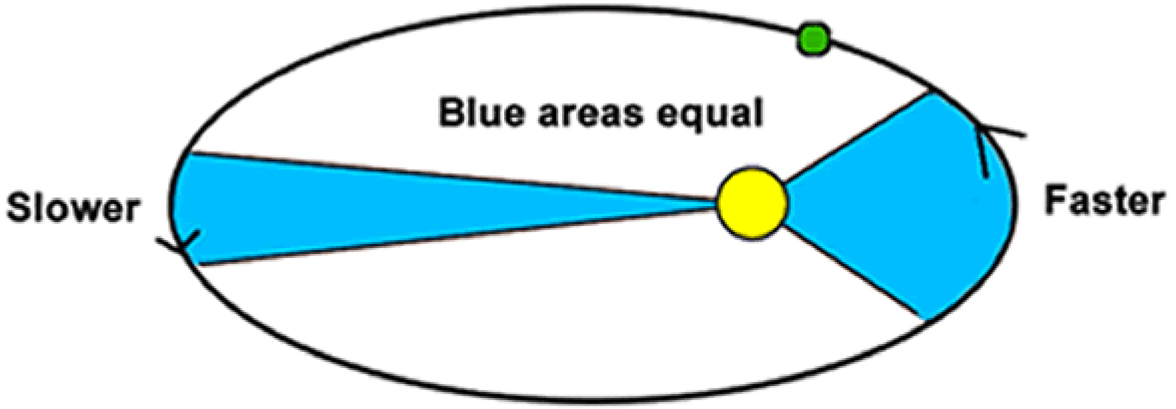
Because in Newton's days scientist had no feel or knowledge about statistical estimation theory.

Hooke made some experiments in 1680 but was put-off by the large spread of the falling objects – just like in 1804



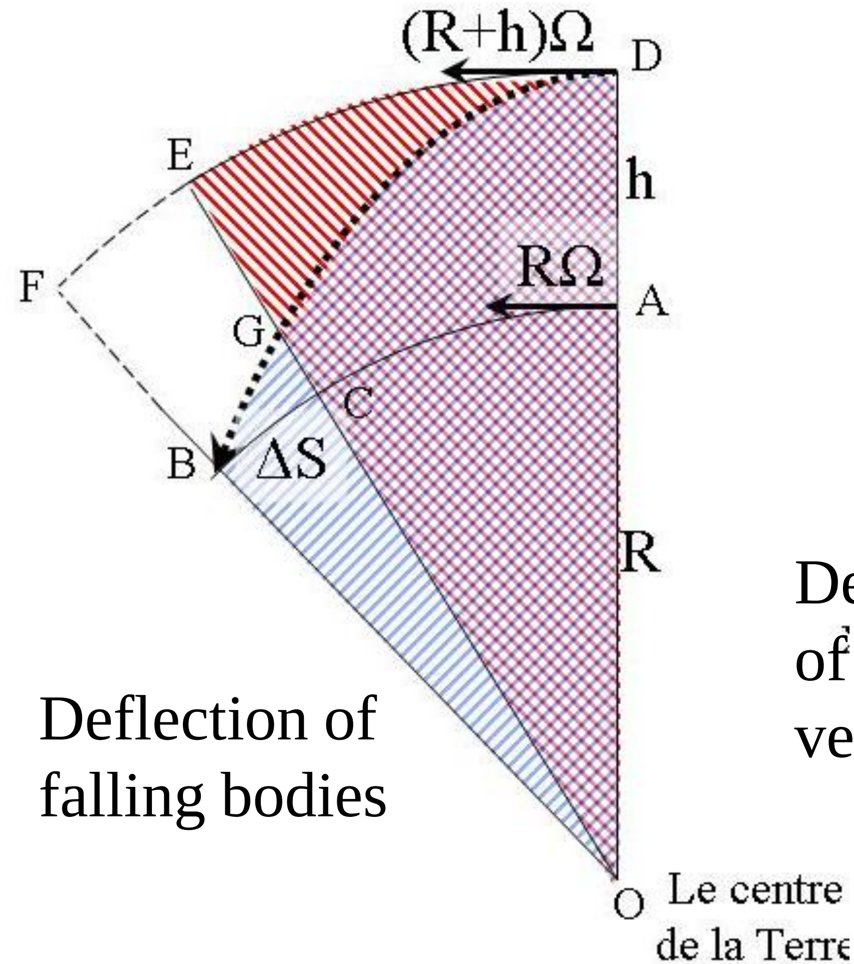
But in 1804 scientists had some feel and knowledge about statistics, the value of averages and how to calculate them

