Albert Einstein and Frau Schrödinger's tea cup

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There is a widespread idea that classical physics is outdated and therefore simple and easy, while modern physics, in the forefront of science, is very complicated and difficult. This historical example will show that this is not necessarily the case.

1. Introduction

We tend to associate Albert Einstein (1879-1955) with his special theories of relativity and Erwin Schrödinger (1887-1961) with his quantum wave mechanics. Here we will meet them in the context of what is going on in a cup of tea!

1.1 Frau Schrödinger's question

It is autumn 1925 and we are in Berlin. Erwin Schrödinger is in the midst of developing his new wave theory when Albert Einstein pays a visit to the family. Perhaps it was just a social visit; perhaps he wanted to discuss Erwin's new ideas. Anyhow, when tea is served Frau Schrödinger is curious to know why the tea leaves, when the tea water is stirred around, always tend to collect in the center of the cup. Her famous husband could not answer, perhaps Professor Einstein could explain?

Indeed he could. It is often forgotten than Einstein was not only a great theoretician. For about five years, 1904-1909, he was employed at the Patent Office in Bern dealing with very practical problems. He would later say that this practice had also benefited his research.

1.2 Einstein's solution

To understand the gist of Einstein's explanation we must realize some peculiar behaviours of friction. When Frau Schrödinger stirred the water in the tea cup it was seen to move around with a uniform speed. But this was not true for the water closest to the sides and at the bottom of the cup. In a thin "boundary layer" closest to the cup, as with all solids, the velocity of any liquid or gas goes down to zero. i.e. molecules in direct contact with the solid surface remain fixed to it.

This was discovered in 1904 by the German aerodynamicist Ludwig Prandtl (1875-1953).To quote the American scientist Richard Feynman, this insight made the science of fluid dynamics start to deal with "wet water", i.e. with viscosity effects included, and not just with "dry water", where the viscosity was ignored.

Einstein now explained to the Schrödingers how a centrifugal force acts on the rotating water. This force is proportional to the square of the velocity and thus, because of the friction, becomes weaker, in particular closest to the bottom of the cup. This will result in a circular movement of the liquid which can be seen through the movement of the tea leaves (figure 1).

***Figure 1* from Einstein's 1926 article.** *The uneven distribution of the centrifugal forces generates an inward flow near the bottom that gathers tea leaves in a small heap.*

1.3 Meandering of rivers

The story could have ended here hadn't Einstein found what he considered to be a wider application to his explanation, a mechanism that contributes to the meandering of rivers. When water in a river flows through a bend it will follow a rotational motion. This "primary" motion will, just as in the tea cup, through friction towards the river banks, generate a "secondary" flow that will cause some erosion along the banks *on both sides*.

However, in the mid-1800's a Baltic-German scientist Karl Ernst Ritter von Baer (1792-1876) during travels in Siberia had noticed that the big rivers tended to be eroded on their *right hand* side. This he explained this to be due to the earth rotation which causes a deflection of moving objects to the right (on the Northern Hemisphere, to the left on the Southern). This so called "Baer's Law" would lead to an increase of the erosion on the right hand side of the river and weaken it on the left. Over time the riverbeds would change to the right and this would thus explain the meandering of the rivers.

***Figure 2:* Karl Ernst Ritter von Baer (1792-1876)** *(image WikiCommons | Voyages de la Commission scientifique du Nord, en Scandinavie, en Laponie, au Spitzberg et aux Feröe, Artus Bertrand, Paris, 1852 | digitization by Norwegian National Library).*

1.4 "Baer's Law"

"Baer's Law" has since then figured on and off in the scientific debate. Various authors suggested that the influence of the tendency was small and that other factors (e.g., wind and eolian deposition) could account for stream course asymmetry, but no convincing rebuttal of the law has ever been produced. The present view seems to be that the meandering of rivers is mainly a random process where the earth's rotation play no or a very minor role.

