The Coriolis Effect: Four centuries of conflict between common sense and mathematics, Part I: A history to 1885

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> The deflective force due to the earth's rotation, which is the key to the explanation of many phenomena in connection with the winds and the currents of the ocean, does not seem to be understood by meteorologists and writers on physical geography—William Ferrel[1]

Introduction: the 1905 debate

One hundred years ago the German journal *Annalen der Physik*, the same 1905 volume in which Albert Einstein published his first five ground breaking articles, provided a forum for a debate between three physicists, A. Denizot, M.P. Rudzki and L. Tesař on the correct interpretation of the Coriolis force[2]. The debate was complicated by many side issues, but the main problem was this: if the Foucault pendulum was oscillating, as it was often assumed[3], with its plane of swing fixed relative to the stars, why then was not the period the same, 23 hours and 56 minutes, everywhere on earth and not only at the poles? Instead it was 28 hours in Helsinki, 30 hours in Paris and 48 hours in Casablanca, i.e. the sidereal day divided by the sine of latitude. At the equator the period was infinite; there was no deflection. This could only mean that the plane of swing indeed was turning relative the stars. But how could then, as it was assumed, a "fictitious" inertial force be responsible for the turning?

One hundred years later, Einstein's five papers published in 1905 in *Annalen der Physik* are commonly used in undergraduate physics education whereas teachers and students, just like Denizot, Rudzki and Tesař, still struggle to come to terms with the Coriolis effect. This essay will sketch the complex and contradictory historical development of understanding the Coriolis Effect to about 1885. The continuing confusion since then is another story, but is undoubtedly related to our "Aristotelian" common sense. The reader's attention is directed to the copious endnotes for additional details.

The Coriolis effect – the basic mathematics

At the outset it seems appropriate to remind ourselves what is generally agreed on with respect to the deflective mechanism in rotating systems. A mass particle (m) that is *stationary* in a rotating system ($\mathbf{\Omega}$) at a distance $\mathbf{R}$ from the center of rotation, appears to an observer taking part in the rotation, to be affected by a centrifugal force $\mathbf{C} = - m(\mathbf{\Omega}\times(\mathbf{\Omega}\times\mathbf{R})$. If the particle is not stationary but moves ($\mathbf{V_r}$) relative to the rotating system, it appears to be affected by an additional force $\mathbf{F} = -2m\mathbf{\Omega}\times\mathbf{V_r}$. The cross product indicates that $\mathbf{F}$ is perpendicular both to the relative motion $\mathbf{V_r}$ and to the rotational axis $\mathbf{\Omega}$. For this reason, and not only because the force is inertial, the Coriolis force does not do any work, i.e. it does not change the speed (kinetic energy) of the body, only the direction of its motion. The statement that the Coriolis force "does not do any work" should not be misunderstood to mean that it "doesn't do anything"[4].

The cross product also tells us that only motions, or components of motions, perpendicular to $\mathbf{\Omega}$ are deflected. This will help us to explain why the Coriolis force on a rotating planet varies with the sine of latitude φ, $\mathbf{F}= -2m\mathbf{\Omega}\sin\varphi\mathbf{V_r}$, the "sine law." Since the Coriolis force is perpendicular to $\mathbf{V_r}$ a body in constant relative horizontal motion is driven into a circular path, or "inertia circle," with radius $R=V_r/2\Omega$ and a period of $\tau=\pi/\Omega$. At latitude 43º where $2\Omega\sin\varphi$ is approximately equal to $10^{-4}s^{-1}$, a motion of 10 m/s would correspond to an inertia circle of 100 km radius. The clearest example in nature of the Coriolis effect is inertia oscillations in the oceans (fig.1). Other clear examples involve equatorial upwelling, Taylor columns, gyroscopes and Lagrange points[5].

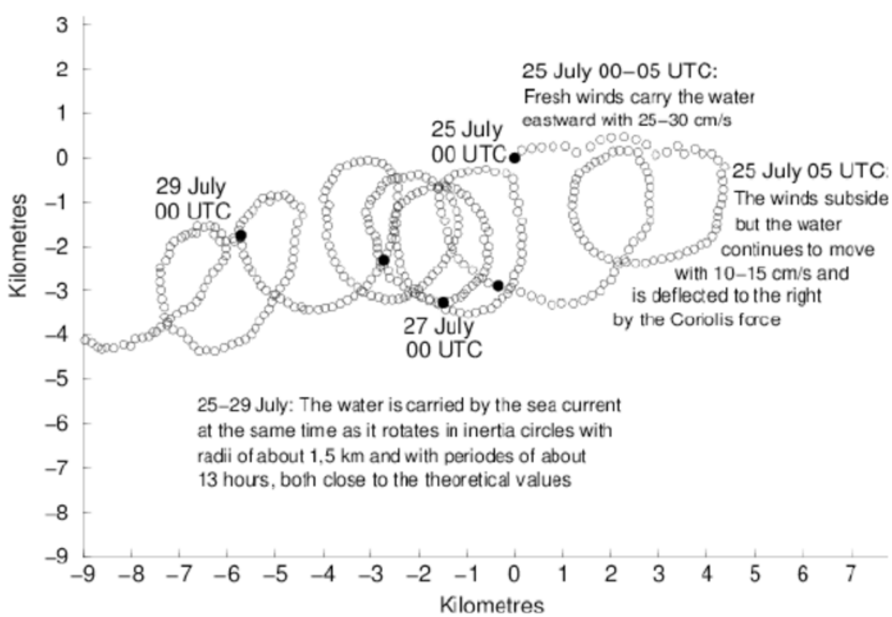

Fig. 1. Drifting buoys set in motion by strong winds tend, when the wind has decreased, to move under inertia and follow approximately inertia circles—in the case of steady ocean currents, cycloids. The example is taken from oceanographic measurements taken in summer 1969 in the Baltic Sea just southeast of Stockholm (Courtesy Barry Broman at the oceanographic department at SMHI).

In contrast to "normal" inertia, which resists changes in a body's motion, the Coriolis inertial force resists displacements by trying to return the body by a circular motion to the origin (fig. 2). *Any mathematical derivation or intuitive explanations of the Coriolis force, which is in conflict with the notion of the inertia circle motion, is therefore misleading, incomplete or wrong.*

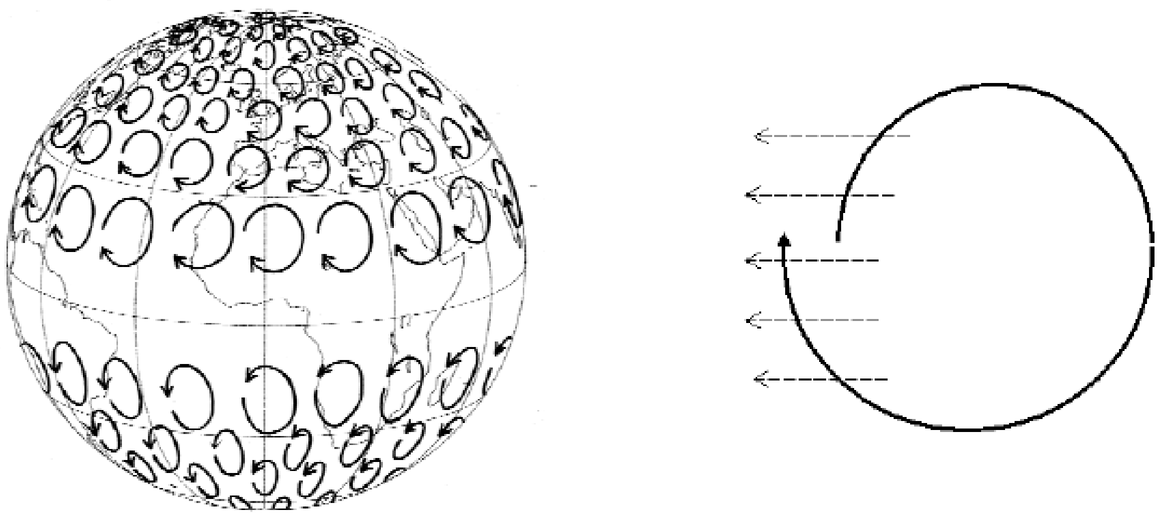

Fig. 2. a) The Coriolis force tends to restore a body to its initial position. This hinders the geographical displacement of air masses. The vortices and jet streams are the consequences of two opposing forces, one (the pressure gradient force) trying to equalize large-scale density contrasts, the other (the Coriolis force) trying to restore them. b) Due to the latitudinal variation of the Coriolis force, the inertia circles are actually spirals transporting mass westward, the so-called β-effect.

As a consequence of this latitudinal variation the inertial horizontal motion will be more curved in higher latitudes than in lower and lead to a westward migration of successive inertial evolutions. This "β-effect" accounts partly for the dynamics of large-scale planetary (Rossby) waves and the asymmetry of the Gulf Stream. But the Coriolis effect is only one part of a three dimensional deflective mechanism discovered and discussed at separate historical epochs:

1. the horizontal deflection of vertical motion in the 17th and early 19th century,
2. the vertical deflection of horizontal motion (the Eötvös effect) in the late 19th and early 20th century, and
3. the horizontal deflection of horizontal motion (the Coriolis effect) from the early18th century until our times.

Let us consider each of these mechanisms in greater detail.

Horizontal deflection of vertical motion

During the 17th century the possible deflection of falling objects was considered a means of proving or disproving the Copernican theory that Earth rotates and not the stars. The debate became known in England through a memorandum by David Gregory in 1668. In November 1679 Robert Hooke, in his capacity as newly elected Secretary of the Royal Society, tried to draw Isaac Newton into a discussion on the motions of the planets and comets. But Newton had something else on his mind, “a fancy of my own,” that the horizontal deflection of objects dropped from a high altitude could stand as proof of the Earth's rotation. Newton had just returned from a long vacation at his family home in Lincolnshire where he might have been inspired by watching apples fall in the garden[6].