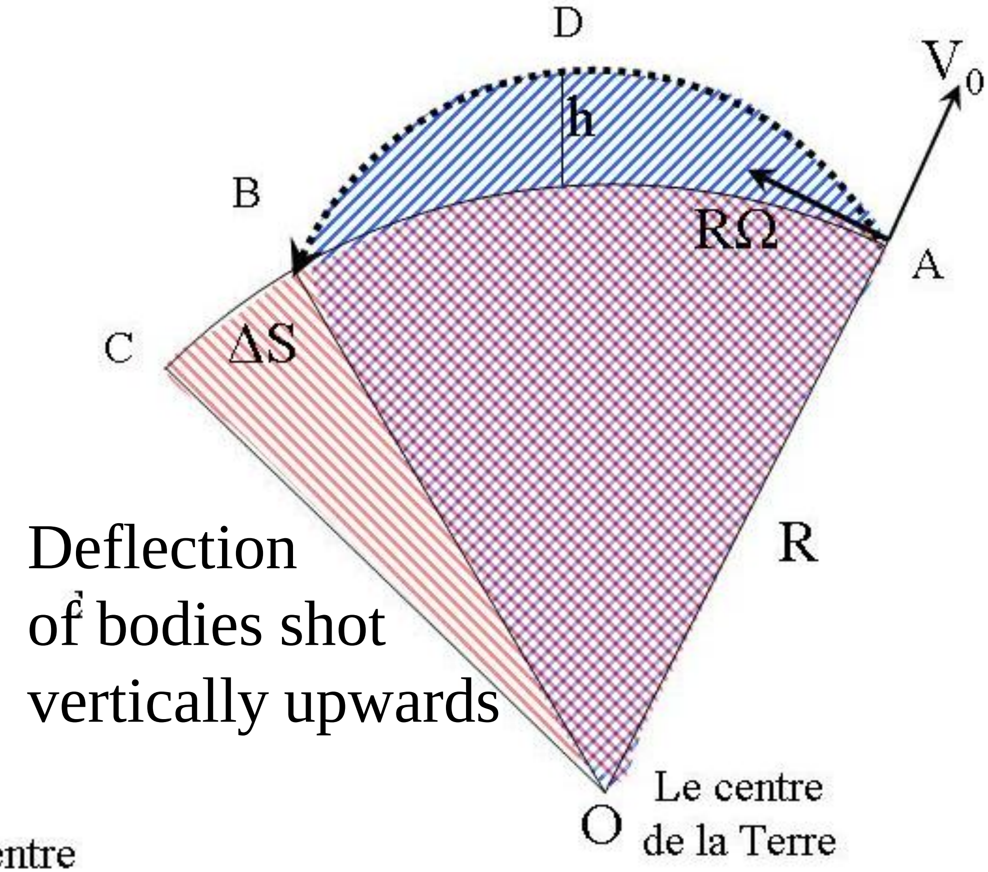
Also Johannes Kepler (1571-1630) could have derived the Coriolis acceleration from his 2nd Law



An imaginary line joining a planet and the sun sweeps out an equal area of space in equal amounts of time.

How Johannes Kepler could have derived the Coriolis acceleration by using his 2nd law





But he didn't realise the law was also valid for "earthly" objects

-Isn’t it true that the Coriolis force is only a fictitious force?

-Yes, that is true!

-Isn’t it also true that the Coriolis force cannot do any work?

