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The Coriolis force and the geostrophic wind

(Coriolis Part 5)

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"I always find my pen sticks to the paper and refuses to move when I try to draw an isobar across the equator."

(Shaw 1923, p. 51)

There seem to be many ways to misunderstand the Coriolis force. Some textbooks explain the Coriolis effect by assuming a priori that a body, moving over the earth's surface, unaffected by any other real forces than gravitation, will follow a great circle.* From this they draw the conclusion that when an aircraft following a great circle, initially heading straight eastwards along a latitude circle, deviates to the right, relative to this latitude circle, this is caused by the Coriolis force (see *e.g.* Ahrens 1985, p. 241). But everything that is going to the right in the Northern Hemisphere is not doing so because of the Coriolis effect. The same aeroplane heading westwards along the same great circle will be deviated to the left relative to the latitude circle (Fig. 1).

As we learned from G. I. Taylor's experiments (Persson 2001), the peculiarity with the Coriolis force is not primarily that it 'deflects', but rather that it resists all displacements by forcing every moving object into a circular motion, thereby trying to restore the original conditions (Batchelor 1967, p. 556). In the atmosphere and oceans any parcel of air or water will, if it is in frictionless unaccelerated motion, V, follow, not a great circle, but a so-called 'inertia circle'. The radius of this circle, R, is V/f, where f is the Coriolis parameter ($2\Omega\sin\phi$, where Ω is the angular velocity of the earth and ϕ the latitude).

* This was also what L. F. Richardson assumed a priori when he was thinking about the Coriolis force in winter 1945/46 (Persson 2000). This erroneous assumption led him into contradictions, which he was unable to sort out, and he therefore never published anything on this subject.

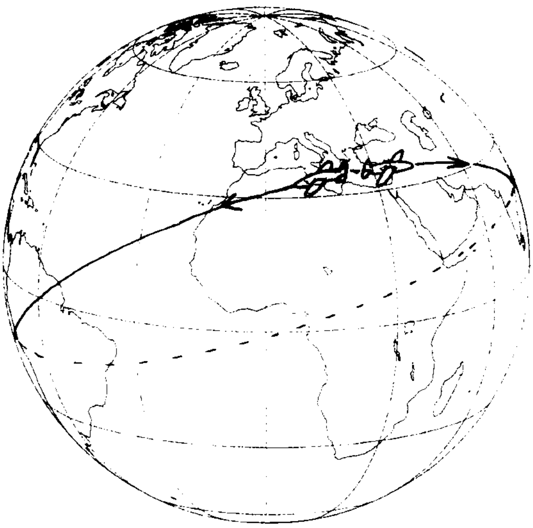

Fig. 1 The deviation of an aeroplane initially moving along a latitude, following a great circle, has nothing to do with the Coriolis effect since the deviation is always to the south in the Northern Hemisphere for westward and for eastward motion

At mid-latitudes, where f is typically $10^{-4}\,s^{-1}$, the radii will take surprisingly small values. A bird or an air parcel passively drifting northward from Heathrow (at 52°N) at 2–3 m s^{-1}, under the influence of the Coriolis force alone, will follow a circular path with a radius of curvature of 25–30 km, more or less following the M25! If the speed is 10 m s^{-1}, the circular path will have a radius of 100 km and the bird will reach no further than the Pas de Calais region of northern France on its continental day trip before it returns to south-east England after 18 hours* (Fig. 2).

In fact, the air in the free atmosphere would, due to the Coriolis force, have very large difficulties in going any great distance. Wind speeds of 30 m s^{-1}, typical for the large-scale jet streams in the upper troposphere, would, in the mid-latitudes, confine air parcels to an area smaller than France. Even in the equatorial