The exchange of letters that followed during the winter 1679-80 shows that Newton had not yet ascquired a deeper understanding of celestial mechanics. His first idea was that a falling object would follow a trajectory that, in principle, approaches the centre of the earth in a *spiral*. Thanks to Hooke, he came to realize that that the fall of the body must be treated as an *elliptic* orbit with the centre of the Earth in one of its foci (fig. 3).

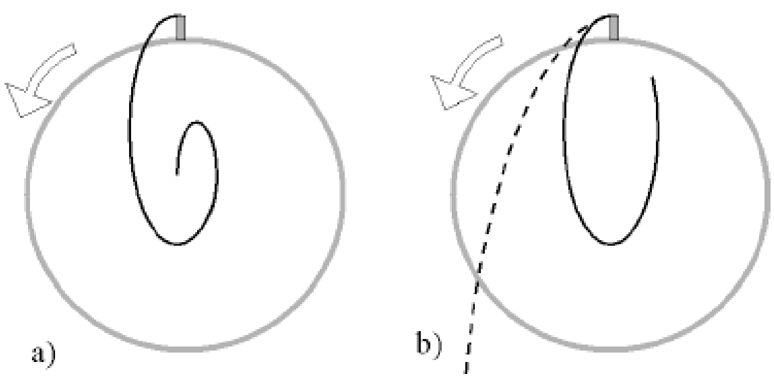

Fig. 3. a) Newton’s first intuitive idea was that the trajectory of a falling object would spiral towards the centre of the earth, b) just considering conservation of absolute velocity would result in a parabolic path (dashed line), while the true trajectory would be an ellipse (solid line).

From the insight that a falling object in absolute space follows the same type of orbit as any of the planets or comets around the Sun, it was possible for Newton to infer that the motions of all terrestrial and extra-terrestrial bodies might be controlled by the same mechanism, *universal gravitation*. When Newton was looking for what we would now call the Coriolis effect, he found the laws of motion[7].

More than a century after Newton, in 1803, an experiment was conducted in Schlebusch, Germany that attracted the interest of the scientific community. Twenty-nine iron pebbles were dropped into a 90-meter deep mineshaft. The average deflection was estimated to be 8.5 mm. Before the event the 24-year Carl Friedrich Gauss and the 53-year Pierre Simon de Laplace volunteered to calculate the theoretically expected deflection. Both came up with an expected deflection of 8.8 mm by deriving the full three-dimensional equation for motions on a rotating earth. They specifically pointed out what mechanisms were responsible for the deflection. Gauss and Laplace must therefore be regarded as the first scientists to contribute to the understanding of the Coriolis effect and the proof of the rotation of the earth. In 1831 the experiment was repeated in a 158.5 m deep mine in Freiburg, Saxony. From 106 drops an average deflection of 28.3 mm was estimated, close to the theoretical value of 27.5 mm[8].

Well into the 20th century there was a controversy over a possible slight southward deflection, which turned up in some experiments and derivations. The heart of the matter depends on how we define "vertical". Due to the non-spherical shape of the Earth the upper part of a plumb line is at a slightly (very slightly!) higher latitude than the plumb itself[9].

Vertical deflection of horizontal motion (The Eötvös Effect)

In the early twentieth century a German team from the Institute of Geodesy in Potsdam carried out gravity measurements on moving ships in the Atlantic, Indian and Pacific Oceans. While studying the results the Hungarian nobleman and physicist Lorand Roland Eötvös (1848-1919) noticed that the readings were lower when the boat moved eastwards, higher when it moved westward. He identified this as primarily a consequence of the rotation of the earth.

To demonstrate the effect, Eötvös constructed a balance with a horizontal axis, where, instead of pans, weights are attached to the end of the arms. When the balance is rotated the weight moving towards the west will become heavier, the one moving towards the east lighter and will deflect from its state of equilibrium. This proof of the earth's rotation is perhaps of greater significance than Foucault's pendulum experiment since it also works on the equator.

In 1908 new measurements were made in the Black Sea on two ships, one moving eastward and one westward (fig. 4). The results substantiated Eötvös' claim. Since then geodesists use the correction formula

$$a_r = 2\Omega u \cos\varphi + \frac{u^2 + v^2}{R}$$

where a_r is the relative acceleration, R is the radius of the earth. The first term is the vertical Coriolis effect, the second term reflects the upward centrifugal effect of moving over any spherical surface, also non-rotating ones.

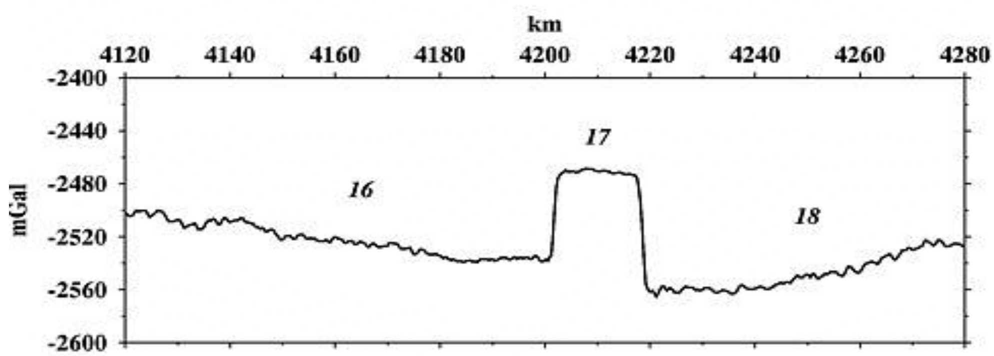

Fig. 4. The Eötvös effect measured by a French research vessel in the South Indian Ocean. The ship is first moving slowly in a westerly direction (16), then faster westward (17), and finally slowly eastward (18). The units on the y-axis indicate gravity and are inversely proportional to the ship's weight. Figure courtesy of Dr Helen Hebert, Laboratoire de Détection et de Géophysique, Bruyères-le-Chatel, France.

The possible relevance of the "Eötvös effect" for meteorology was discussed ten years *before* Eötvös' discovery. In 1894-97 the Swedish meteorologist Nils Ekholm hypothesized that the vertical deflection of horizontal motion played an important role in atmospheric dynamics[10].

Horizontal deflection of horizontal motion (before Coriolis)

In 1735 George Hadley (1686-1768) suggested that, since the surface of the earth at the equator moved faster than the surface at higher latitudes, air that moved towards the equator would gradually lag behind and be observed as a NE wind north of the equator and a SE wind south of the equator (fig. 5). Hadley's model, although a great step forward for its time, is incorrect for three reasons:

1. Bodies moving under frictionless conditions on the surface of a rotating planet will not conserve their absolute velocity.
2. Even if they did, Hadley's scenario will mathematically explain only half the Coriolis force.
3. Finally, Hadley's explanation suggests that the deflection only occurs for meridional motion. The fact that the observed winds in the Tropics were only a fraction of what Hadley's model suggested, was explained by the effect of friction.

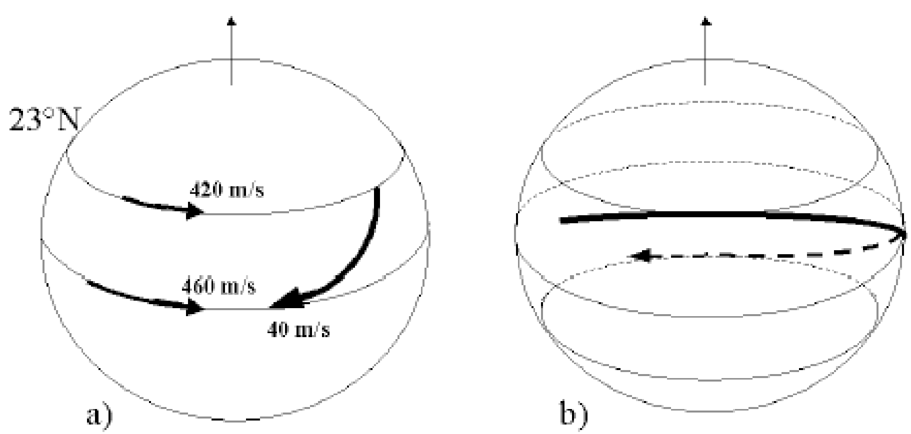

Fig. 5. Two erroneous images of the deflection mechanism: a) conservation of absolute velocity and b) motion along great circles. The latter appears to work for eastward motion, but not for westward motion.

Some years after Hadley, in 1742, the French mathematician A.C. Clairaut (1713-65) discussed the deflection of relative motion on a flat rotating platform, also in terms of conservation of absolute velocity. He therefore obtained the same underestimation as Hadley.[11]

Pierre Simone Laplace (1749-1827) is often considered to be the "true" discoverer of the Coriolis effect since his 1775-76 papers on the equations of motion on a rotating planet contain the 2Ω-term[12]. But Laplace did not make any correct physical interpretation of this term. On the contrary, in his physical explanations of the Trade winds he used Hadley's erroneous model[13].

It is not clear if Laplace in 1775-76 knew about Hadley's 1735 paper or if he independently had reached the same "common sense" explanation. It is normally thought that Hadley's paper lay dormant till the end of the 18th century when John Dalton (1766-1844) championed it in 1793. According to Dalton the Swiss scientist Jean André De Luc (1727-1817), who lived in England, had thought along the same lines some 15 years earlier[14].