Baer was not alone presenting a "law" describing the consequences of the earth's rotation. In 1856, the Dutch meteorologist Buys Ballot formulated his "law" according to which, with the wind in your back, there will, due to the earth's rotation, be a low pressure system to the left and a high pressure system to the right. At about the same time an American school teacher, William Ferrel, formulated a general mathematical "law" according to which any parcel of air or water moving over the earth's surface would be deflected to the right (on the Northern Hemisphere, to the left on the Southern).

None of them seem to have been aware that a French scientist Gaspard Gustav Coriolis (1794-1843) already in 1835 had given a general solution to the deflection, nowadays called "the Coriolis Effect".

2. The scientific value of Einstein's tea cup explanation

On 7 January 1926 Einstein had a presentation at the Prussian Academy on "The cause of the formation of meanders in the courses of rivers and of the so-called Baer's law". It was also publicized in the periodical "*Die Naturwissenschaften*" (The Natural Sciences) in March 1926 (Vol. 14, p. 223).

We will leave aside the details of Einstein's explanations of meandering rivers and focus on the tea leave model.

1. Was his explanation, so outside from everything else he did, really his idea?
2. How has it been judged by scientists dealing with rotational phenomena in fluid mechanics, meteorology and oceanography?
3. What type of atmospheric and oceanographic phenomenon does Einstein's tea cup help to explain?

Although Einstein's explanation is incomplete in some respects, it actually seems to have had some influence on the sciences of fluid mechanics, meteorology and oceanography.

2.1 Was it really Einstein's idea?

Immediately after reading Einstein's article Ludwig Prandtl sent a letter to *"Die Naturwissenschaften"* . He quoted other scientists to show that that the basic idea in Einstein's article was not new. "Secondary flows" developing due to friction had been discussed by J. Isaachsen in a periodical "*Civilingenieur*" in 1896 and later in "*Zeitschrift der Verein Deutsche Ingenieure*" in 1911.

More intriguingly, already in 1857 the British professor James Thomson (brother to John Thomson, more well-known as Lord Kelvin), had used the tea cup analogy in almost identical words. In a talk *"Grand current of atmospheric circulation"* he had imagined a shallow vessel with a flat bottom, filled with water. When the water was stirred around "a few tea leaves taken from a teapot" would visually indicate how the leaves collects in the center:

> *They are evidently carried there by a current determined towards the centre along the bottom in consequence of the centrifugal force of the lowest stratum of the water being diminished in reference to strata above*

through a diminution of velocity of rotation in the lowest stratum by friction on the bottom.

The particles being heavier than the water, must, in respect of their density, have more centrifugal force than the water immediately in contact with them; and must therefore in this respect have a tendency to fly outwards from the centre, but the flow of water towards the centre overcomes this tendency and carries them inwards; and thus is the flow of water towards the centre in the stratum in contact with the bottom palpably manifested.

Thomson was of course not aware of the discovery of the "boundary layer" but intuitively he took the frictional effect correctly into account. Nor is it likely that that Einstein had read or was aware of Thomson's 1857 paper. Thanks to Thomson, Einstein or somebody else, the image of tea leaves in a cup of stirred water has since then figured as a conceptual model for fluid dynamists and meteorologists.

2.2 Was Einstein's explanation correct?

An indication that Einstein had himself figured out the tea cup explanation is that it is not quite correct, or rather it is incomplete. He explained the "secondary circulation" only as a result of changes in the centrifugal forces. But the centrifugal forces are what we call "fictitious", a result of inertia, and cannot create motion, kinetic energy; for this we need a conversion from potential energy.

The needed potential energy is created when the upper water surface, due to the centrifugal effects, takes a concave, parabolic form. Then slightly more water will gather at the sides of the cup compared to the center. The water at the sides will therefore be under slightly more pressure from the water above, than the water at the center. As a result there will in the lowest layer be a radial inward, pressure gradient force which will balance the outward centrifugal force.

