Back to basics:

Coriolis: Part 2 – The Coriolis force according to Coriolis

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In his professional activity Gaspard Gustave Coriolis (Fig. 1) was never concerned with the atmosphere, not even with the rotating earth. His interest was to promote the Industrial Revolution in early nineteenth-century France. It was during his study of machines, their forces and energy exchanges, that he made his discovery of the deflective force.

Coriolis was both a victim and an offspring of the French Revolution. He was born on 21 May in the fateful year 1792 in Paris to a small aristocratic family. His father, Jean-Baptiste-Elzéar, who had been a captain in Louis XVI's guard, was ruined by the political turmoil and, to save his own life, had to flee to Nantes, where he became a businessman. The young Gaspard showed early remarkable mathematical gifts. At 18 he was admitted to l'École Polytechnique and at 20 he continued as an engineering student at l'École Ponts et Chaussées. He showed great talent as a teacher, and in 1816 was employed by the school (see Persson 1998a,b for more details about Coriolis's life).

Educated workers

The early nineteenth century was a time of change, with the Industrial Revolution in full swing and French industry lagging behind the British. A radical and patriotic movement developed within l'École Polytechnique to promote technical development by educating workers, craftsmen and engineers in 'mechanique rationelle' to make them understand the functioning of machines in order to improve them. In his 1829 book *Calcul de l'effet des machines* Coriolis presented mechanics in a way that could be used by the industry. The book was also a milestone in the general development of physics since it established for the first time the correct relation between potential and kinetic energy, and showed that their sum remained constant in the absence of any external force (Grattan-Guinness 1997, p. 449). It was another 20 years after Coriolis's discovery before W. Rankine coined the phrase 'kinetic energy' in 1853 and Lord Kelvin and P. G. Tait the phrase 'potential energy' in 1862. Progress in science is not always as rapid as we tend to imagine*.

Billiard game

When Coriolis worked on a new edition of his book he became interested in applying the notion of kinetic energy and work to rotating systems. The impetus came not only from the demands of the technical development. L'École Polytechnique was (and is) a military school led by a general. In Coriolis's day this commander was a keen billiard player and he commissioned Coriolis to look into the problem of how and why the balls moved and bounced as they did. In 1832 Coriolis's investigations on rotating systems resulted in a book *Théorie mathématique du jeu de billiard* (Coriolis 1832, 1990) and a paper which deals with the energetics of a rotating system. Three years later came *Sur les équations du mouvement relatif des systèmes de corps* (Coriolis 1835, 1990; see also Dugas 1955, p. 374), where the 'deflective force' explicitly appears for the first time.

The difficulty for a modern reader to take in

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* For example, there is no mention of energy in Newton's work. It was not until around 1750, when measurements of the shape of the earth had finally confirmed Newton's theory, that the notion of forces was generally accepted.

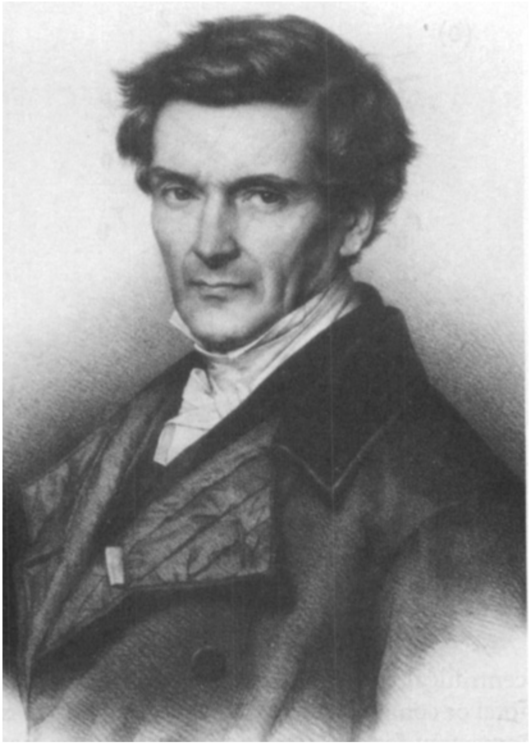

Fig. 1 Gaspard Gustave Coriolis (MP © Archives de l'Académie des Sciences, Paris)

Coriolis's 165-year old text does not rest so much with the language or the mathematical conventions, as with the early nineteenth-century technology to which he refers, such as water mills and other machines containing rotating parts. These parts are subjected to centrifugal forces which must be known to prevent disruption of the machinery. To make it easier for readers brought up in twentieth-century technology to appreciate Coriolis's discussion I will make use of an idea from another eminent scientist, Lewis F. Richardson (1881–1953).