Hadley's explanation was later adopted by the German meteorologist Heinrich W. Dove (1803-79) and became known as the Dove-Hadley theory. Dove gained his reputation from his "Law of the wind turning" (Drehungsgesetz) according to which the wind locally tended to change from S to W to N to E to S, i.e. locally to the right. This "law" only reflected the climatological fact that most cyclones travel eastward.

In 1843 the American Charles Tracy tried to show that the deflection was also valid for east-west motion. Erroneously, he thought the spherical shape of the earth was the prime reason for the deflection. He therefore argued that inertial motion should follow a great circle and for that reason eastward motion deviated to the south, to the right.[15] Tracy evaded the embarrassing fact that that his model suggests that westward motion is deflected to the *left* (fig. 5b).

Gaspard Gustave Coriolis and "his" force

At the start of the Industrial Revolution a radical and patriotic movement developed in France to promote technical development by educating workers, craftsmen and engineers in "mechanique rationelle." Gaspard Gustave Coriolis (1792-1843), a well-respected teacher at l'Ecole Polytechnique in Paris, published in 1829 a textbook which presented mechanics in a way that could be used by industry. Here we find for the first time the correct expression for kinetic energy, $mv^2/2$. Two years later he established the relation between potential and kinetic energy in a rotating system[16].

In 1835 Coriolis published the paper that would make his name famous: "Sur les equations du mouvement relatif des systemes de corps," where the "deflective force" explicitly appears. The problem Coriolis set out to solve was related to the design of certain types of machines with separate parts, moving relative to the rotation. Coriolis showed that the total inertial force is the sum of two inertial forces, the common centrifugal force $\Omega^2 R$ and the "compound centrifugal force" $2\Omega V_r$, the "Coriolis force"(fig.6) [17]. This is in agreement with the standard equation

$$m\mathbf{a_r} = m\mathbf{a} - 2m\mathbf{\Omega}\times\mathbf{V_r} - m\mathbf{\Omega}\times(\mathbf{\Omega}\times\mathbf{R})$$

where the last two terms for inertial motion ($m\mathbf{a}=0$) represents the total inertial force.

Coriolis was not as interested in "his" force as much as we are. He only valued it in combination with the common centrifugal force. In this view the Coriolis force is the *difference* between two inertial forces, or rather the part of the total inertial force, which is not explained by the common centrifugal force (fig.6).

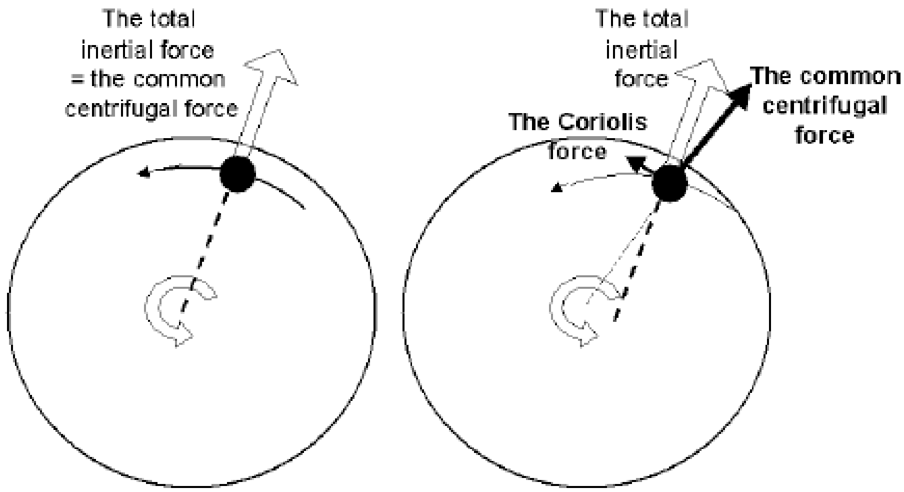

Fig. 6. An object fixed to a rotating platform follows a curved trajectory and is affected by a total inertial force, which is the “common” centrifugal force. The body can move along the same trajectory, also as a consequence of a combination of the rotation and the motion relative the platform. The total inertial force is the same, but is now the sum of the common centrifugal force and the “Coriolis force.”

French investigations before and after the Foucault experiment 1851

Coriolis’ 1835 paper directly influenced Simon P. Poisson (1781-1840) who, a few years later, made an analysis on the deflection of artillery shells[18]. Coriolis’ and Poisson’s papers were highly mathematical, however, and were not easily accessible. In 1847 the French mathematician Joseph L. F. Bertrand (1822-1900) suggested to the French Academy a “simplified” derivation. He made two common sense, but erroneous, assumptions: a) conservation of absolute velocity and b) the deflective acceleration on a rotating turntable is constant and only due to the Coriolis effect[19]. The first assumption underestimates the Coriolis effect and the second overestimates it - *so the errors cancel out* (fig.7). Bertrand’s derivation became popular and entered meteorology in the 1880s. If we today are grappling to understand the Coriolis effect, one source of confusion is this “simple” but deceptive derivation, which appears to justify two frequent misconceptions.

On pages 6 and 21-24 in his 1838 paper Poisson ruled out any effect on a swinging pendulum. This was refuted by Foucault's historical pendulum experiment in 1851, which is often quoted as a *clear* observational evidence of the Coriolis effect, since it is thought that the swing of plane is fixed versus the stars. As discussed above, the plane of swing indeed turns versus the stars. That means that a real force is doing work, the component of gravitation perpendicular to Ω. Only at the poles is this component zero[20].

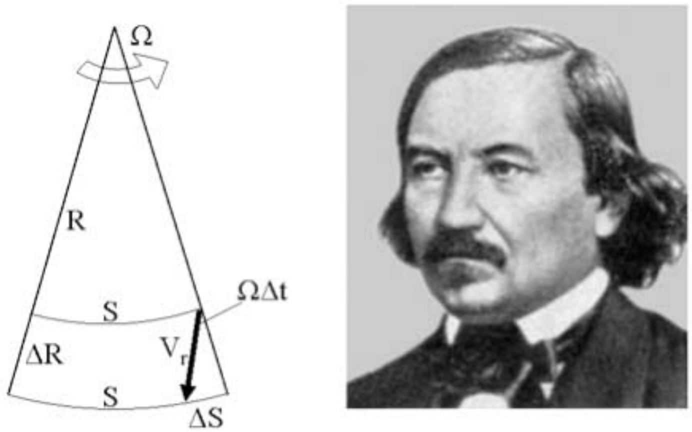

Fig. 7. Joseph Bertrand and his "simplified" geometrical derivation. An object on a turntable at a distance R from the centre of rotation is moving radially outwards with a relative velocity $V_r = \Delta R/\Delta t$. Due to the rotation Ω the relative velocity is supposed, due to an erroneous assumption about conservation of velocity, to be subject to a deflective acceleration a, which erroneously is assumed constant. The deflected distance ΔS during Δt can be expressed both as $\Delta S = a(\Delta t)^2/2$ and $\Delta S = \Omega\, \Delta R \Delta t$ which yields $a = 2\Omega V_r$.

Anyone looking for a "simplified" derivation would have been wise to consult the British mathematician O'Brien, one of the early proponents of vector notations. He made in April 1852 what seems to be the first algebraic derivation of the Coriolis force by making use of the relation $\frac{d\mathbf{A}}{dt} = \left(\frac{d\mathbf{A}}{dt}\right)_r + \mathbf{\Omega} \times \mathbf{A}$ which applied on a position vector **R** yields $\frac{d\mathbf{R}}{dt} = \left(\frac{d\mathbf{R}}{dt}\right)_r + \mathbf{\Omega} \times \mathbf{R}$ or $\mathbf{V} = \mathbf{V}_r + \mathbf{\Omega} \times \mathbf{R}$ and applied on a velocity vector $\mathbf{V} = \left(\frac{d\mathbf{R}}{dt}\right)_r$ yields

$$\frac{d\mathbf{V}}{dt} = \left(\frac{d\mathbf{V}}{dt}\right)_r + \mathbf{\Omega} \times \mathbf{V} = \left(\frac{d\mathbf{V}_r}{dt}\right)_r + \left(\frac{d(\mathbf{\Omega} \times \mathbf{r})}{dt}\right)_r + \mathbf{\Omega} \times \mathbf{V}_r + \mathbf{\Omega} \times (\mathbf{\Omega} \times \mathbf{r})$$

which simplifies into

$$\frac{d\mathbf{V}}{dt} = \left(\frac{d\mathbf{V}}{dt}\right)_r + 2\mathbf{\Omega} \times \mathbf{V}_r + \mathbf{\Omega} \times (\mathbf{\Omega} \times \mathbf{r}) \quad \text{and} \quad \left(\frac{d\mathbf{V}}{dt}\right)_r = \frac{d\mathbf{V}}{dt} - 2\mathbf{\Omega} \times \mathbf{V}_r - \mathbf{\Omega} \times (\mathbf{\Omega} \times \mathbf{r})$$

where the term $-2\Omega \times V_r$ in O'Brien's words, was "the force which must be supposed to act as a correction for the neglected rotation"[21].

The mechanical and geophysical debates around 1860

In autumn 1859 the French Academy had a comprehensive debate about the effects of the earth's rotation on terrestrial motion. The triggering factor seems to have been a inference by the Baltic-German naturalist Karl Ernst von Baer (1792-1876) that the meandering of the north-south running Siberian rivers was due to the rotation of the earth[22]. Von Baer, who was a firm believer in the Hadley-Dove model, rejected any notion that the rotation of the earth had any effect on the rivers which flow from east to west. The French Academy had problems in tallying this with the meandering of east-west flowing rivers like the Seine and Loire. It might have been the meanders of their own rivers that sowed the first seeds of doubt in French minds about the correctness of Hadley's model.