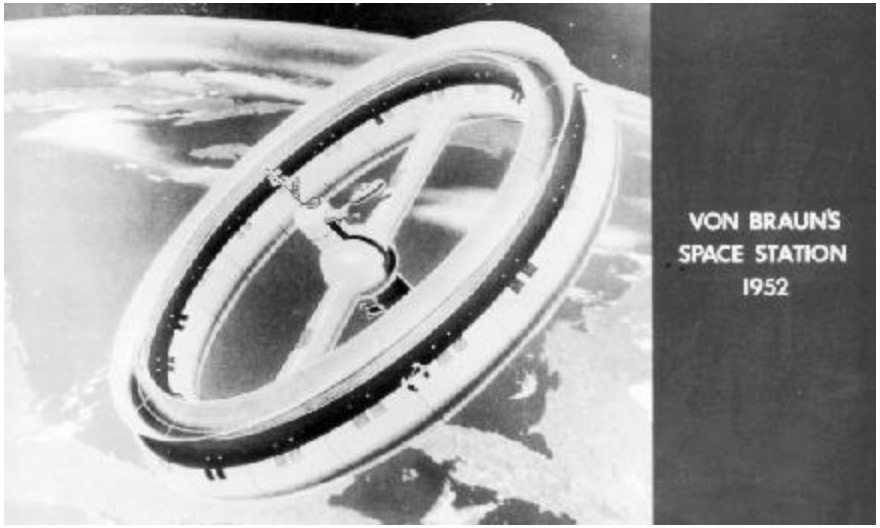
-That is absolutely true, it is always directed perpendicular to the motion and can only change its direction, not its speed (its kinetic energy)

-So isn’t the Coriolis Effect just an optical illusion???

-No because when cannot use “fictitious” and “work” in their colloquial, “everyday” meanings

Being "fictitious" and unable to "do work" does not mean the Coriolis Effect can be seen as an "illusion"

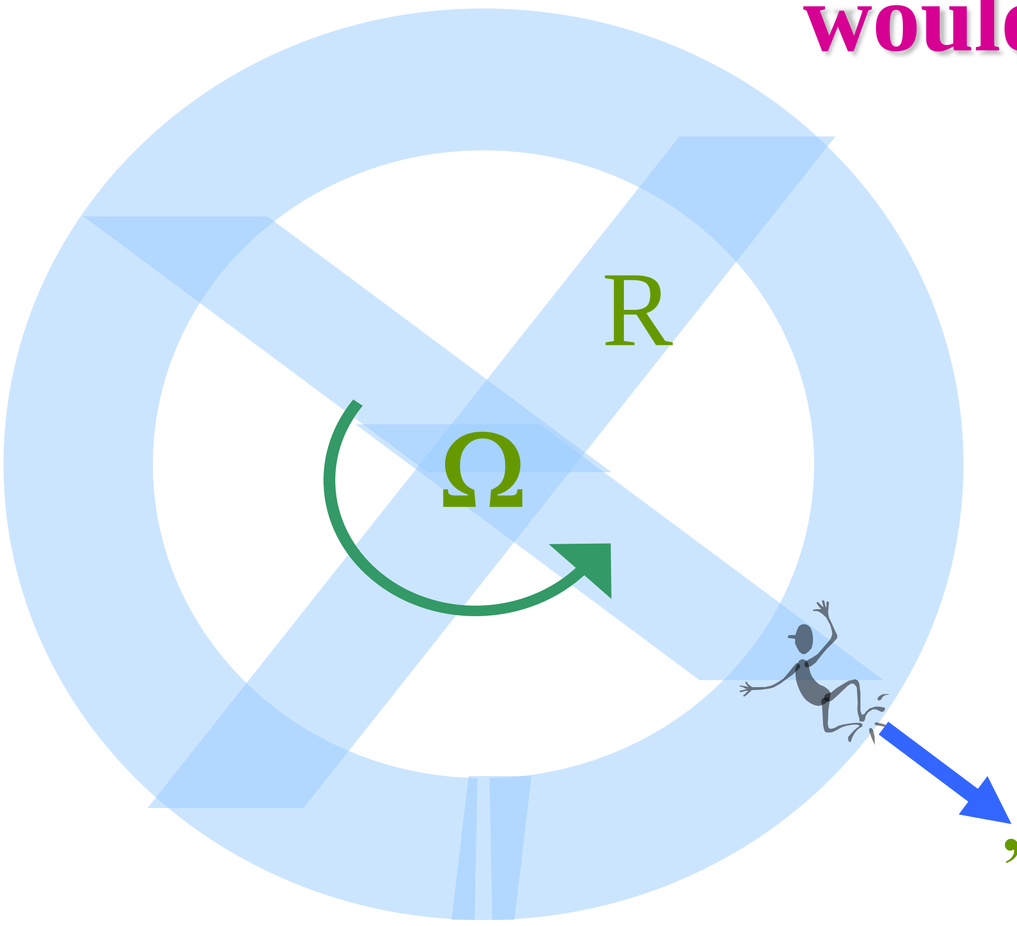
One example: In the 1950's and 1960's planned to create artificial gravity on their space stations by letting them rotate. This was nicely depicted in Stanley Kubrick's 1969 movie "2001 - A Space Odyssey":