However, closest to the bottom of the cup, this inward directed pressure gradient force will be stronger than the outward directed centrifugal force weakened because of the friction against the bottom. This imbalance will lead to a net inward acceleration of the water. The water will converge at the center and then rise, leaving the heavier tea leaves to gather at the bottom (figure 2).

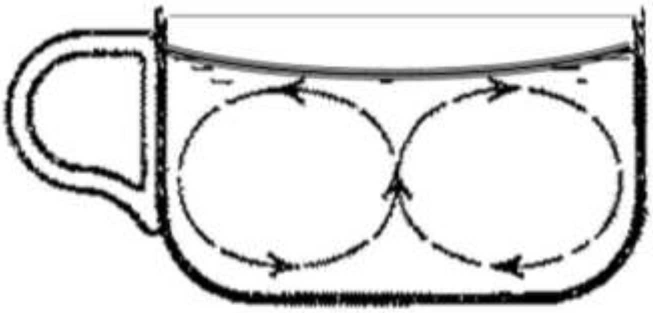

***Figure 3:* The complete physical explanation of Frau Schrödinger's tea cup problem:** *the creation of an upper parabolic surface due to the centrifugal effect gives rise to radially inward pressure gradient forces that, at the bottom of the cup, where friction has slowed down the motion, accelerates the water and the tea leaves towards the centre.*

In Einstein's own image (figure 1) the upper surface is horizontal which clearly is not physically realistic. He might have considered, although it is not likely, the special case of a solid upper lid preventing a parabolic surface to form. This will, however, not change the result since the lid will provide precisely the centripetal force that will give the inward acceleration of the fluid. Or in other words: when the spinning fluid "wants to" generate a parabolic upper surface, the rigid lid pushes back on the fluid to prevent this, providing an equivalent radial pressure gradient.

2.3 Further developments of Einstein's tea cup idea

In the hands of fluid dynamicists, meteorologists and oceanographers Einstein's tea cup model has been corrected and further developed to illustrate different processes. Indeed, the leading textbook in dynamic meteorology, by Professor James R. Holton at Washington University in Seattle, used Einstein's tea cup model to illustrate how atmospheric spinning vortices are affected by surface friction.

2.3.1 Molecular viscosity is not enough to weaken a low

Most weather systems develop over water areas and weaken when they move in over land. This appears natural since the seas and oceans, when not ice covered, supply heat and moisture to the weather systems. The sea surfaces are also smoother than the land surfaces and do not retard the motions as much through frictional effects.

In the free atmosphere the air moves in a very frictionless environment. The weather systems are often stretching all the way up to the top of the troposphere, around 10 kilometers. If friction would act as we might think it does, spreading from the surface upward thanks to molecules in adjacent layers "rubbing" against each other, it would take a few weeks to fill a low pressure system. Instead surface friction communicates its influence in a much more subtle, ingenious and efficient way that fills a low pressure system in a few days.

Take a typical atmospheric large-scale cyclonic vortex, with winds rotating anti-clockwise around a center with lowest pressure in the center, just as with the cup (figure 4).

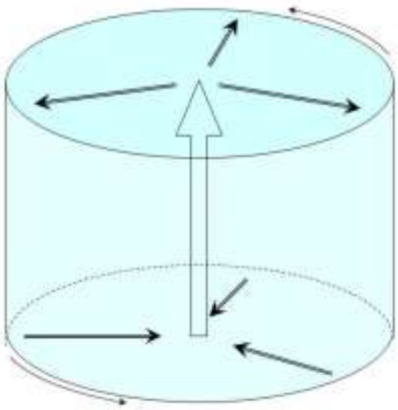

***Figure 4:* The same as figure 2, but in a perspective which makes the upper and bottom surfaces visible.** *When rising tea water at the centre of rotation reaches the upper surface it spreads out radially in all directions. Note that in the atmosphere an oceans this 1:1 proportion between height and horizontal length should be rescaled to something much "flatter", rather like 1:100.*

When the air, with friction slowing it down, converges into the low, it rises, just as the water in the center of the tea cup. When air reaches to the top of the low pressure system it behaves just as the water that reaches to top of the tea cup, it spreads out radially.