Jacques Babinet (1794-1872) admitted that "everybody and first of all he" had been "completely wrong" not to realize that the deflective mechanism worked for all directions, not just north-south. Joseph Bertrand, on the other hand, denied there was any deflection of east-west motion. Babinet tried to derive the deflection of east-west motion, but made an error and only got $\Omega V_r \sin\varphi$, half of the correct value. Charles E. Delaunay (1816-72), a prestigious astronomer and author of an influential textbook in mechanics, then made a comprehensive pedagogic presentation. He reminded the audience that the "weight" of a stationary body was the combination of the gravitational attraction and the centrifugal effect of the earth's rotation. But when the body moved relative to the earth "things change completely":

> The compound centrifugal force goes to combine its effect with the one that is due to the effect of the weight of the body. And the result thereof in the movement are the changes which reveal to us the existence of the rotation of the earth.

Delaunay showed, in part by quoting Coriolis' 1835 paper, that the deflection worked in all directions and was proportional to 2Ω, double the angular velocity. He also suggested that flowing water in a canal would have an inclined surface with a higher level on the right side. The incorrigible Bertrand found the concept of "fictitious force" useless since it could not explain "real causes." He was also critical of Delaunay's explanation of gravity and finally called out that "everybody seemed to admit the absence of any measurable influence of the earth's rotation on the flow of water."

Guillame Piobert (1793-1871) reminded the audience about Poisson's 1838 work, which showed that the deflection was to the right. Babinet listed the Foucault's pendulum, the deflection of projectiles, falling objects and many other physical examples of the deflective mechanism. One was "the effect of the wind on a lake, that according to Mr. Foucault's law, tends to impose a movement always directed in the same direction, independent of the direction of the wind"[23]. Both Babinet, and after him Charles Combes (1801-72), presented an expression for the inclination of river surfaces which was essentially the geostrophic balance (fig.8). Combes introduced the concept of inertia circle and showed that that at 45° latitude a 3 m/s motion would move around in a circle of 29 kilometers radius [24].

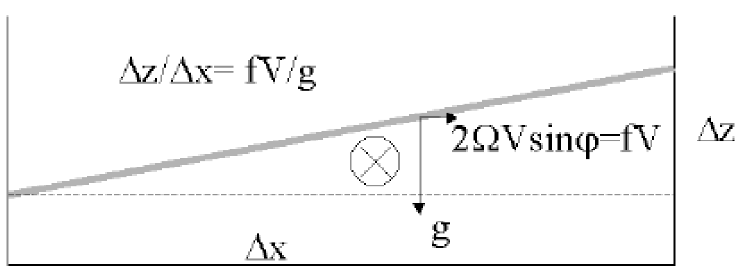

Fig. 8. The slope of the surface of river with a flow V is proportional, per unit mass, to the Coriolis force (fV) and the weight of the water (g), which yields the geostrophic relation V=g/f dz/dx.

The French discussions on deflection of flowing water in rivers are related to the problem of the sideways acceleration of *constrained* motion, like trains on rails. This was taken up by the Austrian-Russian scientist Nikolai D. Braschmann (1796-1866) and promoted by the German professor Georg Adolph Erman (1806-77)[25].

William Ferrel and the geophysical implications of Foucault's experiment

At this time an unknown schoolteacher in Nashville, Tennessee, USA applied the equations of motion on a rotating sphere to meteorological problems, in particular the global circulation. William Ferrel (1817-91) a farmer's son from Pennsylvania was in his late 30s when two books challenged him to venture into a new direction. One was Laplace's *Mechanique Celeste*; the other was Matthew F. Maury's 1855 *Physical Geography of the Sea* which Ferrel found unscientific. In 1856 Ferrel argued, in the first of a series of articles, that the motion of the atmosphere was governed by four mechanisms: the change of density distribution due to differential heating, the flow of air from high pressure to low pressure and the two "forces" due to the earth's rotation, both known to Ferrel from Laplace's tidal equations.

One of these forces Ferrel recognized from "Hadley's theory", about which the reader was "no doubt familiar." He had not yet discovered Hadley's error in assuming conservation of absolute motion and only criticized him for having disregarded the deflection of east-west motion. Ferrel identified "a new force" as the unbalanced centrifugal force due to the combination of the earth's eastward rotation and any east-west relative motion. Ferrel's two forces we today know as the east-west and the north-south components of the Coriolis effect. Due to an erroneous derivation or misunderstanding of Laplace's equations, Ferrel had got the impression that first force was smaller than the second by a factor of $\cos\varphi$.

Any misinterpretations were soon rectified in two brief papers published in *The Astronomical Journal* in January 1858 where he correctly derived expressions for the deflective mechanism in all three dimensions. He stated what became known as "Ferrel's Law": *If a body is moving in any direction, there is a force arising from the earth's rotation, which always deflects it to the right in the northern hemisphere, and to the left on the southern*. He briefly discussed the deflection of projectiles and established that falling objects only deviate in the east-west direction.

Ferrel seems to be the first scientist to identify the inertia circle motion: "*If a body receives a motion in any direction, it describes the circumference of a circle, if the range of motion is small, the radius of which is determined by [$V/2\Omega\sin\varphi$]; and the time of its performing a revolution is equal to the time of the earth's rotation divided by twice the sine of the latitude.*" He realized that the larger the range of motion, the more it deviates from a circle. But from the fact that the curve must always be symmetrical on each side of the central median, he wrongly assumed that the body would return to the point from which it started, and thus did not discover the west drift caused by the β-effect.

One year later Ferrel published a detailed mathematical derivation not only of the deflective mechanism but also of the possible consequences for the general circulation of the atmosphere. At the end he makes the important observation that the effect of the earth's rotation is to constrain the air mass flow by inertia circle motion, in particular holding back the exchange between lower and higher latitudes:

The motion towards the poles in the upper regions causes an eastward motion which gives rise to a force toward the equator, and which, consequently, counteracts the motion toward the poles, and the motion toward the equator produces a westward motion which gives rise to a force acting in the direction of the poles, which counteracts the motion toward the equator.

From this insight he was able to infer that if the rotation decreased there would be "a sweeping hurricane from the pole to the equator." He also ascribed "the maximum accumulation of the atmosphere near the parallel of 30" (the subtropical highs) to "the heaping up of the atmosphere"[26].

Growing opposition to the Hadley model, 1860-80

By 1860 it was mathematically established that the deflective effect of the earth's rotation was $2\Omega V\sin\varphi$, that it worked in all directions, and that it drew a moving body into an inertial circle with radius $V_r/2\Omega\sin\varphi$. Still, Hadley's explanation dominated the literature, in particular in Germany where H.W. Dove was still supreme. But the criticism was growing.

In 1869 Adolph Mühry (1810-88) rejected the prevailing "mathematical conception" of deflection due to meridional rotation differences, since they were based on inertial motion from an impulse and did not consider the real physical processes. According to Mühry, "One cannot compare the motions of the air with a fired cannonball."[27] In the 1860's Ferrel's work started to become noticed in Europe. Dove, who could not distinguish between local and individual derivatives, claimed that his "law", which indicated a local turning to the right of the wind, was consistent with "Ferrel's law". Some questioned if the rotation of the earth had any influence on the weather at all since the Hadley-Dove explanation predicted 40 m/s easterly winds at the equator.

A great step forward in the development of meteorology in Germany was the founding of the Deutsche Seewarte in Hamburg in 1875. Its first director Georg von Neumayer (1826-1909) encouraged the Norwegian meteorologist Henrik Mohn (1835-1916) to issue a German edition of his book *Om vind og Vejr.* When Mohn in 1876, together with his countryman C.W. Guldberg, published *Études sur les Mouvements de l'Atmosphère*, Julius Hann (1839-1921), editor of the Vienna-based *Zeitschrift für Meteorologie*, requested and was given a more accessible German version.

In 1877 Professor Joseph Finger at University of Vienna set out to derive the equations of motion on a non-spherical, spheroid earth. In 1877 Carl Benoni, professor in Lemberg (then in Austrian Poland, later Lwow in Poland and now Lviv in Ukraine) in defense of Dove, stated that Ferrel's Law was "obviously incorrect". It was, according to Benoni, "completely clear" that, when the air flows along any latitude circle on the earth's surface, the rotational speed does not change and "consequently there can be no deflection due to the earth's rotation"[28].

Two years later, in 1879, Dove died and it is perhaps no coincidence that the first attempt to seriously question the Dove-Hadley's explanation now saw the light of day in Germany. By this time Köppen was director of the research department at Deutsche Seewarte, which became the leading centre for synoptic and dynamic meteorology. One of their scientists was Adolf Sprung (1848-1909), who rose to become the leading theoretician in meteorology.

Adolph Sprung's contributions

In 1879-80 Adolph Sprung made three contributions to the understanding of the Coriolis effect.[29] He succeeded in that which Ferrel had failed to do: make the correct interpretation of the latitudinal dependence of the Coriolis force resulting in a westward spiraling trajectory (fig. 9).