Richardson's unfinished experiment

Richardson spent some time during the winter of 1945/46 thinking about the Coriolis force. Two years earlier he had moved to Kilmun, north-west of Glasgow. The shortest route is a beautiful drive by winding roads along the shores of several lochs (Ashford 1985, p. 193). Perhaps he was travelling on some public transport on these winding roads when, on 7 December 1945, he came to think out this rather unusual experiment to measure the Coriolis deflection:

> "Try to walk straight, and rapidly, along the corridor of a tram or bus whilst the vehicle is turning a corner. There is of course always some centrifugal force on a passenger standing at rest relatively to the vehicle, whilst it goes round the corner" (Richardson 1946).

Before we return to Richardson let us make sure what we mean by a centrifugal force. First of all, it is not a real force but a consequence of a body's inertia, its 'urge' to continue its movement although the ground under it is rotating or moving away under it. As we all know from travelling in cars, what determines the centrifugal force is the speed of the vehicle, u_0, and the sharpness of the turning. The latter is measured as the radius, r_0, of a circle with the same curvature. The mathematical expression for the centrifugal acceleration is u_0^2/r_0. It is also important to know that the centrifugal force is always pointing perpendicular to the trajectory of the motion (Fig. 2(a)).

What Richardson set out to measure in 1945 was: how will the centrifugal force be changed if he is not standing still, but moving around in the bus?

> "But the experiment is to ask whether there is another force which depends on the passenger walking straight along the vehicle, and whether this force can be reversed in direction by reversing the walk, provided it is sufficiently rapid?" (Richardson 1946).

Richardson even drew up a form to make notes on how he was deflected depending on whether the vehicle turned to the right or left, and whether he was walking forwards or backwards in the corridor (Fig. 2(b)). It is not known if Richardson ever made his experiment and it is doubtful if it would have yielded any useful information. What interests us here is that his idea, eccentric as it might have been, was actually on the same line of thought as Coriolis's 110 years earlier.

Coriolis's compound centrifugal force

For the same reason that the centrifugal force on a passenger in a bus also depends on the

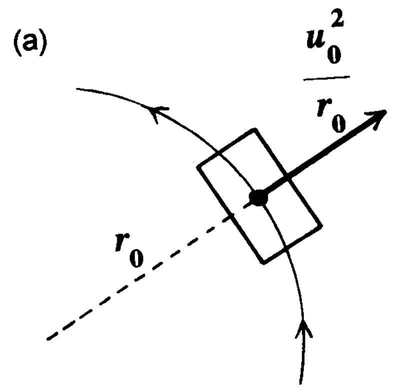

Fig. 2(a) A passenger standing in a vehicle moving in a curved motion will experience a centrifugal acceleration, u_0^2/r_0, *pointing perpendicular to the trajectory of the motion and determined by the tangential velocity,* u_0, *and the radius of curvature,* r_0, *at every instant*