https://www.youtube.com/watch?v=q3oHmVhviO8

https://www.youtube.com/watch?v=1wJQ5UrAsIY&ebc=ANyPxKo4CqF8_xFhOGFvxKcYafafA0yy4qJOLEyy9E-Ar-6ou7TNub_e9DNKLtfamKKTqQ_HhYpnX_z5ZZG8mZpbPrLBqQgTkA

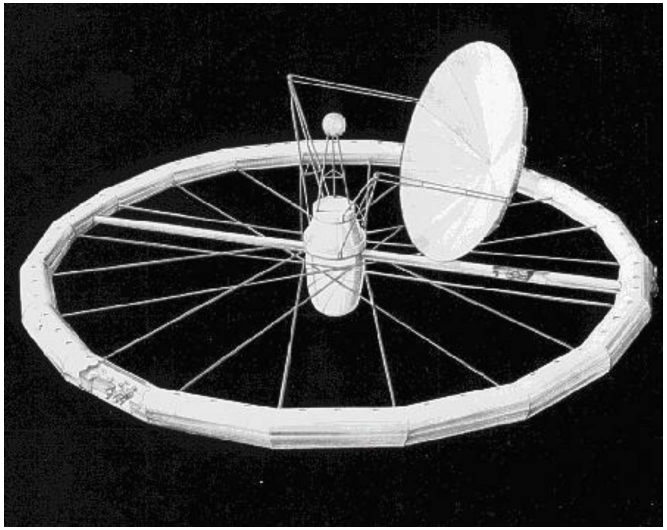
But the astro- and cosmonauts would get sea sick!



For R=100 m it needs a rotation 300 faster than the earth's to provide a ”g” = 9.81 m/s^2

That means 300 times stronger Coriolis forces!

$$\text{"g"} = \Omega^2 R$$



So the Coriolis force might be "fictitious" and unable to do "work" but it was still able to thwart the American and Soviet plans to create artificial gravity on manned space stations – which is still an unsolved problem

Is it a too provocative title?

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And coming to optical illusions: I found this on the web

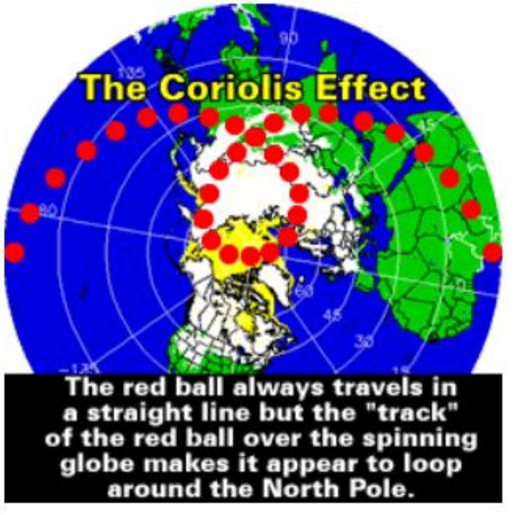
← Planetary Rotation 3: Mars, Earth and Venus