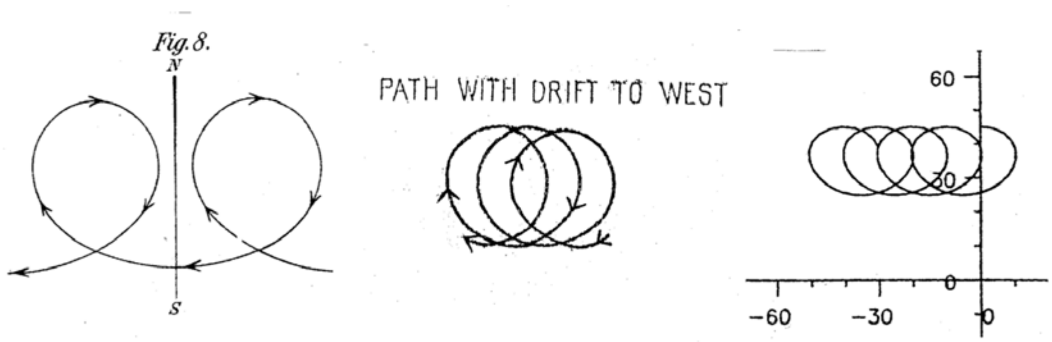

Fig. 9. Inertial motion, away from the equatorial region, according to, from left to right Sprung (1879), Whipple (1917) and Paldor (1988).[30]

Sprung generalized the concept of the Coriolis force by showing it to be, in the spirit of Coriolis (1835), but probably unaware of his work, an extension of the centrifugal force. He did so by deriving the equations for a relative motion on a flat turntable. He then gave the turntable a parabolic form and showed how this neutralized the common centrifugal force and left only the Coriolis force driving the moving object into inertial circles (fig. 10).[31]

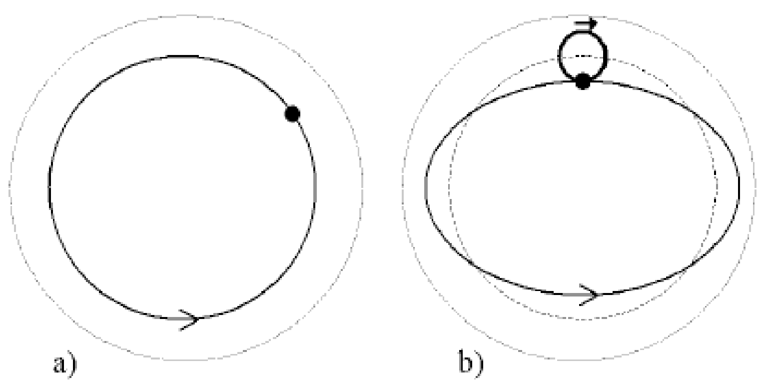

Fig. 10. Absolute and relative motion of a ball in a parabolic shaped turntable rotating anti-clockwise. a) A ball, stationary in the rotating system, appears from outside to be moving in a circle (full line); b) the ball has been given an impetus and is in the rotating system moving in an clockwise inertia circle, from outside it appears to be moving in an ellipse. (The vertical movement of the ball is neglected since it introduces a slow anticlockwise precession of the ellipse).

Sprung then considered a rotating spheroid and showed that, although the centrifugal force Ω^2R changed by a factor $\sqrt{1-\varepsilon^2\sin^2\varphi}^{-1}$ (ε= the eccentricity), the Coriolis force was not affected. Since a component of gravitation, due to the non-spherical form of the earth, always balanced the centrifugal force in the horizontal plane, whatever its magnitude, the only extra fictitious force that had to be taken into account was the Coriolis force (fig. 11).

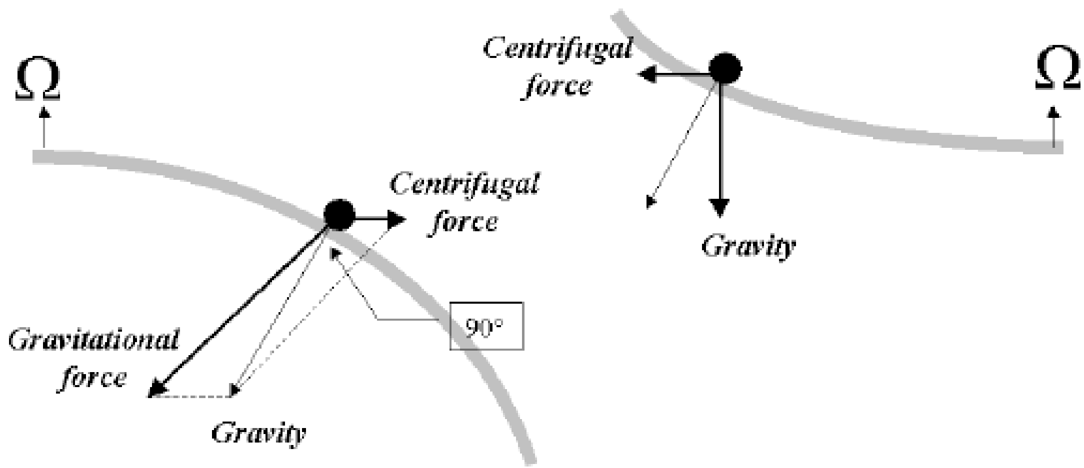

Fig. 11. For a stationary object on a rotating planet or a rotating parabolic surface the horizontal components of the centrifugal force is balanced by the horizontal component of the gravitational force of the planet and, on the turntable, the horizontal component of the weight of the body. In both cases the component of gravitation and gravity, perpendicular to the rotational axis, equals the centrifugal force.

Finally, Sprung suggested a different mathematical way to deal with the Coriolis effect: *abandon the notion of relative motion and derive the acceleration in a fixed system*. In other words: find the acceleration to *prevent* a relative motion from being deflected! This acceleration, achieved by some real force is, by some convention, called the Coriolis acceleration and written $+2\Omega\times V_r$. The derivation is simple and Newton could have done it by the same Euclidean method he used to find the centripetal acceleration in “Principia” (fig.12).

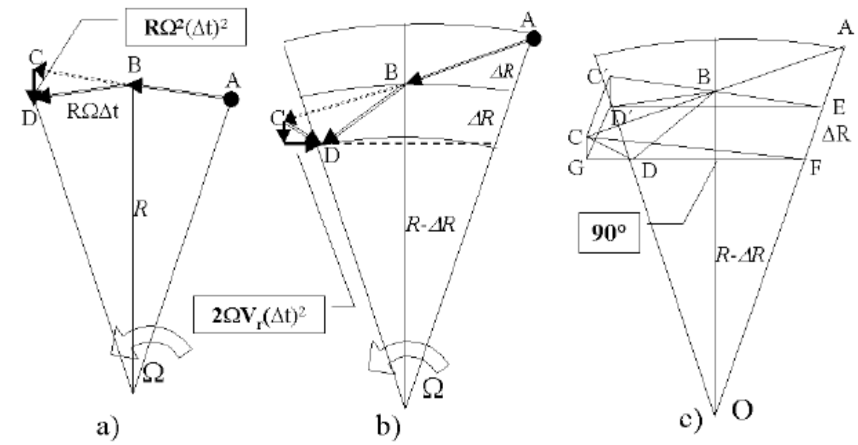

Fig. 12. a) Newton's derivation of the centripetal acceleration: a body is over time Δt by the rotation Ω carried from A to B and would, by pure inertia, in the next time interval have continued to C, had it not been affected by a centripetal impetus which brought it to D. By simple geometry one gets $CD=R\Omega^2(\Delta t)^2$. In case b) the body is also moving radially with relative velocity V=ΔR/Δt and would have continued from B to C, had it not been affected by a centripetal impetus which brought it to D. Since ACF is proportional to ABE and AF=2AE, it follows that CF=2BE≈2ΩRΔt and since DF ≈ 2Ω(R-ΔR)Δt it follows that GD=GF-DF≈CF-DF=2ΩΔRΔt=$2\Omega V(\Delta t)^2$. It can easily be shown that D´EFG is a parallelogram with GF perpendicular to OB, so GD ⊥ BO and the Coriolis acceleration is perpendicular to the relative motion – but to the left!

Sprung does not seem to have been aware that Leonard Euler already in 1749 derived analytically what was essentially the Coriolis acceleration (fig. 13).[32]

Otons l'une de ces équations de l'autre, & nous aurons:
$(2drd\Phi + rdd\Phi)(\tan\Phi + \cot\Phi) = 0$
ou bien $2drd\Phi + rdd\Phi = 0$
Multiplions la premiere par cot Φ & la feconde par tang Φ, & nous aurons en les ajoutant enfemble :
$(ddr - rd\Phi^2)(\cot\Phi + \tan\Phi) = -\frac{1}{2}Vdt^2(\cot\Phi + \tan\Phi)$
ou bien $ddr - rd\Phi^2 = -\frac{1}{2}Vdt^2$.

Fig. 13. Leonard Euler's 1749 derivation of the Coriolis acceleration (2drdφ) and the so-called Euler acceleration (rddφ), which is the acceleration due to variations in the angular velocity.

If we multiply Euler's equation $2\frac{dr}{dt}\frac{d\varphi}{dt}+r\frac{d^2\varphi}{dt^2}=0$ with r and integrate, we will get $r^2\frac{d^2\varphi}{dt^2}=c$ which is today called "angular momentum conservation," but is in fact only Kepler's second law, "The Area Law" ("Flächensatz" or "Loi des Aires"), which is a scalar version of angular momentum conservation.

Another 120 years of conflict between mathematics and common sense

After Dove's death the effects of the rotation of the earth was very much "in the air." In 1879, when Sprung published his first monograph, a colleague Max Thiesen at the Prussian Meteorological Institute in Berlin, reviewed Finger's 1877 and Ferrel's 1859-1860 papers. Like Ferrel he regarded the deflection as a consequence of an imbalanced centrifugal force. He also used the concept of inertial circle[33].

Thiesen's article started a debate with Sprung which soon came to involve other meteorologists and physicists like Fr. Roth in Buxtehude, H. Bruns in Leipzig, K. Weirauch in Dorpart (Tartu in today's Estonia) and Julius Hann. Hann was at this time working on his *Handbuch der Klimatologie* and Sprung, upon the request of von Neumayer, on a groundbreaking textbook on dynamic meteorology. The debate made to a large extent reference to the motions of the atmosphere and oceans, both to illustrate certain properties of the Coriolis effect, but also to explain the atmospheric motions. Since this was done without always distinguishing between forced and inertial motion, the discussion could become quite confused.