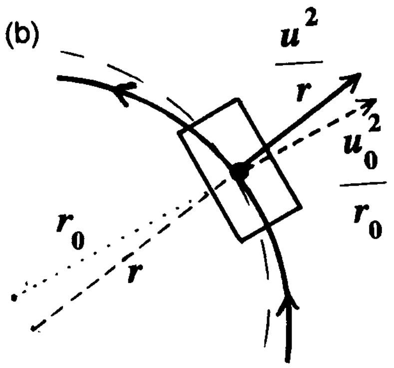

Fig. 2(b) If a passenger is moving radially inside the vehicle, in this case from left to right, his velocity, u, *trajectory, and radius of curvature,* r, *will be slightly different and thus the centrifugal force acting on them. The difference will be a component pointing to the right of his movement inside the vehicle. It is this difference which accounts for the Coriolis effect and which L. F. Richardson was intent on measuring in 1945.*

passenger's movements inside the bus, so does the centrifugal force on any body moving within a rotating system. Its velocity and/or trajectory will no longer be the same as if it had been fixed to the rotating system. Consequently the centrifugal force will be different in magnitude and/or direction compared to a stationary body. It is this difference which accounts for the 'Coriolis effect' (Fig. 3).

Remember from Part 1 (Persson 2000) that the man walking tangentially on a merry-go-round was subject to a centrifugal acceleration of about 30 m s^{-2}. Depending on whether he moved with or against the rotation this would increase or decrease to 36 or 23 m s^{-2} respectively. The difference of 6–7 m s^{-2} is the contribution from the Coriolis effect. The deflection for radial motion is due to a change of the centrifugal acceleration, because of a change to the radius of curvature. A radial motion will be seen from outside to follow a differently curved trajectory (Fig. 4).

The total centrifugal force, C, on a body moving within the rotating system can therefore be decomposed into two components, one being the radial centrifugal force, which we normally call the centrifugal force, $m\omega^2 r$. The other component, directed from the centre of rotation, Coriolis found to be $2m\omega\mathbf{V}$. This is the Coriolis force (Fig. 5).

It is worth noting that Coriolis was not interested in 'his' force as much as we are. He only valued it in combination with the radial centrifugal force, to be able to compute the total or compound centrifugal force, which was important for the designs of the machines that he was interested in.

'Fictitious force'

In his 1835 paper Coriolis (1835) was not content with a mere geometrical treatment of the problem, but provided a thorough dynamical analysis, which even took into account the forces that were needed to make the body remain within the rotating system. Thanks to this he was able to make the important observation that since the deflection is always at right angles to the motion, and has no component along the movement, it can only deflect the motion, not increase or decrease the velocity and thereby the kinetic energy. Changing the kinetic energy of a body is called in mechanics 'doing work' on the body. One important lemma to Coriolis's paper is therefore that the Coriolis force cannot do work.

The fundamental reason that prevents the Coriolis force from doing work is that it, like the centrifugal force, is a consequence of inertia. It is therefore not a 'real' force such as gravity or electromagnetic forces which can increase or decrease the velocity of a body. Meteorological textbooks go to great lengths to

