Back to basics:
Coriolis: Part 1 – What is the Coriolis force?

Anders Persson
Reading, Berkshire

Being 'basic' does not always mean 'easy'. This is particularly true for one of the basic concepts in meteorology, the Coriolis force, named after a French nineteenth-century mathematician. Another name is the 'deflective force', which tells us what it is all about: on our rotating earth it will try to make any body moving over its surface deviate to the right in the Northern Hemisphere, to the left in the Southern Hemisphere. The magnitude of the Coriolis force determines much of the character of the dynamics of the oceans and atmosphere, from the formation of monsoons and cyclones to the set-up of local sea-breezes (Persson 1997). The strength of the Coriolis force is proportional to the mass of the body, m, its velocity, $\mathbf{V}$, and the so-called Coriolis parameter, $f = 2\Omega\sin\phi$, where Ω is the angular velocity of the earth (2π/one day = $7.292\ 10^{-5}\,s^{-1}$) and ϕ the latitude. The Coriolis parameter, f, is zero at the equator and increases towards the poles. At European latitudes the Coriolis force, $f\mathbf{V}m$, is just strong enough to have a possible influence on the outcome of a golf tournament (Fig. 1).

For a long time the Coriolis force was derived in a rather complicated way using trigonometrical and geometrical techniques. It is only during the last 30–40 years that it has been replaced by a vector algebraic derivation which makes use of the kinematic relation between an acceleration in a fixed system and in a rotating system (French 1971, p. 521; Batchelor 1967, p. 139; Pedlosky 1979, p. 16). The expression for the Coriolis force is then $-2m\boldsymbol{\omega}\times\mathbf{V}$ where $\boldsymbol{\omega}$ is the angular velocity of the rotating system.*

* Mathematically inclined readers should note the convention to express the Coriolis acceleration as $+2\boldsymbol{\omega}\times\mathbf{V}$ and the Coriolis force as $-2m\boldsymbol{\omega}\times\mathbf{V}$, which makes them point in opposite directions. This has confused generations of meteorologists, some of whom in their writings have made it appear as if the Coriolis deflection is to the *left* (Scorer 1958, p. 21, 1997, p. 113; Green 1998, p. 29; Meteorological Office 1991, p. 74). Physically, $+2\boldsymbol{\omega}\times\mathbf{V}$ should be understood as the acceleration we have to impose on a moving body by some real force in order to keep it in a steady course, preventing it from being deflected (Lambe 1959, p. 239).

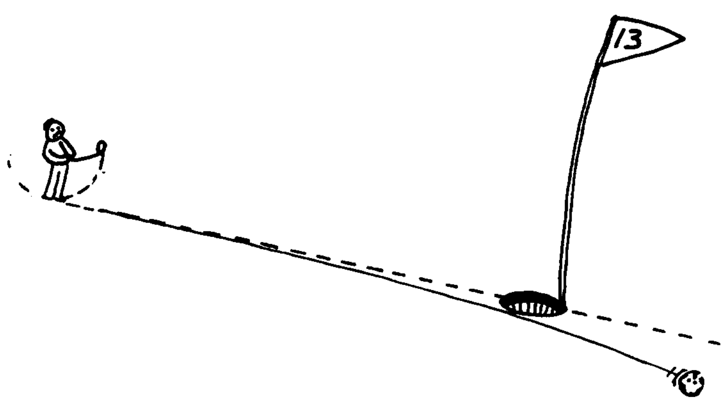

Fig. 1 A golf ball putted on a frictionless green in central Europe (at latitute 43°N where f = $10^{-4}\,s^{-1}$) with a speed of 2 m s^{-1} will, after 15 m, have deviated 5 mm to the right, enough to risk missing the hole. In southern Europe the deviation would be just 4 mm, in northernmost Scandinavia it would be 7mm.

Invoking vector algebra facilitates our understanding in a surprisingly simple manner. The cross-product between two vectors takes its largest value when they are perpendicular and is zero when they are parallel. Since the vector which represents the spin of the earth, $\boldsymbol{\omega}$, is parallel to the earth's axis, it follows that the Coriolis force is strongest for all motions, $\mathbf{V}$, perpendicular to the axis, and vanishes for all motions parallel to it. So in the mid-latitudes the Coriolis force acts strongest on air rising equatorward or sinking poleward, and is weak or vanishes for air rising poleward or sinking equatorward (Fig. 2).