Inventions and Deceptions – Gravitational Mass Attraction →

Inventions and Deceptions – Coriolis Effect

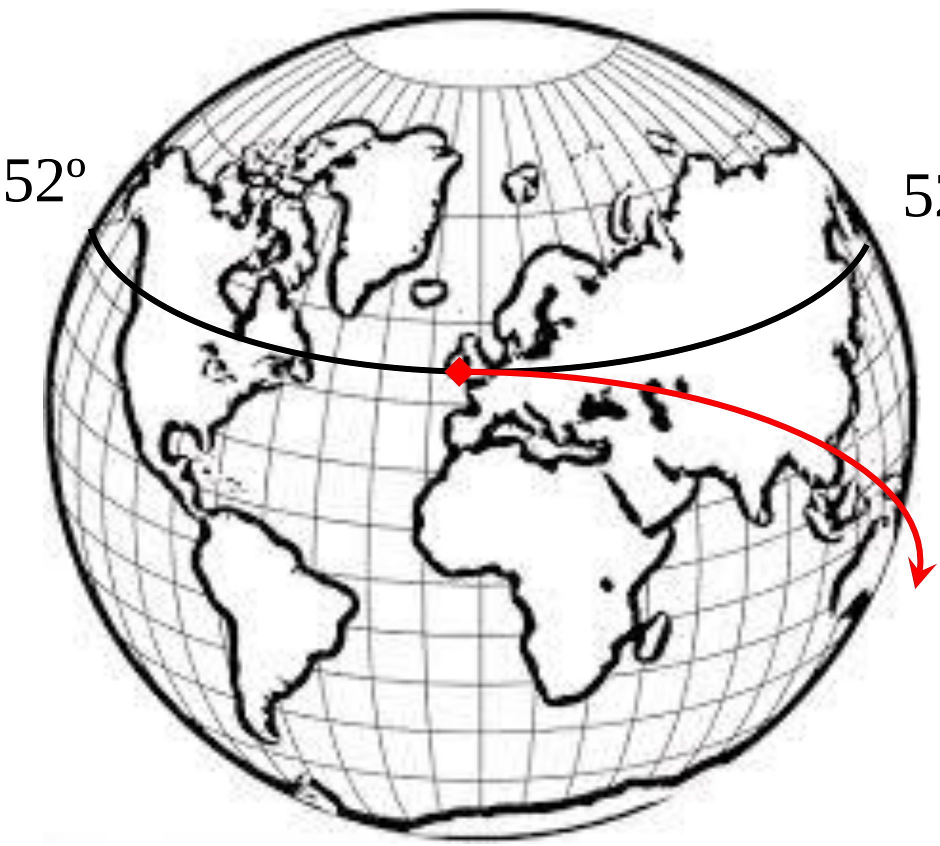
Posted on January 29, 2013

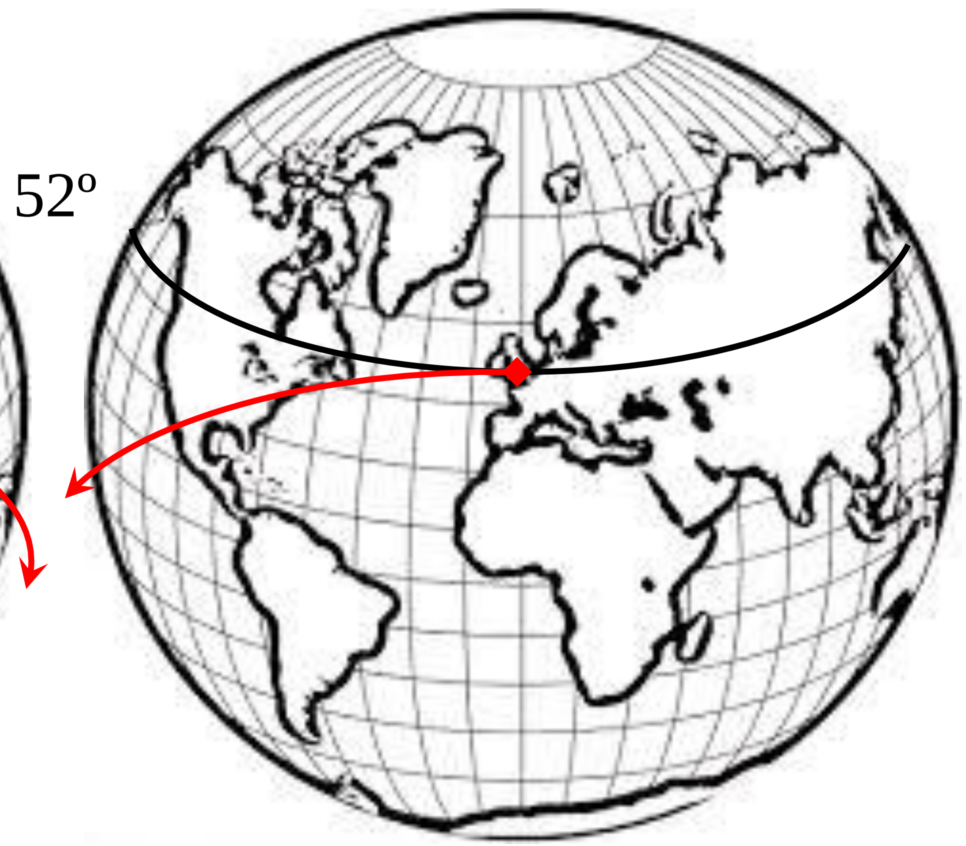
The Coriolis Effect is a very real optical illusion that appears to deflect a moving object from its trajectory when it is viewed in the context of a rotating reference frame.