Among the "paradoxes" that served to complicate the debate (excluding "mysteries" with the Foucault pendulum):

1. The "Coriolis force" does not depend on the radius of the earth, which therefore can be treated as a perfect sphere. But on a rotating spherical planet every object would be accelerated towards the equator - so there would be no "Coriolis effect."
2. Common sense tells us that it is through friction that a body "knows" it is moving over a rotating surface. But how much friction is "needed"? The Coriolis force is after all an inertial force and friction would complicate the mathematics…
3. Hadley's model implied 40 m/s Trade winds. But we know that his principle of conservation of absolute velocity was wrong, whereas the principle of conservation of absolute angular momentum is correct. But this principle yields 80 m/s Trade winds!

The German debate is interesting and thought provoking, but we have to stop here. The development up to 1885 can be treated as historical since the problems have been resolved. However, those discussed since *are still unsolved or at least controversial*. By 1885 almost everything about the Coriolis effect was known and widely published. The following 120 years, i.e. up to now, have seen a constant repetition of the discussions and debates of the preceding 120 years, with interesting additions provided by new technological proposals such as a rotating space station.[34]. In general we meet the same attempt then as now to reconcile mathematics with Hadley's, Bertrand's and others' flawed but intuitively appealing "common sense" explanations and conceptual models. One can wonder why?

It is often said that dynamic meteorology is difficult because of its mathematics, which contains non-linear differential equations. But while the non-linearity makes *predictions* difficult because of the "Butterfly Effect," the mathematics of the Coriolis effect is not particularly difficult and is *linear*. Euler's equation has been used in celestial mechanics for 250 years without causing confusion and endless debates. But these equations and concepts relate to an *absolute* motion, whereas the Coriolis force relates to *relative* motion, which seems to be difficult to comprehend intuitively. Even more out of reach of everyday life is frictionless motion.

Correspondents to the *American Journal of Physics* have noted that university students cherish naïve, Aristotelian ideas about how and why things move. For example, many students believe that forces keep bodies in motion and, conversely, that in the absence of forces bodies are at rest.[35] There are no reasons to assume that students in meteorology are immune to this "Aristotelian physics" as it has been called. The crux of the matter does not lay in the mathematics but in our common senses which are still Aristotelian.

Endnotes

[1] William Ferrel, *Nature* 4 (1871).

[2] The controversy started at the University of Cracow (then in Austria) over a publication by A. Denizot, "Theorie der relativen Bewegung mit einer Anwendung auf das Problem der Bewegung eines Körpers an der Oberfläche der rotierenden Erde sowie auf den Foucaultschen Pendelversuch," *Bull. Acad. Sci. d. Cracovie* (Dec. 6, 1904): 449. Critical comments were made by M.P. Rudzki in the same publication in March 1905. Then followed a debate in *Phys. Z.* from April 1905 to September 1906 and in *Ann. d. Phys.* from July 1905 to February 1906 between A. Denizot, L. Tezař and M.P. Rudzki. At the end a retired professor M. Koppe from Budapest was drawn in. The debate seems to have ended in Denizot's favor because in 1913 he published a book, *Das Foucaultsche Pendel* (Leipzig and Berlin, 1913), referenced in *Meteorol. Z.* 37 (1920): 101 and in *Ann. D. Hydrog*. 50 (1922): 152.

[3] There are numerous examples during the 150 years since the first experiment. For modern examples see *Encyclopedia Britannica* 8 (1974), 524, "…the pendulum was remaining faithful to its original arc, with respect to space, but the Earth was moving under it due to the Earth's rotation." In a recent book about the Foucault pendulum contains the following: "[The] plane of oscillation could not change as the Earth rotated under it…. [the] Foucault pendulum keeps its plane of oscillation independent of the rotation of the earth." A.D. Aczel, *Pendulum, Léon Foucault and the Triumph of Science* (New York: Washington Square Press, 2003), 102, 168, see also 155 and 166.

[4] Textbooks often go to great lengths to tell the reader that the Coriolis force is "fictitious," "artificial," a "pseudo force," or even a "mental construct." The equally "fictitious" centrifugal force is rarely talked about in this way. This might easily mislead a student into believing that some "fictitious" forces are more fictitious than others. A statement to the effect that the Coriolis force is indeed more fictitious than the centrifugal force, a "real physical force", can be found in professor W.D. McComb's university textbook *Dynamics and Relativity* (Oxford: Oxford University Press, 1999), 145.

[5] For Taylor columns see A. Persson, "The Obstructive Coriolis Force," *Weather* 50 (2001): 204-09 and G.K. Batchelor, *An Introduction to Fluid Dynamics* (Cambridge: Cambridge Univ. Press, 1967), 556-57. For gyroscopes see A. Sommerfeld, *Mechanics* (New York: Academic Press, 1952), 168-69 and R. Lyttleton, *The Comets and Their Origin* (Cambridge: Cambridge Univ. Press, 1953), 17. A large number of asteroids are trapped in Jupiter's two stable Lagrange points. See B.W. Carroll and D.A. Ostlie, *An Introduction to Modern Astrophysics* (Reading, Mass.: Addison-Wesley, 1996), 688, fn. 5); D. Hestenes, *New Foundations for Classical Mechanics* (Dordrecht: D. Reidel, 1999); C.D. Murray and S.F. Demott, *Solar System Dynamics* (Cambridge: Cambridge Univ. Press, 1999); and N.J. Cornish, "The Lagrange Points," http://map.gsfc.nasa.gov/m_mm/ob_techorbit1.html .

[6] A. Persson, "Proving that the earth rotates: The Coriolis force and Newton's falling apple," *Weather* 58 (2003): 264-72. A. Armitage, "The deviation of falling bodies," *Ann. Sci.* 5 (1947): 342-51; A. Koyré, "A documentary history of the problem of fall from Kepler to Newton," *Trans. Amer. Phil. Soc.* 45 (1955): 329-95; H.L. Burstyn, "Early explanations of the role of the Earth's rotation in the circulation of the atmosphere," *Isis* 52 (1966):167-87.

[7] Newton, like Galileo (and many others later) had a *mental image* about the deflection of being an effect of conservation of absolute velocity, which yields a 50 percent larger deflection.

See the letter by L. Falk, "Deflection of a falling body," *Amer. J. Phys.* 51 (1983): 872, for a case when the Swedish school authorities made the same error in an nationwide examination test.

[8] J.F. Benzenberg, *Versuche über das Gesetz des Falles, über den Widerstand der Luft und über die Umdrehung der Erde nebst der Geschichte aller früheren Versuche von Galiläi bis auf Guglielmini* (Dortmund: Mallinckrodt, 1804), 349-92; C.F. Gauss, "Fundamentalgleichungen für die Bewegung schwere Körper auf der rotirenden Erde" (1804) in *Werke* 5 (Hildesheim: Königliche Gesellschaft der Wissenschaften zu Göttingen, 1973), 495-503; P.S. Laplace, "Mémoire sur le mouvement d'un corps qui tombe d'une grande hauteur," *Bull. Soc. philométrique de Paris* 3, no. 75 (1803): 109-15; also in Laplace, *Oeuvres complètes* 14 (Paris, 1893), 267-77.

Later work includes F. Reich, "Fallversuche über die Umdrehung der Erde," *Pogg. Ann.* 29 (1833): 494; J.F. Benzenberg, *Versuche über die Umdrehung der Erde aufs neue berechnet* (Düsseldorf, 1845); W.C. Redfield, "Effects of the earth's rotation upon falling bodies and upon the atmosphere," *Amer. J. Sci.* (1847): 283-84; J.G. Hagen, *La rotation de la terre, ses preuves mécaniques anciennes et nouvelles* (Roma: Tipografia poliglotta vaticana, 1911, 1912); D.R. Stirling, "The eastward deflection of a falling object," *Amer. J. Phys.* 51 (1983): 236 (see also *Amer. J. Phys.* 52 (1984): 562; J.M. Potgieter, "An exact solution for horizontal deflection of a falling object," *Amer. J. Phys.* 51 (1983): 257; R.H. Romer, "Foucault, Reich and the mines of Freiberg," *Amer. J. Phys.* 51 (1983): 683.

[9] E.H. Hall, "Do objects fall south?" *Phys. Rev.* 17 (1903): 179, 245-46; A.P. French, "The deflection of falling objects," *Amer. J. Phys.* 52 (1984): 199; E. Reddingius, Comment on "The eastward deflection of a falling object," *Amer. J. Phys.* 52 (1984): 562; D.R. Sterling, Reply to "Comment on 'The eastward deflection of a falling object," *Amer. J. Phys.* 52 (1984): 563; E.A. Desloge, "Horizontal deflection of a falling object," *Amer. J. Phys.* 54 (1985): 581-82; E. Belorizky and J. Sivardière, "Comments on the horizontal deflection of a falling object," *Amer. J. Phys.* 56 (1987): 1103; and E.A. Desloge, "Further comment on the horizontal deflection of a falling object," *Amer. J. Phys.* 58 (1989): 282-84.

[10] R. Eötvös, "Experimenteller Nachweis der Schwereänderung, die ein auf normal geformte Erdoberfläche in östlichen oder westlichen Richtung bewegter Körper durch dieser Bewegung erleidet," *Ann. d. Phys.* 59 (1919): 743-52. O. Hecker, *Bestimmung der Schwerkraft auf dem Schwarzen Meere und an dessen Kuste, sowie neue Ausgleichung der Schwerkraft auf dem Atlantischen, Indischen under Grossen Ozean* (Berlin: Stankiewicz, 1910). For an early insight into the Eötvös effect see W.M. Davis, "The deflective effect of the earth's rotation," *Amer. Meteorol. J.* (April 1885): 516-24. Also, "Eötvös the Scientist," http://www.elgi.hu/museum/elatud_.htm and József Ádám, "Geodesy in Hungary and the Relation to IAG around the turn of 19th/20th Century: A historical review" http://www.gfy.ku.dk/~iag/HB2000/part1/historic.htm . For meteorological applications of the Eötvös effect see articles by Ekholm and Sprung, later also Köppen and Hermann in *Meteorol. Z.* (1894-97). A Hungarian meteorologist brought up the matter in 1923: E. Szolnoki, "Die Anwendung des Eötvöseffektes in der Atmosphäre," *Meteorol. Z.* 40 (1923): 28-29.