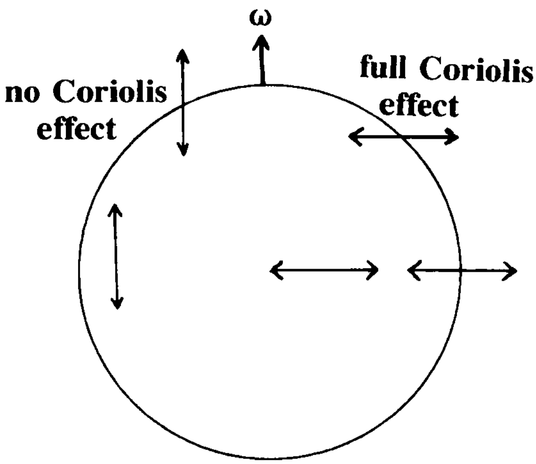

Fig. 2 The Coriolis deflection takes its maximal values for motions perpendicular to the earth's axis and vanishes for motions parallel to the earth's axis. The Coriolis effect is, contrary to popular belief, also active on the equator where vertical motion will be deflected zonally, zonal motion deflected in the vertical.

However, a correctly performed mathematical derivation is not enough; one must also have an intuitive grasp of what is physically going on. It is when textbooks, after a rigorous mathematical derivation, try to do this that all the confusions come to the surface. In France teachers in meteorology and oceanography are encountering "serious difficulties" with their students over the introduction of the Coriolis acceleration (Genty 1994), and in the USA leading oceanographers have criticised the "incomplete explanations" of the Coriolis force that abound in popular books and magazines: "The sense of frustration that overcomes those who try to understand explanations of meteorological and oceanographical phenomena can thus be accounted for" (Stommel and Moore 1989, p. 2).

This is nothing new. An American meteorologist admitted 50 years ago that the meteorological concept he, as a teacher, found "least satisfactorily" presented in textbooks was the Coriolis effect (McDonald 1953), and in 1920 the chief of the US Weather Bureau wrote about the deflection due to the earth's rotation: "It is very suggestive of the profound obscurity of this subject to recognize that it has occupied the attention of scientists for fully 200 years; nevertheless several of the most recent writings contain erroneous statements concerning its application in both meteorology and astronomy" (Marvin 1920, p. 567).

In the leading German meteorological journal, *Meteorologische Zeitschrift*, there were intense debates about the proper understanding of the deflective force from the 1880s to around 1925. I have not seen the Coriolis force being debated in any British publication – so this is a first!

The reason why so many attempts to physically explain the Coriolis effect fail is because they try to make a direct physical interpretation of the mathematical derivations, and in doing so choose the wrong system or apply inadequate physical principles. There seem to be three different ways of getting it wrong, and these will be discussed in the following sections.

First confusion: Relative motion

One can often read that a body moving over the earth's surface is deflected because it moves into latitudes which rotate with different speeds. This explanation, based on the principle of conservation of velocity, has its roots way back in the early eighteenth century when George Hadley (1685–1768) speculated about the causes of the trade winds. Hadley noticed that, since the circumference of the earth at the equator is about 3332 km larger than the circumference at the Tropics of Cancer and Capricorn (23°N and 23°S respectively), the surface of the earth at the equator, and its air, moves faster than the surface of the earth at the higher latitude:

"From which it follows, that the Air, as it moves from the Tropics towards the Equator, having a less Velocity than the Parts of the Earth it arrives at, will have a relative Motion contrary to that of the diurnal Motion of the Earth in those Parts, which being combined with the Motion towards the Equator, a N.E. Wind will be produc'd on this Side of the Equator, and a S.E. on the other" (from Hadley's paper reproduced in Shaw 1979).

Hadley's reasoning appears, at first sight, alright since it yields a deflection in the correct direction. However, a closer mathematical examination reveals that it accounts for only half of the deflection. His explanation also gives the impression that the deflection affects only north–south motion, when in reality it works for motions in all directions. In any case, Hadley's explanation was a scientific advance at its time since, almost for the first time, it suggested the rotation of the earth as an important mechanism in the atmospheric circulation. Before Hadley there were all kinds of strange explanations of the trade winds, such as exhalations from the sargasso weed in the subtropical parts of the oceans.