2.3.2 The effect of the earth's rotation

Both the tea cup and the low pressure system are on the rotating earth and therefore subject to the Coriolis Effect. But now there is a crucial difference. The Coriolis Effect in the tea water is negligible; the cup is far too small. But the atmospheric low pressure systems are much bigger "tea cups". During its long motion outward the air will be highly affected by the Coriolis force, turning the motion to the right, on the Northern Hemisphere, to the left on the Southern (figure 5).

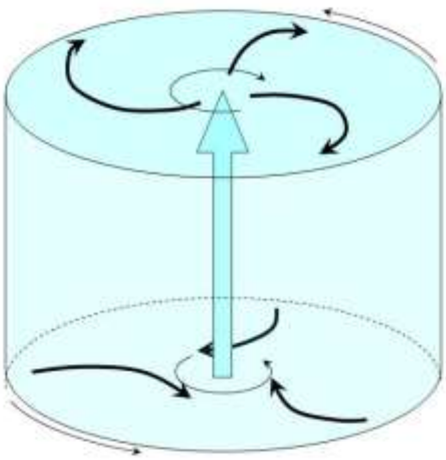

***Figure 5:* The sequence of events leading to a rapid slowing down of an atmospheric vortex:** *friction at the surface of the low pressure system creates a so called "secondary circulation" which will counter the prevailing circulation and make the low pressure system vanish in a few days.*

This deflection tries to generate a circulation *in the opposite direction*, a clockwise (anticyclonic) circulation in the otherwise anticlockwise (cyclonic) circulation. This "secondary" counter rotating circulation will now weaken the "primary" circulation and ultimately make it disappear in a few days.

But this is not the end of the story. Sometimes the surface friction does not slow down the motion, but is rather instrumental in keeping it going and may even strengthen it.

2.3.3 The winter low that brought snow to the 1964 Olympics

In January 1964 the Austrians were preparing for the Winter Olympics 29 January - 9 February in Innsbruck. However, there was hardly any snow and a vigorous and persistent high pressure area was covering Europe. In the churches the Austrians were praying for snow.

Then on 5 January, three weeks before the games, a small low pressure vortex with anticlockwise (cyclonic) circulation[1] was formed over southern Scandinavia. During the following week it moved from Jutland over the southern Baltic Sea. It then started to penetrate the high pressure clockwise (anticyclonic)

1. The author assumes that the reader is familiar with the common notion of the wind blowing parallel to the isolines of pressure, the full lines in figure 6.

circulation over the continent. On 10 January it deposited some snow in the mountains around Innsbruck, the ski resort Obergurgl got 4-5 cm (figure 6).

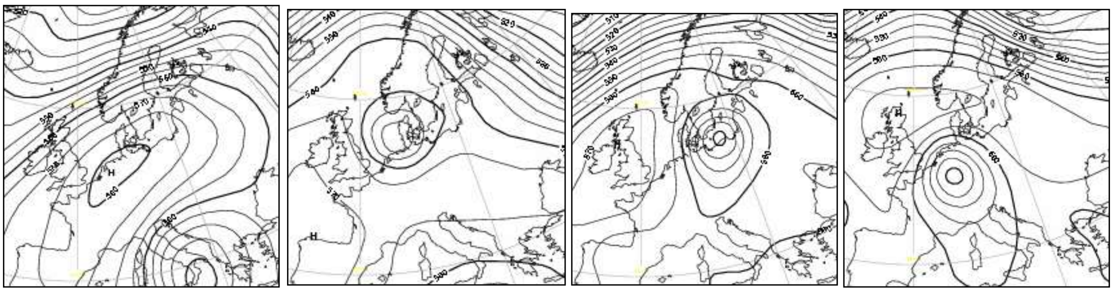

Figure 6: The flow pattern at 5-6 km height over Europe from left to right on 4, 6, 8 and 10 January 1964. *The anticyclone in the first image (4 January) with its centre over The Netherlands has clockwise wind circulation, with for example strong south-westerly winds over the British Isles and Scandinavia. The small vortex moving from Denmark (6 January) over southern Baltic sea (8 January) to C Europe 10 January) has anticlockwise wind circulation with for example strong north easterly winds over The Netherlands.*