[11] G. Hadley, "On the cause of the general trade winds," *Phil. Trans. Roy. Soc. of London,* 34 (1735): 58-62, reprinted in C. Abbe, *The Mechanics of the Earth's Atmosphere, Smithsonian Misc. Collections* 51, No. 1 (1910). D.B. Shaw, Meteorology over the tropical oceans, Roy. Meteorol. Soc. (1979); A.C. Clairaut, "Sur quelques principes donnant la solution

d'un grand nombre de problèmes," *Mém. Acad. Sci. Berlin*, pt. 1 (1742): 370-72; R. Dugas, *A History of Mechanics* (New York: Dover, 1955). It has been claimed, for example by M. Jacobi, "Immanuel Kant und die Lehre von den Winden," *Meteorol. Z.* 20 (1903): 419-21, that Kant discovered the same principle in 1756, independently of Hadley. I have not read I. Kant, "Neue Anmerkungen zur Erläuterung der Theorie der Winde," (1756) from Kant, *Physische Geographie* 4 (Mainz: Vollmer, 1805), 37-53, but the extracts by Jakobi are not convincing. C. Truesdell, *Essays in the History of Mechanics* (Berlin: Springer Verlag,1968), 131 confirms that Clairaut did not calculate the relative acceleration correctly.

[12] P.S. Laplace, "Recherches sur plusieurs points du système du monde," *Mém. Acad. Roy. d. Sci.* 88 (1775): 75-182; P.S. Laplace, "Recherches sur plusieurs points du système du monde," *Mém. Acad. Roy. d. Sci.* 89 (1776): 177-264. About twenty years later, Laplace divided the works, which are essentially technical, although they also contain some qualitative discussions, into two parts. One, *Mécanique Céleste*, became the technical part; *Système du Monde* became a purely descriptive part. Each of these works went through several editions during Laplace's lifetime. When reference is now made to the Systeme, what is meant is the descriptive account first published in 1796, not the original memories of 1775-76, which have approximately the same title! I am much indebted to professor George W. Platzman for having clarified this to me in 1999. The mathematical derivation is in Memoir 2, pp. 187-89, Article XXII in the 1796 edition, reprinted in Laplace, *Oeuvres complètes* 9. The qualitative discussion is in Memoir 1, p. 90, Article II.

[13] "The speed of a molecule of the fluid is supposed to remain the same in the latitudinal direction…The closer the air is to the pole, the smaller is its actual speed. Consequently, as it approaches the equator, it must turn slower than the corresponding part of the Earth…Thus, for an observer believing to be at rest, the wind seems to blow in a direction opposed to the one of the rotation i.e. from east to west: indeed, it is the direction of the Trade winds." P.S. Laplace, "Recherches sur plusieurs points du système du monde," *Oeuvres complètes* 9 (Paris, 1893), 69-183, 187-310.

[14] John Dalton, *Meteorological Observations and Essays*, 2nd ed. (Manchester: Harrison and Crosfield, 1834), 85, quoted in N. Shaw, *Manual of Meteorology* 1 (Cambridge: Cambridge Univ. Press, 1926), 123 and 289-90. Dalton also provides a reference, perhaps to J.A. de Luc's *Lettres Physiques…* 5 (Paris, 1779-80), Let. CXIV. See also Davis, "Deflective effect of the earth's rotation."

[15] H.W. Dove, "Über den Einfluss der Drehung der Erde auf die Strömungen ihrer Atmosphäre," *Pogg. Ann. Phys. Chem.* 36 (1835): 321-51, transl. "On the influence of the rotation of the earth on the currents of the atmosphere: being outlines of a general theory of winds, *Phil. Mag.* 11 (1837): 227-39, 353-63. See J. Hann, *Meteorol. Z.* 1883**, 18** 176-77 and C. Abbe, *Mechanics of the Earth's Atmosphere*. W.M. Davis, "An early statement of the deflective effect of the earth's rotation," *Science* 1, no. 4 (1883): 98.

[16] P. Costabel, "Coriolis, Gaspard Gustave de," *Dictionary of Scientific Biography* 3 (New York: Scribner's, 1961), 416-19; G.G. Coriolis, "Mémoire sur le principe des forces vives dans les mouvements relatifs des machines" (on the principle of kinetic energy in the relative movement of machines), *J. Ecole Polytech.* 13 (1832): 268-302; I. Grattan-Guinness, *The Fontana History of the Mathematical Sciences* (London: Fontana, 1997), 330, 449; T.S. Kuhn, "Energy conservation as an example of simultaneous discovery," *The Essential Tension:*

Selected Studies in Scientific Tradition and Change (Chicago: Univ. of Chicago Press, 1977), 66-104.

[17] G.G. Coriolis, "Mémoire sur les équations du mouvement relatif des systèmes de corps," *J. Ecole Polytech.* 15 (1835): 142-54; G.G. Coriolis, "Mémoire sur les équations du mouvement relatif des systèmes de corps," *Compt. rend.* 2 (1836): 172-74. See R. Dugas *History of Mechanics*, 374-83 and also R. Dugas, "Sur l'origine du théorème de Coriolis," *Revue Scientifique* 5-6 (1941): 267-70. Coriolis' most important papers have recently been re-published in France: G.G. Coriolis, *Théorie mathematique des effets du jeu de billard* (Paris: J. Gabay, 1990, 1835).

[18] S.D. Poisson, "Extrait de la première partie d'un mémoire," *Compt. rend.* 5 (1837): 660-67. S.D. Poisson, "Sur le mouvement des projectiles dans l'air, en avant égard a la rotation de la terre," *J. Ecole Polytech.* 18 (1838): 1-69, translated by F. Waldo and C. Abbe, *Mechanics of the Earth's Atmosphere*,8-15.

[19] J. Bertrand, "Mémoire sur la theorie des mouvements relatifs, *Compt. rend.* 24 (1847): 141-42; J. Bertrand, "Théorie des mouvements relatifs," *J. Ecole Polytech.* 29 (1848): 149-54. Bertrand criticized Coriolis, who had died five years earlier, for not acknowledging Clairaut's 1742 work. A widely read textbook later took up Bertrand's argument: E.J. Routh, *Dynamics of a system of rigid bodies* 2 (London: Macmillan, 1905), 23-24.

[20] See J.G. Hagen, *La rotation de la terre,* and W. Tobin, *The Life and Science of Leon Foucault: The Man Who Proved the Earth Rotates* (Cambridge: Cambridge Univ. Press, 2003). I am much indebted to professor Norman A. Phillips for having clarified this common misunderstanding. See N.A. Phillips, "Ce qui fait tourner le pendule de Foucault par rapport aux étoiles," *La Météorologie* 34 (Aug. 2001), 38-44.

[21] M. O'Brien, "On symbolic Forms derived from the Conception of the Translation of a Directed Magnitude," *Proc. Roy. Soc. London* (1851): 161-206 (Coriolis force derivation is on pp. 192-97). Ten years later the same derivation was repeated by A. Cohen, "On the Differential Coefficients and Determinants of Lines, and their application to Analytical Mechanics," *Phil. Trans. Roy. Soc. London* 152 (1862): 176-79, 469-510. For a background into O'Brien's and other pioneers of the vector formalism, see N.J. Crowe *A History of Vector Analysis* (New York: Dover, 1967, 1985). For a modern derivation, see A.P. French, *Newtonian mechanics, the M.I.T. Introductory physics series* (New York: Norton, 1971), 520-24.

[22] The idea was first proposed by the Siberian scientist Slowzow in 1827 (see W. Köppen, "Die vorherrschenden Winde und das Baer'sche Gesetz der Flussbretten," *Meteorol. Z.* 7 (1890): 34-35 and 180-83). K.E. v. Baer and Bergsträsser, "Das Seichterwerden der Flussmünungen im allgemein und inbesondere der Wolgamündung," *Astrachaner Gouvernments Nachrichten* (Oct. 1856): 149-51. K.E. v. Baer, "Warum bei underen Flüssen, die nach Norden oder Süden fliessen das rechte Ufter steil unde das linke flach ist," *Marienearchives* (1857): 110-126; K.E. v. Baer, "Über ein allgemeines Gesetz in der Gestaltung von Flussbetten," *Bull. Acad. Imp. d. Sci. St. Pétersbourg* 2 (1860): 1-49, 218-250, 353-382 and the same in *Kaspische Stud.* 7 (1860): 1-6. E. Dunker, "Über den Einfluss der Rotation der Erde auf den Lauf des Flüsse," *Z. Gesammter Naturwiss,* n.f.11 (1875).

In 1926 Albert Einstein entered the debate and explained the meandering as a consequence of an induced secondary circulation perpendicular to the flow of the river. A. Einstein, "Die Ursache der Mäanderbildung der Flußläufe und des sogenannten Baerschen

Gestezes," *Die Natur-Wissenschaften* 11 (1926): 223-24. It is not clear if Einstein had a clear grasp of the Coriolis effect. One who obviously had not was Einstein's friend and Nobel Prize Lauredate Max Born. See J. Tessmann, "Coriolis and consolation, *Amer. J. Phys.* 56 (1987): 382. J.A. Van der Akker, "Coriolis and consolation," *Amer. J. Phys.* 56 (1987): 1063. A. Einstein, *The Born-Einstein Letters, Correspondence between Albert Einstein and Max and Hedwig Born from 1916 to 1955* (New York: Walker, 1971), 141-43.