What is difficult to understand is why, 265 years later, many prestigious publications, for example the *Oxford paperback encyclopedia, The Oxford concise dictionary of science* and the *Oxford dictionary of physics*, continue to promote Hadley's outdated and incomplete explanation and almost copy his misleading mathematical discussion (see Fig. 3):

"The daily rotation of the earth means that in 24 hours a point on its equator moves a distance of some 40 000 kilometres giving it a tangential velocity of about 1670 kilometres per hour. A point at the latitude of, say, Rome, travels a shorter distance in the same time and therefore has a lower tangential velocity – about 1340 km/h. Air over the equator has the full tangential velocity of 1670 km/h and as it travels north, say, it will retain this velocity; to an observer outside the earth this would be clear. However, to an observer in Rome it appears to be moving eastward, because the earth at that point is moving eastward more slowly than the air.

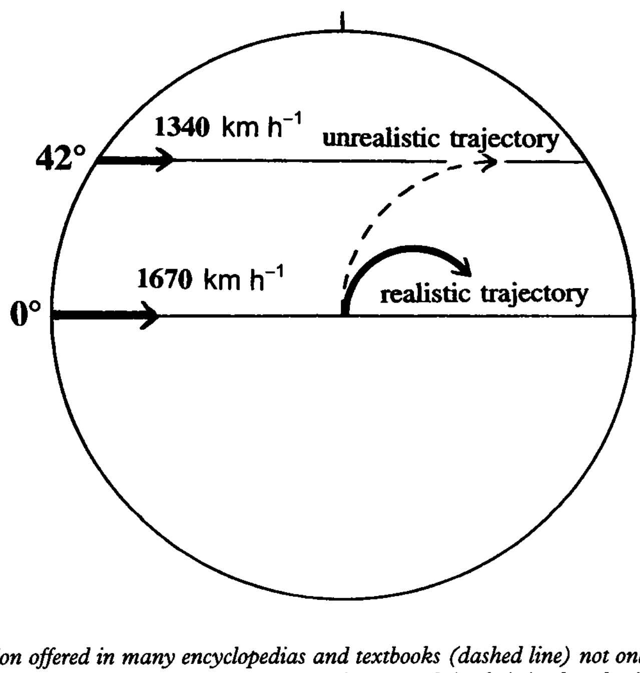

Fig. 3 The explanation offered in many encyclopedias and textbooks (dashed line) not only underestimates the Coriolis force by half and leaves the deflection due to east–west motion unexplained, it is also physically unrealistic. Any airflow from the equator would have enormous difficulties in moving meridionally and would be deflected back due to the strength of the Coriolis force (solid line). A body moving with a velocity of, for example, 10 m s^{-1} from 8°N or S would turn back towards the equator before reaching 10°N or S (Brunt 1934, p. 162, 1941, p. 166).

The Coriolis force (which is quite fictitious) is the force that a naïve [*sic*!] observer thinks is needed to push the air eastward."

Students in Oxford are well advised to notice that a boat rowing with a speed of $6\,m\,s^{-1}$ will, during 2 minutes, due to the earth's rotation, be deflected almost 5 m to the right, not just 2.5 m which the eighteenth-century explanation would yield. Incidently, the Cambridge dictionaries provide an up-to-date and correct presentation of the Coriolis effect.

Second confusion: The Foucault pendulum

Another way to explain the Coriolis force physically is to imagine that we are at the North Pole rigging up an apparatus for a so-called Foucault pendulum. Standing beside the pendulum we will see how the plane of swing slowly changes orientation due to the Coriolis force acting on the bob. It will take almost exactly 24 hours (23 h 56 min) for the plane of swing to come back to its original position. Since the Coriolis force weakens the further we come from the poles, the longer the progression takes: at London's latitude it takes 30 hours, at Cairo's 48 hours. At the equator, where there is no deflection, the progression will take an infinitely long time. So far so good.

What is *not* correct is to say that due to its inertia the plane of swing of the pendulum will remain faithful to its original arc relative to the fixed stars, while the ground is moving under it due to the earth's rotation (see, for example, *Encyclopedia Britannica*, "Gyroscope, History" or Gordon *et al.* 1998, p. 77). This is true only at the poles. If the pendulum "remains faithful to its original arc" also at other latitudes, it would have taken 24 hours to complete a period there also. In reality the pendulum motion is affected by the earth's gravitational pull that is constantly changing direction relative to the fixed stars as the earth spins around its axis. The pendulum bob is therefore not moving under inertia and the arc of swing is slowly progressing relative to the fixed stars – except at the poles.