Then the small vortex turned west and dissipated some days later over the English Channel. Now friction against the warm sea surface contributed to weaken the circulation instead of maintaining it as had been the case over the cold continental land surface.

But how could this, low pressure, anticlockwise and *cyclonic* vortex during a week survive in a high pressure, clockwise *anticyclonic* environment over cold snowy land surfaces?

2.3.4 How can the wind circulation change with height?

With a anticlockwise circulating (cyclonic) low pressure vortex at 5-6 km height one would perhaps expected a similar circulation at the bottom of the atmosphere, at the earth's surface. In this case there was nothing of the kind, rather the opposite, a weak clockwise (anticyclonic) slightly high pressure circulation. This complexity of the atmospheric dynamics demands a short clarification.

The concept "pressure" in the context of "low pressure" and "high pressure" systems, refers to the **weight** of all the air above. This weight is independent of the temperature or density, it is just the "gravitational pressure" of all the molecules above. This pressure naturally decreases with height.

However, the air's temperature, or rather its horizontal variation, determines how much the pressure and thus the flow pattern changes in the vertical. Even if the pressure at the earth's surface may be uniform in all directions and the flow weak or irregular, if the air in a certain vertical column is colder than in another vertical column the warmer surroundings the pressure will decrease more rapidly (figure 7) than in the warmer column.

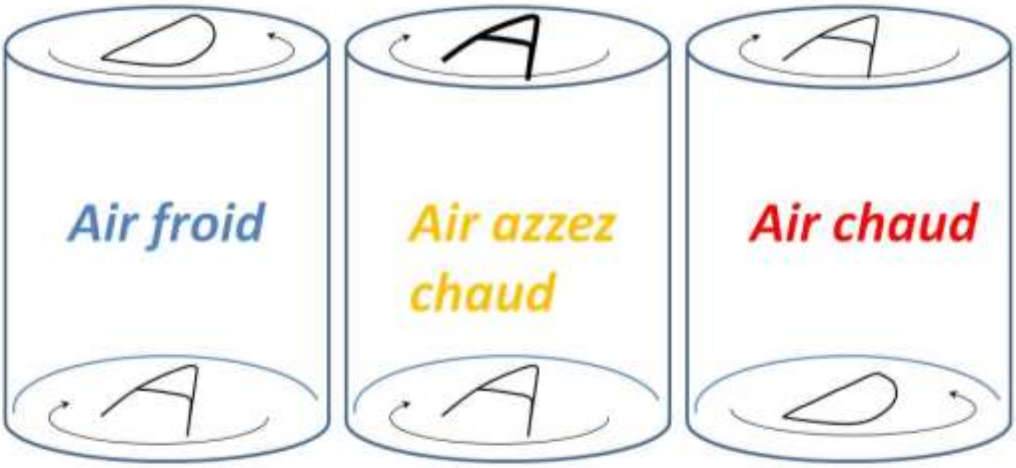

Figure 7:* A schematic illustration of how the temperature in an air column may change the circulation.** ***Left: *Cold, dense and heavy air has caused an area of maximum high pressure at the bottom of the atmosphere. But due to its coldness the pressure decreases rather rapidly upwards and creates an area of minimum lower pressure, cyclonic circulation. This is typical for winter time "cold spells".* ***Centre:*** *The air is generally warm so the area of maximum high pressure is due to the unusual large amount of air above the surface. Depending on the degree of warmth the pressure will with increasing height become even higher, more anticyclonic. This is typical for summer time "heat waves".* ***Right:*** *The pressure might be low at the surface, but due to the warm air the pressure decreases vertically more slowly than in the less warm environment which may at upper levels result in a relative maximum of higher pressure. This is typical for young cyclonic systems, in particular tropical cyclones.*