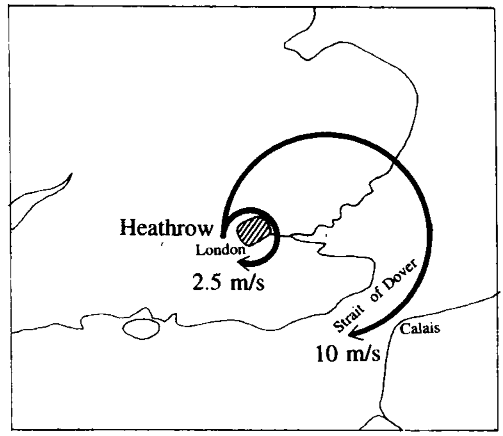

Fig. 2 A parcel of air moving at 2.5 or 10 m s^{-1} northwards from London will be confined over south-eastern England and the Strait of Dover respectively by the Coriolis force

regions, where the horizontal component of the Coriolis force is weak, an air parcel would need speeds of the order of 40–50 m s^{-1} to move under inertia poleward of 20° latitude.† The Coriolis force should therefore obviously be regarded as a strong force. Consequently, another strong force is required in the atmosphere to move air any great distance horizontally. This other strong force is the horizontal pressure gradient force (PGF). The PGF per unit mass is:

$$\frac{PGF}{m} = -\frac{1}{\rho}\frac{\partial p}{\partial n}$$

where $\partial p/\partial n$ is the horizontal variation of pressure, m is the mass of air and ρ is the density.

The PGF

While the Coriolis force always tries to restore differences, the PGF tries to equalise the pressure differences by moving air, or water in the oceans, from areas of higher to lower pressure. What we see on the daily weather maps is, in a nutshell, the eternal struggle between the PGF trying to equalise the pressure distribution and

* The time, τ, to complete a revolution of the circumference of the circle, $2\pi R$, is the same as τV which yields $\tau = 2\pi R/V$ or $\tau = 2\pi/f$ seconds. Since this time can also be written as 12h sinϕ^{-1}, which is the time it takes a Foucault pendulum to complete half a revolution, the name Foucault day (or rather Foucault hour) was introduced in earth sciences by the Norwegian astronomer H. Geelmyuden about 100 years ago.

† The same applies to the oceans. The curved motion of the Gulf Stream is not a direct consequence of the Coriolis force, as is occasionally stated in popular science books. Had this been the case, with its speed of 1 m s^{-1}, the whole Gulf Stream would be confined to an area the size of London!

the Coriolis force, mfV, trying to restore the differences. This struggle sets the air in accelerated motion, $\dot{V}$.

$$\dot{V} = \frac{PGF}{m} - fV. \qquad (1a)$$

With the Coriolis force and the PGF pulling in different directions it is no surprise that they often cancel each other and leave the wind moving by itself with no net acceleration. Such a wind, $\mathbf{V}_g$, moving under a balance between the PGF and the Coriolis force, is called geostrophic.* With $\dot{V} = 0$ Eq. (1a) can be written

$$\mathbf{V}_g = \frac{PGF}{mf}. \qquad (1b)$$

A geostrophic flow is parallel to the lines of equal pressure (isobars). Airflow which is not in geostrophic balance (in strength and/or direction) is termed non-geostrophic. The difference between the geostrophic and non-geostrophic wind is called the ageostrophic wind. The fact that the horizontal component of the Coriolis force is zero at the equator does not, of course, mean that there are no winds or isobars there, just that the relation between the wind and the pressure is not geostrophic; the wind is accelerated directly from high to low pressure.

The geostrophic balance

If there are deviations from geostrophic balance, the PGF and the Coriolis force together provide a mechanism that will act to restore the geostrophic balance by moving air, and thus mass, horizontally between lower and higher pressure.

* The relation between the horizontal pressure distribution and the wind was discovered when the first synoptic weather maps were made in the first half of the nineteenth century. In the 1850s the Dutch meteorologist Buys Ballot defined the famous rule that low pressure is to your left if you stand with the wind in your back (Shelders and Schnuurmanns 1980). The American mathematician William Ferrel confirmed mathematically that this relation was due to the rotation of the earth (Kutzbach 1979). The word 'geostrophic' was coined about 100 years ago by Sir Napier Shaw from Greek *geo* – earth and *strophy* – turning (Gold 1963; Taylor 1963; Ockender 1963).

If the wind is sub-geostrophic, *i.e.* weaker than the geostrophic balance requires, the PGF will accelerate the air towards lower pressure. Mass will be transported from higher to lower pressure and thereby weaken the PGF. At the same time the wind will increase, and so will the Coriolis force, until a new temporary balance with the PGF is established (Fig. 3(a)).