Third confusion: Coriolis on the merry-go-round

The *Encyclopedia Britannica* has a lot of confusing and hilarious references to the Coriolis effect. In its "Easy Reference" it states that the Coriolis effect has no relation to the deflection of the paths of rivers. However, under the entry "Coriolis acceleration" on the same page it is said that it has! The chapter "Mechanics, Classical" (Vol. 11) provides the reader with a correct, but somewhat complicated, exposé of the Coriolis force, but in the chapter "Ballistics" (Vol. 2) we are back to eighteenth-century thinking.

In its chapter about "Motion Sickness" (Vol. 12) *Britannica* warns the reader against being on a ship which "pitches and rocks simultaneously", since this will, due to the

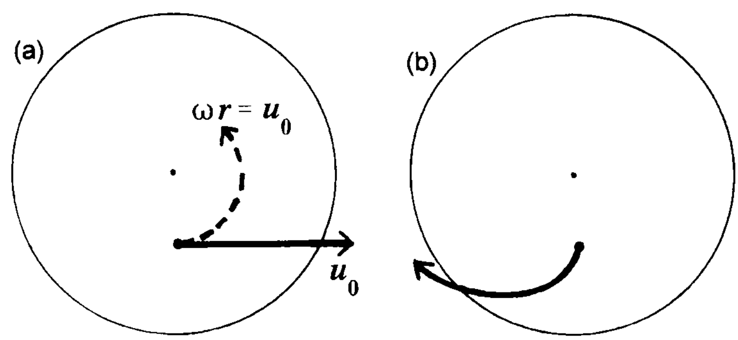

Fig. 4 A disc is rotating with an angular velocity, ω. A body has been fastened to the disc at a distance, r, where it has a tangential velocity, $u_0 = \omega r$. Suddenly the body is set free and glides off a rotating turntable with a speed which is equal to the tangential velocity, u_0. Seen by an observer from outside (a) it follows a straight line; seen by an observer on the turntable (b) it is deflected to the right.

Coriolis effect, upset the ear's balance control. In both cases any discomfort is probably due more to the centrifugal force than the Coriolis force*. A merry-go-round is therefore not the best place to feel the Coriolis effect. A common way to explain the Coriolis deflection is to imagine a ball thrown or rolled over a turntable rotating anticlockwise. From outside, the ball is seen to move in a straight line; seen from the rotating turntable, the ball appears to be deflected to the right. However, most of what we see here is not the Coriolis force at work, but the centrifugal force! If the body had at first been stationary on the disc (and therefore not subjected to any Coriolis effect) and then suddenly fallen off, it would from outside have been seen to follow a straight path, but to an observer on the turntable it would appear to be deflected to the right.

If the body at the same time was moving relative to the disc, the Coriolis effect would indeed come into play (Fig. 4). The man in our previous example walking on the merry-go-round would, if he had walked in the same direction as the rotation, have experienced a centrifugal acceleration of $36\,\mathrm{m\,s^{-2}}$ or, if he had walked against the rotation, $23\,\mathrm{m\,s^{-2}}$. But he would nevertheless have been thrown off the turntable, just slightly quicker or slower (Fig. 5).

* If m is the mass of the body, u its tangential velocity, r the radius of curvature of the motion and $\omega = u/r$ the angular velocity, then the formula for the centrifugal force is mu^2/r or $m\omega^2 r$, and the Coriolis force $2m\omega u$. A man walking on a merry-go-round, making one revolution in 2 seconds ($\omega = \pi$), at a pace of $1\,\mathrm{m\,s^{-1}}$ ($u = 1$) and 3 m from the centre of rotation ($r = 3$), will experience a centrifugal acceleration ($3\pi^2 = 29.6\,\mathrm{m\,s^{-2}}$) that is about five times stronger than the Coriolis acceleration ($2\pi^2 = 6.3\,\mathrm{m\,s^{-2}}$). Only at a distance of 0.6 m from the centre are the two accelerations of equal strength.