Modelling the forces in a vortex

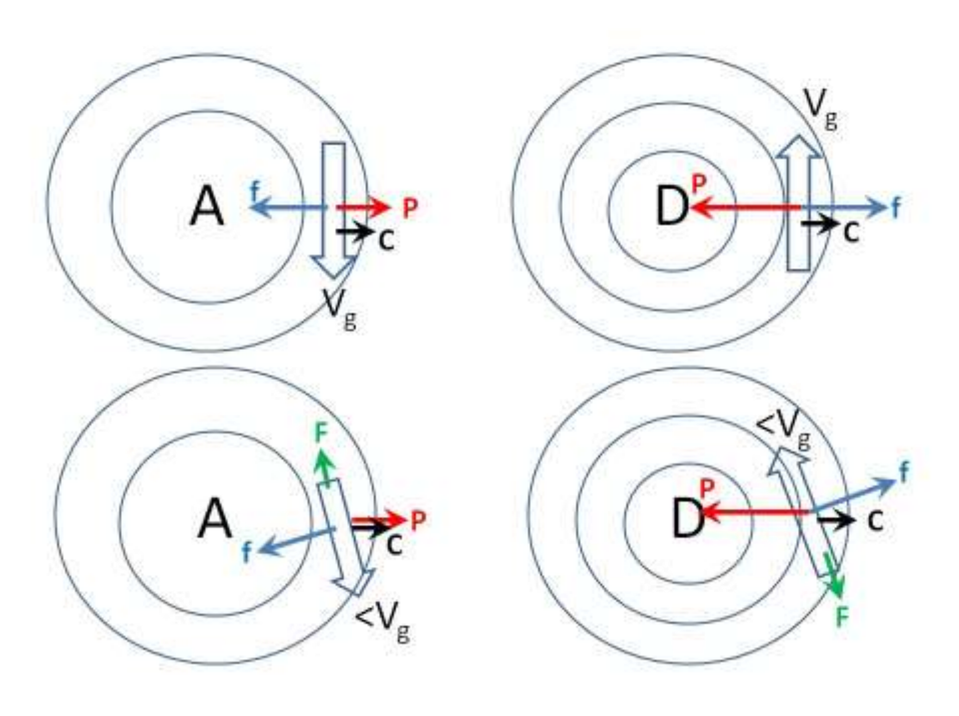

Figure 8

Upper row: Schematic representation of a high pressure (A) and low pressure (D)[2] area. The wind is in a so called gradient wind balance, meaning that it is moving with the pressure gradient[3] force **P**, the Coriolis force **f** and the centrifugal force due to the curvature of the flow **C** balancing each other.
Note that the centrifugal force **C** is always pointing outwards from the circulation, the Coriolis force **f** to the right of the wind direction and the pressure gradient force **P** from high to low pressure. The pressure gradient force **P** and the Coriolis force f therefore always point in opposite directions. The centrifugal force, however, for high pressure (A) systems supports the centrifugal force and for low pressure systems supports the Coriolis force. This explains why the pressure gradients tend to be stronger with low pressure systems and weaker with high pressure systems[4].

Lower row: The same as above, but with friction **F** included[5]. The friction obviously slows down the motion $\mathbf{V_g}$. The weakening of the wind, which weakens the Coriolis force **f** has the effect of the pressure gradient force getting an "upper hand". This diverts the wind towards lower pressure both for high (A) and low pressure (D) systems, outward from the former, inward for the latter.

2. I have used the French symbols D for Depression and A for Anticyclone.
3. Pressure gradient means the degree the pressure varies in horizontal direction.
4. Indeed, low pressure systems can have very low pressure at its centre, 50-60 hPa lower than the environment, whereas high pressure systems only differ by 20-30 hPa from the neighbourhood.
5. In contrast to solid objects, friction with gaseous or liquid substances does not have to be counter-parallel (anti-parallel) to the motion, but I have made it so here.