The increase in wind means an increase in kinetic energy, which is taken from the potential energy 'stored' in the horizontal pressure differences. To change the velocity and thereby the kinetic energy of a body, a force is required. In the language of mechanics this force is said to 'do work' on the body. In this case of the wind the work is done by the PGF and is positive since the wind is increasing.

Now the correct understanding of 'work' becomes crucial. It is a common source of confusion that because the Coriolis force cannot 'do work', it is not doing anything! 'Work' is defined as the product of the distance a body is moving and the force acting on it in the direction of motion.

When a ball rolls downhill it speeds up, potential energy is converted to kinetic and positive work is done by gravity. When the ball, due to inertia, rolls uphill and slows down, work is done – not by inertia, but by gravity doing negative work since now the motion and the force point in opposite directions.

If the wind is super-geostrophic, *i.e.* stronger than the geostrophic balance requires, the Coriolis force will drive the air towards higher pressure. In doing so, mass will be transported from low to high pressure and thereby increase the PGF. At the same time the wind weakens, and so does the Coriolis force, until a new temporary balance with the PGF is established (Fig. 3(b)).

The decrease in the wind and thereby its kinetic energy means that again work is done; the question is – by which force? Since it is the Coriolis force that drives the wind towards higher pressure and thereby slows it down, one would assume that the Coriolis force does the work. But the Coriolis force, being just a reflection of inertia, cannot do work. Instead it is, as before, the PGF which does the work, although now the work is negative, since the wind is decelerating.

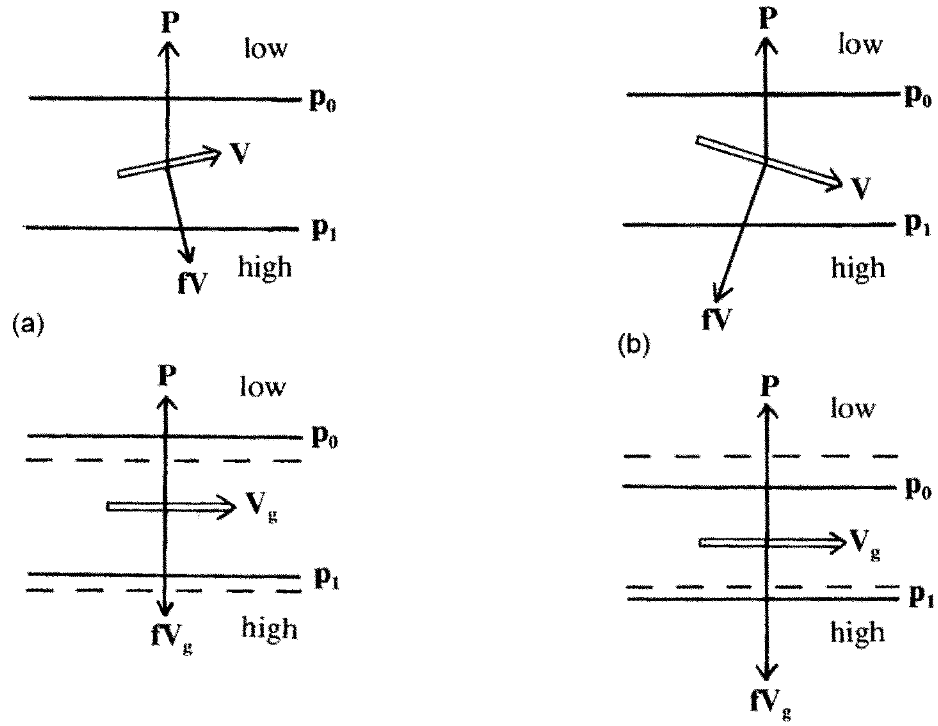

Fig. 3 Deviation from geostrophic balance will create restoring forces. (a) If the wind is weaker than the geostrophic balance requires, the pressure gradient force will accelerate it towards lower pressure until a new balance is established. (b) If the wind is stronger than the geostrophic balance requires, the Coriolis force will drive it towards higher pressure until a new balance is established. Dashed lines indicate the isobars at the initial time.