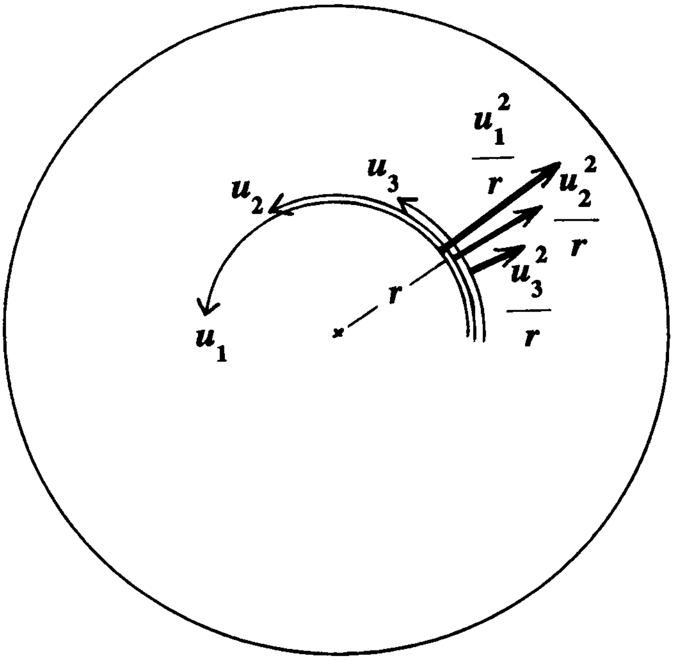

Fig. 3 A body fixed on a turntable rotating anticlockwise at a distance, r, *from the centre of rotation moves tangentially with a velocity,* u_2 *(seen from outside), and is affected by a centrifugal acceleration* u_2^2/r. *If the body also moves tangentially within the turntable, the centrifugal force will increase if the motion is in the direction of the rotation* ($u_1 > u_2$), *or decrease if the motion is against the direction of rotation* ($u_3 < u_2$). *This will increase or weaken the centrifugal accelerations which constitute the Coriolis effect. Note that in either case a body will be thrown off the turntable, the Coriolis effect just makes this happen slightly quicker or slower depending on the motion within the turntable.*

impress on the reader that the Coriolis force due to its inertial nature is 'fictitious', 'artificial', a 'pseudo force' or even a 'mental construct'. The centrifugal force, which of course is equally 'fictitious', is rarely talked about in this way. This might easily mislead an innocent reader into believing that some 'fictitious' forces are more fictitious than others.

Coriolis's last years

After his publication in 1835 of the nature of the deflective force, Coriolis's career continued to rise. He was promoted to higher positions at l'École Polytechnique and was elected into the Academie de Science. In 1843 his health got worse, but he struggled on with his scientific work. His main aspiration was to finalise an update of his 1829 book on mechanics, now with the title *Traité de la mechanique des corps solides*. He died on 19 September 1843 and was buried in the Montparnasse cemetery. All could have ended there and then, but a chain of events would bring his name into a broader scientific environment.

Foucault's pendulum

In the 1840s Jean Bernard Léon Foucault (1819–68), a pioneer in astronomical photography, brought into practice a clock with a conical pendulum. Foucault noticed that the steel rod to support the bob of its pendulum tended to maintain its plane of vibration when the lathe was rotated by hand. This unexpected behaviour of the rod suggested an experimental demonstration of the earth's rotation using a much larger pendulum. When the bob was set free the plane of swing slowly, very slowly, rotated clockwise and completed a circle in about 32 hours. Foucault's experiment attracted wide scientific and popular attention, was soon repeated all over the world, and came to have a revolutionary effect on physics in general and meteorology in particular. It was from learning about Foucault's success, and reading