[23] To a modern reader this sounds like the "Ekman effect."

[24] A. Perrot, "Nouvelle expérience pour rendre manifeste le mouvement de rotation de la terre," *Compt. rend.* 49 (1859): 637-38. J. Babinet, "Influence du mouvement de rotation de la terre sur le cours des rivières," *Compt. rend.* 49 (1859): 638-41. J. Bertrand, "Note relative à l'influence de la rotation de la terre sur la direction des cours d'eau," *Compt rend.* 49 (1859): 658-59; Réponse de M. Babinet, 659; J. Bertrand, "Second Note sur l'influence du mouvement de la terre, *Compt. rend.* 49 (1859): 685-86; J. Babinet, J., 1859: "Sur le deplacement vers le nord ou vers le sud d'un mobile qui se meut librement dans une direction perpendiculaire au meridien, *Compt. rend.* 49 (1859): 686-88.

The total centrifugal force C for an object moving eastward with speed u, Babinet calculated to be $C=\Omega(\Omega r + u) = \Omega^2 r + \Omega u$, instead of the correct $C=(\Omega r+u)^2/r=\Omega^2 r+2\Omega u+u^2/R$; C. Delaunay, "Remarques concernant la question de l'influence de la rotation de la terre sur la direction des courants d'eau," *Compt. rend*. 49 (1859): 688-92. J. Babinet, "Démonstration de la loi de M. Foucault sur la tendance transversale d'un point, qui se déplace à la surface de la terre. Evaluation de la force qui produit dans les rivières la tendance à l'érosion des rives," *Compt. rend.* 49 (1859): 769-75. C. Combes, "Observations au sujet de la communication de M. Perrot et la note de M. Babinet," *Compt. rend.* 49 (1859): 775-80. Even after Babinet's analysis, which von Baer was aware of, he held on to his conviction that only rivers flowing in the north-south direction would be affected.

[25] N. Braschmann, "Sur l'expèrience de M. Perrot," *Bull. Acad. Imp. d. Sci. St. Pétersbourg* (1860): 571 and N. Braschmann, "Note concernant la pression des wagons sur les rails droits et des courants d'eau sur la rive droite du mouvement en vertu de la rotation de la terre," *Compt. rend.* 53 (1861): 1068 ff and in *Cosmos* 19, 661. A. Erman, "The influence of the diurnal rotation of the earth on constrained horizontal motions, either uniform or variable," *Arch. f. Wissenschaftliche Kunde v. Russland* 21 (1862): 52-96, 325-32, transl. C. Abbe, *Mechanics of the Earth's Atmosphere*.

[26] W. Ferrel, "An essay on winds and the currents of the ocean," *Nashville J. Med. Surg.* 11, nos. 4 and 5 (1856): 287-30. His mathematical expressions of the fictitious forces include strangely the radius of the earth; W. Ferrel, W., 1858: "The influence of the earth's rotation upon the relative motion of bodies near its surface," *Astro. J.* 5 (1858): 97-100, 111-114; W. Ferrel, "The motions of fluids and solids relative to the Earth's surface," *Math. Monthly* 1 (1859): 140-48, 210-16, 300-07, 366-73, 379-406, 2 (1859-60): 89-97, 339-46, 374-90; W. Ferrel, *The motions of fluids and solids relative to the earth's surface, comprising applications to the winds and the currents of the ocean* (London and New York, 1860), 72 p.; W. Ferrel, "The motions of fluids and solids relative to the earth's surface," *Amer. J. Sci.* 31 (1861): 27-51, reprinted in *Professional Papers of the Signal Service* 12 (1882): 21-34. See G. Kutzbach, *The Thermal Theory of Cyclones* (Boston: Amer. Meteorol. Soc., 1979), 38-39, fn. 61.

[27] A. Mühry, *Über die Theorie und das allgemeine geographische System der Winde* (Göttingen, 1869). Mühry's necrology is in *Meteorol. Z.* 5 (1888): 410-12. There are also references to an early derivation of the Coriolis effect by Z.B. Buff, "Einfluss der Umdrehung der Erde um ihre Axe auf irdische Bewegungen," *Ann. der Chemie und Pharmacie* 4, suppl. (1865, 1866): 207ff.

[28] C.M. Guldberg and H. Mohn, *Études sur les mouvements de l'atmosphère* 2 vols. (Christiana, 1876, 1880), reprinted in *Norwegian Classical Meteorological Papers Prior to the Bergen School* (Oslo, Norway: Universitetsforlaget, 1966), Coriolis on 30-33. C.M. Guldberg and H. Mohn, "Über die gleichförmige Bewegung der horizontale Luftströme," *Öst. Meteorol. Z.* 12 (1877): 49-60, 177ff, 257ff, 273ff; J. Finger, "Über den Einfluss der Erdrotation auf die parallel zur sphäroidalen Erdoberfläche in beliebigen Bahnen vor sich gehenden Bewegungen, inbesondere auf die Strömungen der Flüsse und Winde," *Sitzungsb. d. Kais. Akad. Wiss. Wiener Akad.* 76 (1877): 67-103; C. Benoni, "Det Einfluss der Achsendrehung der Erde auf das geographische Windsystem," *Petermanns Geograph. Mitteil.* 23 (1877): 93-106. See Kutzbach, *Thermal Theory of Cyclones* for biographies of Sprung, Köppen, v. Neumayer, Hann and others.

[29] A. Sprung, A. 1879: "Studien über den Wind und seine Beziehungen zum Luftdruck zur Mechanik der Luftbewegungen," *Archiv der Deutschen Seewarte* 2 (1879): 1-27, Anhang "Über das Hadleysche Prinzip." A. Sprung, "Die Trägheitscurve auf rotierender Oberflächen als ein Hilfsmittel beim Studium der Luftbewegungen," *Öst. Meteorol. Z.* 15 (1880): 1-16.

[30] F.J.W. Whipple, "The Motion of a Particle on the Surface of a Smooth Rotating Globe," *Phil. Mag.* 33 (1917): 457-71; N. Paldor and P.D. Killworth, "Inertial Trajectories on a Rotating Earth," *J. Atmos. Sci.* 45 (1988): 4013-19. See also W.S. von Arx, *An Introduction to Physical Oceanography* (Reading, Mass.: Addison-Wesley, 1962), 101-02 and G.J. Haltiner and F.L. Martin, *Dynamical and Physical Meteorology* (New York: McGraw-Hill, 1957), 181.

[31] P.T. Bush, D.J. Digiambattista, and D.A. Coats, "Coriolis deflection on the rotating platform," *Amer. J. Phys.* 44 (1976): 883-85; H.A. Daw, "Coriolis lecture demonstration," *Amer. J. Phys.* 55 (1987): 1010-14, fig.3 and discussion on p. 1012; D.T. Durran, D.R., 1996: "An apparatus for demonstrating the inertial oscillation," *Bull Amer. Meteorol. Soc.* 77 (1996): 557-59; A.A. Klebba and H. Stommel, "A simple demonstration of Coriolis force," *Amer. J. Phys.* 19 (1951): 247.

[32] L. Euler, "Recherches sur le mouvement des corps célestes en général," *Histoire de l'academie Royal des Sciences et Belles Lettres*, (1749): 93-143, see p. 106. After Euler had derived the Coriolis acceleration in angular variables he is said to have tried the same in Cartesian coordinates but made an error and only got half of the right answer. See Truesdell, *Essays in the History of Mechanics.* Both French, *Newtonian Mechanics*, 557-60 and D. Acheson, *From Calculus to Chaos, An introduction to dynamics* (Oxford: Oxford Univ. Press, 1997, 2003), 74-77 present the derivation of the Coriolis acceleration with references to Newton.

[33] M. Thiesen, "Über Bewegungen auf der Erdoberfläche," *Öst. Meteorol. Z.* 14 (1879): 203-06; M. Thiesen, [Review of Ferrel, 1877 and Finger, 1877], *Öst. Meteorol. Z.* 14 (1879): 386-91; A. Sprung, "Zur Theorie der oberen Luftströmungen," *Öst. Meteorol. Z.* 15 (1880): 17-21 and *Ann. d. Hydr. u. maritime Meteorol.* 8 (1880): 603; M. Thiesen, "Über Bewegungen auf der Erdoberfläche," *Öst. Meteorol. Z.* 15 (1880): 88-89; A. Sprung, *Lehrbuch der Meteorologie* (Hamburg: Hoffmann und Campe, 1885).

[34] A direct application of Coriolis' 1835 paper came during the 1960s. The Americans and Russians had plans to build a rotating space wheel with the intention to create an artificial gravity through the rotation. The plans were abandoned when it was realized that the rotation necessary to create an artificial gravity close to 9.81 m/s^2 would create Coriolis forces thousand of times stronger than on earth. The crew would suffer from physiologically or psychologically uncomfortable Coriolis effects, machinery with moving or rotating parts like centrifuges and washing machines might break down.

[35] The American-Hungarian physicist Lanzos commented in 1949 that his students knew every detail in an atom-smashing machine, but still were ignorant about the difference between heavy and inertial mass. R.J. Whitaker, "Aristotle is not dead: student understanding of trajectory motion," *Amer. J. Phys.* 51 (1983): 352-57. J. Lythcott, J., 1985: "'Aristotelian' was given as the answer, but what was the question?," *Amer. J. Phys.* 53 (1985): 428-32. McComb's *Dynamics and Relativity* shows on p. 145, in conflict with the author's own mathematical derivations a few pages earlier, but in full agreement with Aristotle, that falling bodies are deflected to the *west*.