Unfortunately, this optical illusion is incorporated into some very dubious scientific theories as if it were a real force. In the older scientific literature this optical illusion is frequently called the Coriolis Force, although the modern mainstream magicians usually try to be more subtle [in their misdirection] by calling it the Coriolis Effect.

The most stupid of all stupid Coriolis explanations: an airplane deflected when taking off eastward along a great circle:





Okay, deflected **to the right** on the Northern Hemisphere

. . . but deflected **to the left** when taking of in the other direction!

Summary:

1. The Coriolis force (acceleration) can be regarded as an extension to the Centrifugal (Centripetal) force for a body moving relative to the rotating system

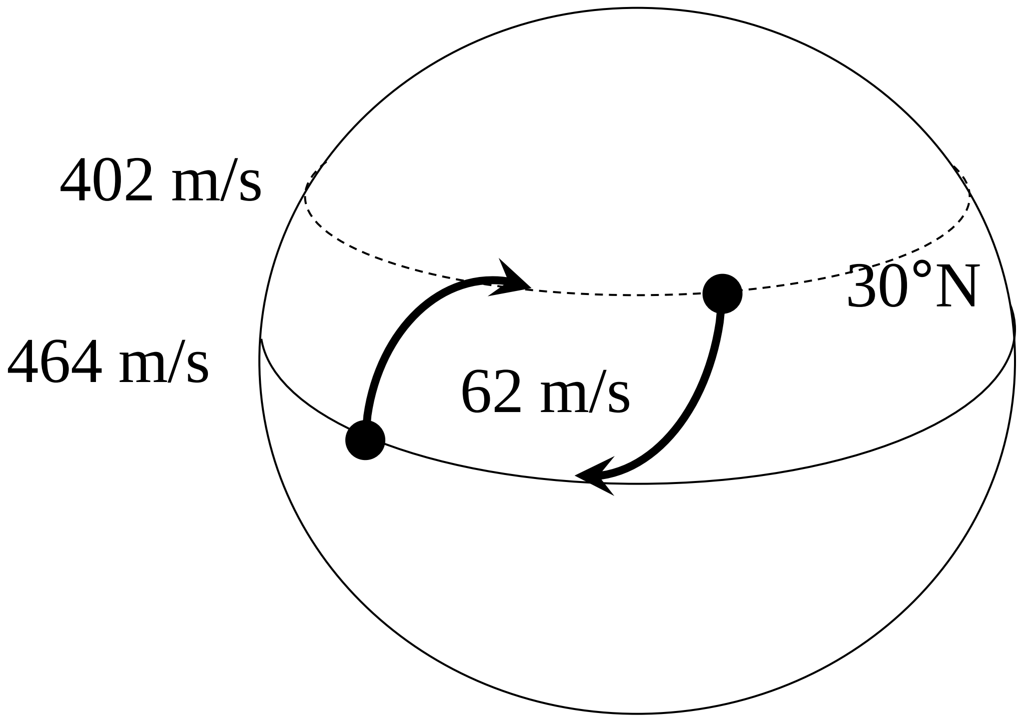
2. The Coriolis force is indeed "fictitious" and unable to "do work" but is therefore not some "optical illusion"

3. The Coriolis Effect could mathematically have been discovered already in the 1600s by Newton and even Kepler, hadn't their insights been blocked by "simple", but profound misconceptions.

What "simple" misconceptions block our visions today?

...perhaps this one?

The popular, but erroneous "Hadley's Principle" using conservation of absolute velocity



Common explanation: the excessive winds are retarded by friction

Break