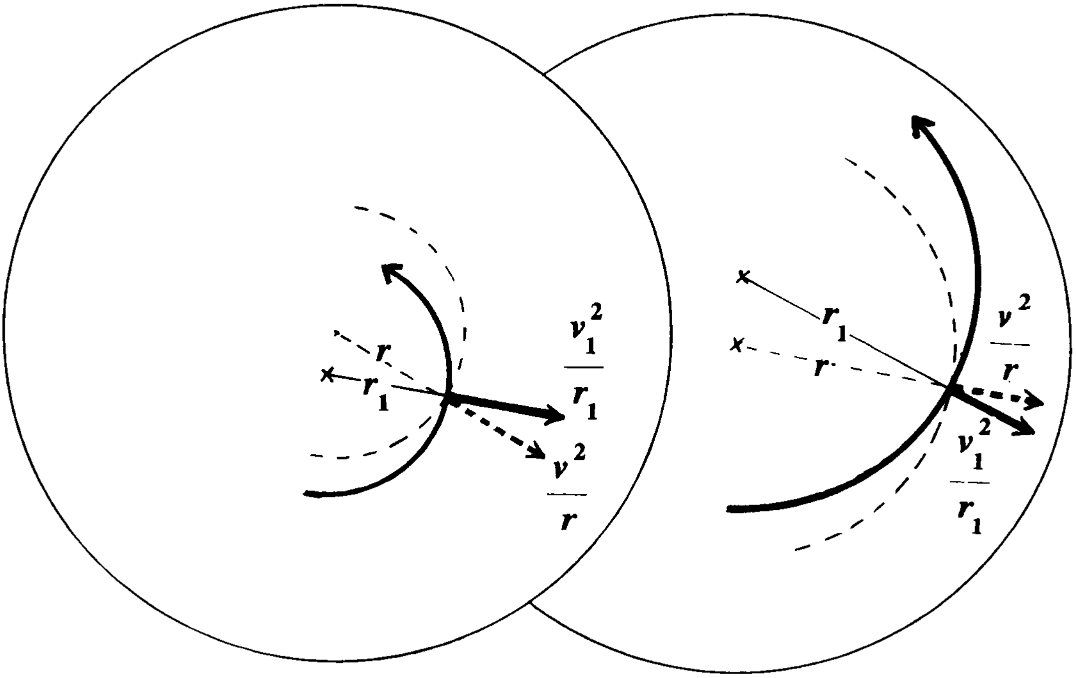

Fig. 4 For radial motion the trajectory of the body's motion is no longer a circle but a spiral. If (a) the motion within the turntable (v_1) is inward, the trajectory will be an inward spiral, with an increased curvature, a shorter radius of curvature ($r_1<r$) and a stronger centrifugal force v_1^2/r_1 which no longer will have a radial direction. If (b) the motion within the turntable (v_2) is outward, the same reasoning will indicate that the centrifugal force is weaker and pointing in a non-radial direction.

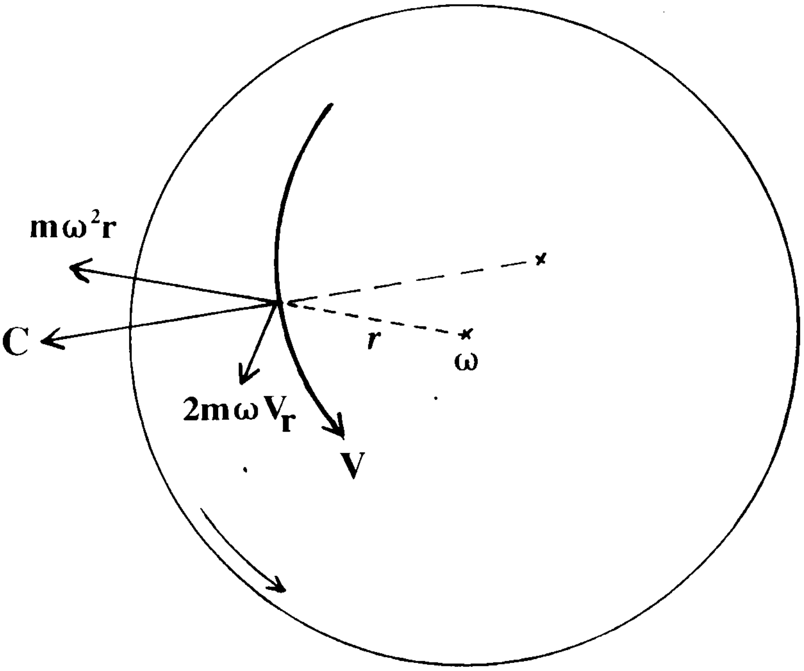

Fig. 5 The relation between the total centrifugal force, the radial centrifugal force and the Coriolis force. For any motion, $V=\omega r+V_r$, *the total centrifugal force, C, can be decomposed into one component pointing radially outwards, the radial centrifugal force* $m\omega^2r$; *the remaining component, pointing at right angles to the motion,* $2m\omega V_r$, *is what we call the Coriolis force.*

Newton's and Pierre Simon Laplace's (1749–1827) works, that made the American William Ferrel (1817–91) conclude in 1856 that the direction of the wind is parallel to the isobars, and that its speed is dependent on the latitude and the horizontal pressure gradient (Khrgian 1970, p. 222; Kutzbach 1979, p. 36–38). At the same time, but independently of Ferrel, the Dutch meteorologist C. H. D. Buys Ballot (1817–90) published his rule based on empirical data, according to which low pressure is to the left if you have the wind in your back.

Where does Coriolis fit in?

Neither Foucault, nor Ferrel, nor Buys Ballot had read Coriolis's paper. The first time that his work was brought into earth sciences was in June 1859 at the French Academy. On this occasion a comprehensive discussion took place on the role of the deflective force in relation to the problems associated with water currents in channels or rivers (Khrgian 1970, p. 222; Kutzbach 1979, p. 92; Gill 1982, pp. 210, 371). But it was a long time before Coriolis's name started to appear in the meteorological literature. When meteorologists in the second half of the nineteenth century were taught about the effects of the rotation of the earth it was the findings of Foucault, Ferrel and Buys Ballot that were referenced.