The gradient wind

The further away from the equator, the better the geostrophic wind agrees with the real wind. There are, however, a few factors which prevent the wind from being geostrophic. One is when the air moves in curved trajectories, for example forced by the pressure field. The air then also becomes affected by a centrifugal force, which disrupts the bilateral balance between the Coriolis force and the PGF. Centrifugal forces are always directed outward from the centre of rotation. The centrifugal force will therefore support either the Coriolis force or the PGF, depending on how the trajectory is curved. This in turn depends both on the shape of the pressure patterns and the changes and movements of these pressure patterns.

In a stationary high-pressure system in the Northern Hemisphere, where the air moves in clockwise trajectories, the centrifugal force will point in the same direction as the PGF. In a stationary low-pressure system, where the air follows anticlockwise trajectories, the centrifugal force will point in the same direction as the Coriolis force. This is why high-pressure systems normally have weaker pressure gradients, *i.e.* fewer isobars, and low-pressure systems generally have stronger pressure gradients which is reflected in more isobars (Fig. 4).

Where the PGF, the Coriolis force and the centrifugal force balance each other, the airflow is called gradient flow. In small, intense vortices, with high speeds and small radii, such as in tropical cyclones, the centrifugal force may exceed the Coriolis force. These are called cyclostrophic flows.

If the pressure field is changing, the relation between the centrifugal force, the PGF and the Coriolis force can be quite different and complicated. Some simple kinematic guidelines can help us to understand why in a moving vortex

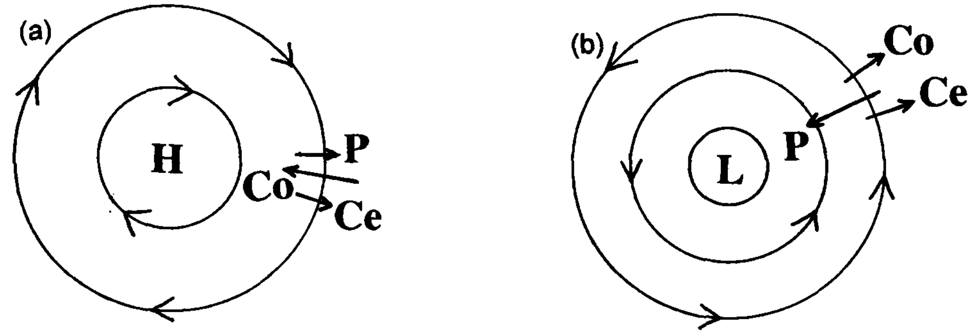

Fig. 4 The gradient wind approximation for anticyclonic and cyclonic flow. (a) For curved air trajectories around a high-pressure area the centrifugal force, Ce, *will support the pressure gradient force,* P. *(b) For curved trajectories around a low-pressure area the centrifugal force will support the Coriolis force,* Co.

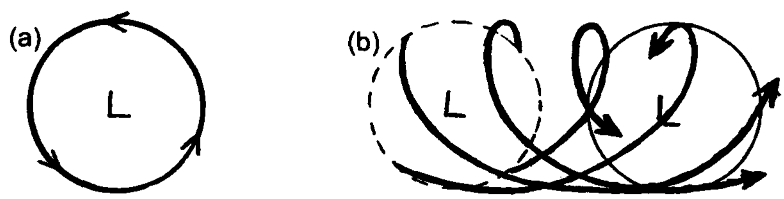

Fig. 5 (a) For a stationary circular vortex the trajectories will be circular; (b) for a circular vortex, moving eastwards (to the right of the picture), the trajectory of an air parcel on the side where the air is moving in the same direction as the vortex will have a less curved trajectory. On the opposite side, where the air is moving in the opposite direction, the trajectory will be more curved.

the trajectories will alternately be more or less curved, compared with those of a stationary vortex (Fig. 5). The winds on the southern (warm) side of a cyclone passing eastwards over the British Isles will be close to geostrophic because the trajectories of the air are rather straight (a large radius of curvature). On the other hand, on the northern side, the winds will be much weaker, although the pressure gradients might be the same, because the trajectories are more curved.