We seem to have come far away from the Coriolis effect, but it was actually when thinking along these lines that Gaspard Gustave Coriolis discovered 'his' force in 1835. In Part 2 I will discuss "The Coriolis force according to Coriolis".

Acknowledgement

At an early stage in my exploration of the Coriolis effect it was David Anderson, François Bouttier, George Platzman, Adrian Simmons and Jim Holton who patiently convinced me that some traditional truths are not wrong, just because they are traditional. At a crucial stage in my learning process it was Norman Phillips who put me on the right track by pointing out the flaws in the popular explanations involving merry-go-rounds and pendulums.

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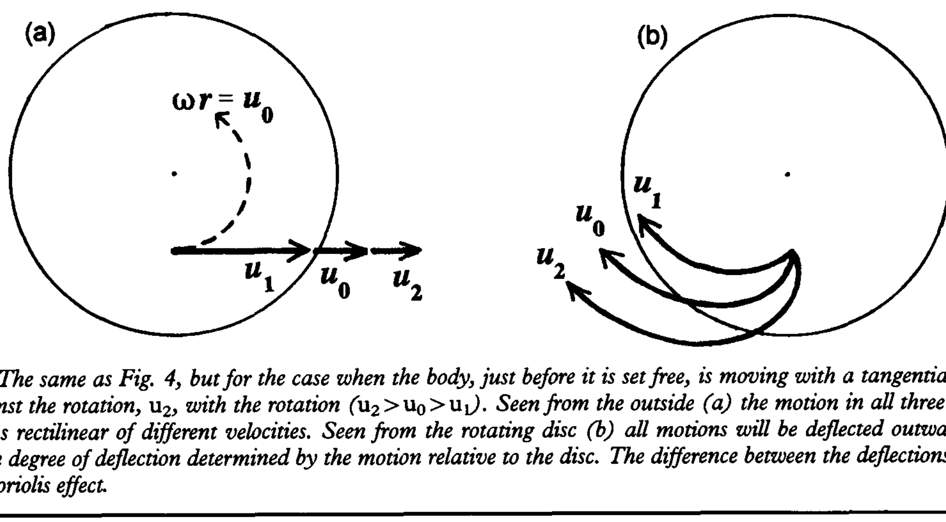

Fig. 5 The same as Fig. 4, but for the case when the body, just before it is set free, is moving with a tangential velocity, u_1, *against the rotation,* u_2, *with the rotation* ($u_2 > u_0 > u_1$). *Seen from the outside (a) the motion in all three cases will appear as rectilinear of different velocities. Seen from the rotating disc (b) all motions will be deflected outward, to the right, the degree of deflection determined by the motion relative to the disc. The difference between the deflections constitutes the Coriolis effect.*

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Correspondence to: Mr A. Persson, 60 Ryhill Way, Lower Earley, Reading, Berkshire RG6 4AZ.

Some comments on long-term trends observed in an east England relative humidity dataset

J. G. Lockwood
University of Leeds

The relative humidity dataset for Cranwell–Waddington, east England, 1920–95, described by Carter and Robertson (1998), is intriguing because they report a persistent overall trend toward less humid conditions throughout the period. This tendency is most marked in the summer, but is observed even in winter months. The relative humidity of moist air is defined as the ratio of the actual vapour pressure to the saturated vapour pressure at the same temperature. Although relative humidity is often regarded as a measure of the drying power of the air, its fundamental significance lies in the specification of the thermodynamic equilibrium between liquid water and water vapour (Monteith and Unsworth 1990). For a surface of pure water, equilibrium is established when the air in contact with the water is saturated so that the relative humidity is 100%. In contrast, when the water is held in a porous medium, such as the soil or plant leaves in a vegetation canopy, the equilibrium relative humidity is usually found to be less than 100%. The situation in the real atmosphere is further complicated by the continual replacement of air near the surface by air from aloft. This arises through convection and turbulence – processes normally most marked during daylight when solar heating is at its maximum. Thus, particularly in daylight, an airstream may travel a considerable distance across a landscape before it becomes even approximately in equilibrium with the surface. Calder (1990) comments that there is considerable uncertainty over the scale and extent of this process. He considers that there are two distinct mechanisms that may influence the process towards equilibrium:

(i) A change in surface characteristics, either through water availability (such as a change in soil moisture) or through aero-