The name 'Coriolis force' was not used until the early 1920s, but then was simply attached to a mechanical effect which had been explored by others. Nothing in today's meteorology would therefore have been different if Coriolis and his work had remained forgotten. The old name 'deflective force' would just have stayed or it would have been named after Foucault, Ferrel or Buys Ballot. The question has indeed been raised whether Gaspard Gustave Coriolis has any place in the science of meteorology (Burstyn 1966; Landsberg 1966). This presentation shows that he is indeed well qualified to lend his name to the Coriolis force. If Coriolis was with us today, he might be one of the few who would understand the deflective force properly!

It is by applying Coriolis's physical approach that we can understand how the Coriolis force acts in the earth–atmosphere system. One would imagine that this would be rather difficult since we are dealing with motion on a rotating ellipsoid. But on the contrary – when we move to our earth things become much more easy! In Part 3 we will consider: "The Coriolis force on the physical earth".

Acknowledgements

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Global and regional climate in 1999

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Global climate

The average temperature near the surface of the earth in 1999 was the 5th highest so far recorded, an estimated 0.33 degC higher than the 1961–90 average (Fig. 1(a)). This is based on air temperature data collected from over 1000 land-based weather stations (Jones *et al.* 1999), plus sea surface temperatures (SSTs) (Parker *et al.* 1995) measured from the Voluntary Observing Fleet of about 7000 ships and 1000 buoys. The year was much less warm on a global average than 1998, which was the warmest in the instrumental record and 0.58 degC warmer than average (Fig. 1(a)). The difference between 1998 and 1999 is statistically significant at the 99% level of confidence; the standard error of recent global, annual averages is about 0.06 degC owing to large gaps in the data coverage, especially in the Arctic and Antarctic (Jones *et al.* 1997).

The annual anomalies of 0.45 and 0.20 degC for the Northern and Southern Hemispheres were 5th and equal 10th highest on record respectively (Figs. 1(b) and 1(c)). The associated standard errors are about 0.07 and 0.10 degC (Jones *et al.* 1997), so that the coolings since 1998 were statistically significant at the 95 and 90% levels respectively. Temperature anomalies in the tropical belt 20°N to 20°S (Fig. 1(d)) were close to the 1961–90 average, but the northern and southern extratropics (Figs. 1(e) and 1(f)) had their second and third warmest years respectively.

Overall global warming at the surface (Fig. 1(a)) is now estimated at 0.6 degC since 1861, or 1901, but owing to shorter-term variations and incomplete sampling these estimates have 95% confidence limits of ±0.2 degC. The cooling from 1998 to 1999 does not signify the reversal of this trend; the reason for the reduced warmth in 1999 was the persistent La Niña event which had developed in the tropical Pacific Ocean late in 1998. No year in the instrumental record with a major La Niña event was as warm globally as 1999.

The annual pattern of temperature differences from 1961–90 climatology (Fig. 2(a)) shows that SST anomalies were lower than −0.5 degC over most of the eastern and central tropical Pacific. Anomalies were lower than the 10th percentile relative to the period 1961–90 (Fig. 2(b)) in much of a swathe from the central equatorial Pacific to the western coast of North America. A warm tongue extending eastwards from Asia at about 30°N was an extension of a belt of warmth about 30° wide extending from North America, across the mid North Atlantic and Eurasia. This pattern of warmth and cold was consistent throughout all the seasons.

In northern winter 1998/99 (Fig. 3), there was a large area of warmth over Asia, and the Atlantic Ocean had a warm region centred over the Azores. Colder than normal areas included a region in the central southern Indian Ocean, and the equatorial Pacific, close to the date line. In northern spring the pattern was similar but northern Asia was cool and there was a small area of unusual warmth centred over Labrador. In northern summer, parts of eastern Europe and the Middle East experienced