The ratio between V^2/L, the centrifugal force per unit mass, and fV, the Coriolis force per unit mass, is V/fL (where L is the radius of curvature of the air's trajectory (approximately the size of the system)) and is called the Rossby number.* The smaller the V/fL, the better the geostrophic approximation. A hurricane with a radius of 300 km at 10° latitude and winds of 30 m s^{-1} has the same high Rossby number (4) as a thunderstorm system with radius 10 km at 43° latitude and average winds of about 4 m s^{-1}. This indicates that the geostrophic approximation is not valid for either of them, but is for a mid-latitude cyclone with a radius of 1000 km and wind of about 10 m s^{-1} as the Rossby number is 0.1.

* Named by Fultz (1951) after the Swedish–American meteorologist C. G. Rossby. The Rossby number (originally discovered by the Russian meteorologist Kibel in 1940) also expresses other relations, such as the ratio between relative vorticity, $\sim V/L$, and absolute vorticity, $\sim f$, the ratio between the scale of a weather system, L, and typical length scale, and the radius of curvature, R, of an inertia circle.

The beta effect

Another complicating factor in atmospheric dynamics is the fact that the Coriolis force varies with latitude. Our bird, passively drifting northwards from Heathrow, will, as a matter of fact, not make a perfect circle, but will return some kilometres to the west of Heathrow. This is because the Coriolis force is stronger on the poleward side of the circular trajectory than on the equatorward, which makes the radius of curvature, $R = V/f$, shorter on the poleward side than on the equatorward (Fig. 6). The tendency to make air drift westwards in a spiral-

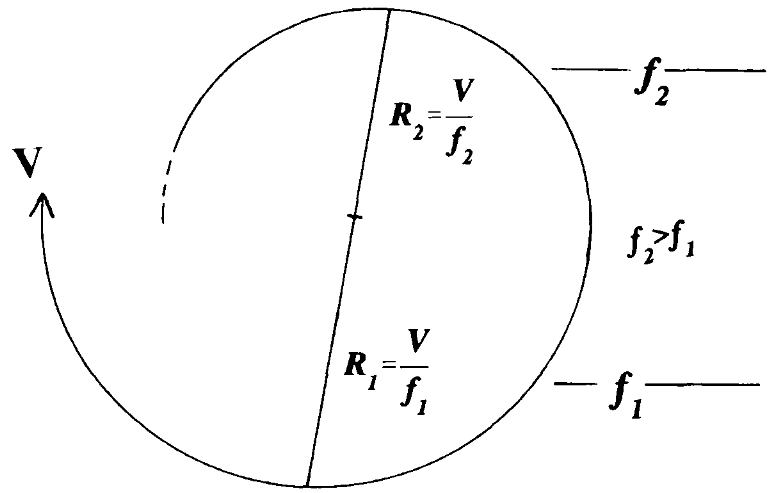

Fig. 6 The westward spiralling motion of a particle moving under inertia. The air parcel moves with speed V *between latitudes with Coriolis parameters* f_1 *and* f_2 *and moves polewards with an approximate radius of curvature* R_2*. Going equatorwards with decreasing Coriolis parameter, the radius of curvature will increase to about* R_1*, and make it arrive at the original latitude somewhat further to the west.*

ling motion leads to a slowing down of the eastward motion of atmospheric weather systems. Some large pressure systems might even become stationary or drift westwards. The beta effect plays an important role in the dynamics of the atmosphere and the oceans, where it is responsible for the asymmetric flow in the Gulf Stream.

In the next paper we will look at another factor which prevents the flow from being geostrophic, namely friction, and ask what snow showers along the East Anglian coast have in common with drifting icebergs, the sea surface temperature over the equatorial Pacific, and a famous Norwegian ambassador in London.

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Correction

The caption to Mr A. K. Overton's photograph which appeared on the front cover of the May issue was incorrect. It should have read: "Sunset through cirrostratus, with contrails, associated with a depression in the North Atlantic, looking towards Polruan from Lantic Bay, Cornwall, at approximately 1915 BST on 6 September 2000."