2.3.5 How can friction increase the circulation?

The vortex in January 1964 was of a similar type as the one to the left in figure 7 with the difference that the high pressure distribution at the bottom of the atmosphere was weak with only a slight anticyclonic circulation.

The weak pressure gradient force at the bottom of the vortex yielded no resisting force to prevent air from flowing out due to frictional effects. This loss of air was compensated by a downward motion in the vortex. But the loss of air at higher levels was in turn at higher levels compensated by an inflow from the sides (figure 8).

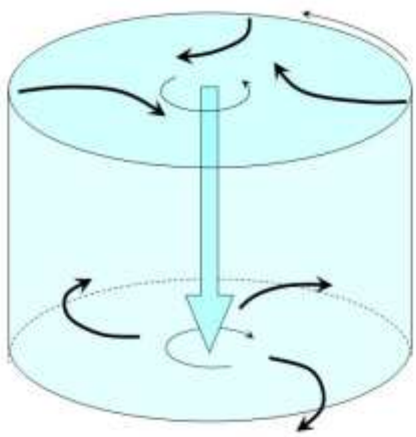

***Figure 8:* Frictional outflow at the bottom of the atmospheric anticyclonic clockwise circulating vortex.** *The bottom outflow or divergence induces a compensating downward motion in the centre, which in turn at upper levels draws in air from the sides. This converging air, affected by the Coriolis Effect, is deflected to the right, which will create an upper cyclonic flow.*

This inflow is of course affected by the earth's rotation. But in contrast to the outflow below, which, when deflected to the right got an *anticyclonic* twist, the upper inflow, deviating to the right, will get a slight *cyclonic* twist – i.e. the same direction as the vortex itself and *therefore maintaining or even strengthening it*.

3. THE FATEFUL YEAR 1926 IN ALBERT EINSTEIN'S LIFE

In winter 1925-26 Albert Einstein's was at the peak of his scientific career. His revolutionary theories about the relation between time and space, the nature of gravitation and the existence of light quanta (photons) had been generally accepted. When he was awarded the Nobel Prize in 1923, it was for the photoelectric effect, but the scientific opinion regarded it as much or even more for his theory of relativity.

But 1925-26 was also a turning point in modern physics with the new ideas from Werner Heisenberg and Erwin Schrödinger, supporting Niels Bohr "Copenhagen School". Einstein had always been ahead of his time – right up to 1925. When he was confronted by these new ideas he could not accept them. His own search for a unified field theory of gravitation and electromagnetism would preoccupy him for the rest of his life but never lead to any results.

4. Epilogue

Receiving the Nobel Prize or any other prestigious award may lead some scientists into depressions, they do not see any challenges ahead that can match the prize; everything else fades into insignificance. Other scientists might start to think too highly of themselves. Encouraged by an admiring surrounding; they become victim of the "halo effect". Einstein obviously reacted in this way and underestimated the complexity of fluid dynamics.

Why should fluid dynamics, a branch of classical mechanics, be as difficult as quantum dynamics or the theory of relativity? Fluid dynamics belongs fully to our everyday world. When we enter the world of modern physics we know, or should know, that we can leave our everyday experiences behind.

The "problem" with classical mechanics is that we cannot do that. We should be able to use our common senses. But if our common senses are poor? If they are based on limited or irrelevant experience? We may for example be very aware of "friction" from our everyday experiences, but that relates to friction *between solid objects* and counts little for friction where fluids and gases are involved.

We may be attracted to modern physics because of the sense of "wonders" it offers. But few branches of physics are so full of "wonders" than rotational dynamics, in particular when gases and fluids are involved. We hope to come back to some of those aspects, in particular since learning more about the "wonders" in classical mechanics can be "taken home" and enrich our everyday, practical life.

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