no green hills nor woods and, during what I took to be a steeply banked turn out to sea, I decided to momentarily transfer my gaze from the tail plane of the leader's aircraft, some 6 ft (2 m) above and 10 ft (3 m) ahead, and look around a little. All I saw was a dense grey, totally without detail, depth or horizon, and the result was immediate vertigo, disorientation and nausea, so that, on the assumption that my gallant leader was not about to dive into the sea, taking us with him, I immediately switched my attention back to the tail plane of his aircraft, upon which my stomach instantly returned to normal!

On the whole, rain has not much affected my flying, but there was one notable exception when, lined up on the middle of the runway ready for take-off, it was most interesting to discover that the rain was so heavy that forward visibility was reduced to less than 50 yards (46 m), only one of the runway centre line markings being visible. To each side was initially grey, blurring into green, while everywhere the rebounding splashes from the rain gave the impression that the runway was a 6 in (15 cm) deep pool of water. However, I was the possessor of a Master Green instrument rating, which gave me the opportunity to make my own decision as to whether or not to take off, and I decided to go. The runway was fitted with a barrier, this being something rather like a tennis net which normally lay on the ground some 75 yards (69 m) from the end of the runway, brought up to the vertical by two hydraulically operated steel posts. Having accelerated to about 80 kn in an enormous cloud of spray, it was not very comforting to hear my chum, who had started his take-off run some 15 seconds earlier, call "Barrier, barrier" over the radio. In the few seconds previous to this call, I had seriously been considering aborting the take-off, but a fully extended barrier net, full of my friends' aircraft, was not an enticing prospect so, realising that there was no option, we just had to keep on going. Fortunately for me, that is still the case, whether power flying or gliding (Fig. 2) I am able to keep on going!

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Proving that the earth rotates: The Coriolis force and Newton's falling apple

(Coriolis Part 9)

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> "All bodies that are put into a direct and simple motion, will so continue to move forward in a straight line, till they are by some other effectual powers deflected and bent into a Motion, describing a Circle, Ellipsis, or some other more compounded Curve Line."
>
> (Robert Hooke, *An attempt to prove the motion of the earth*, 1674.)

In the previous articles on the Coriolis effect we have seen how, due to the earth's rotation, horizontal motions are deflected at right angles horizontally and, for west–east motion, also vertically. To complete the picture we might inquire how vertically dropped or launched objects are deflected – a topic that intrigued scientists for centuries.

During the seventeenth century the possible deflection of falling objects was considered as a

Continued on p. 269.

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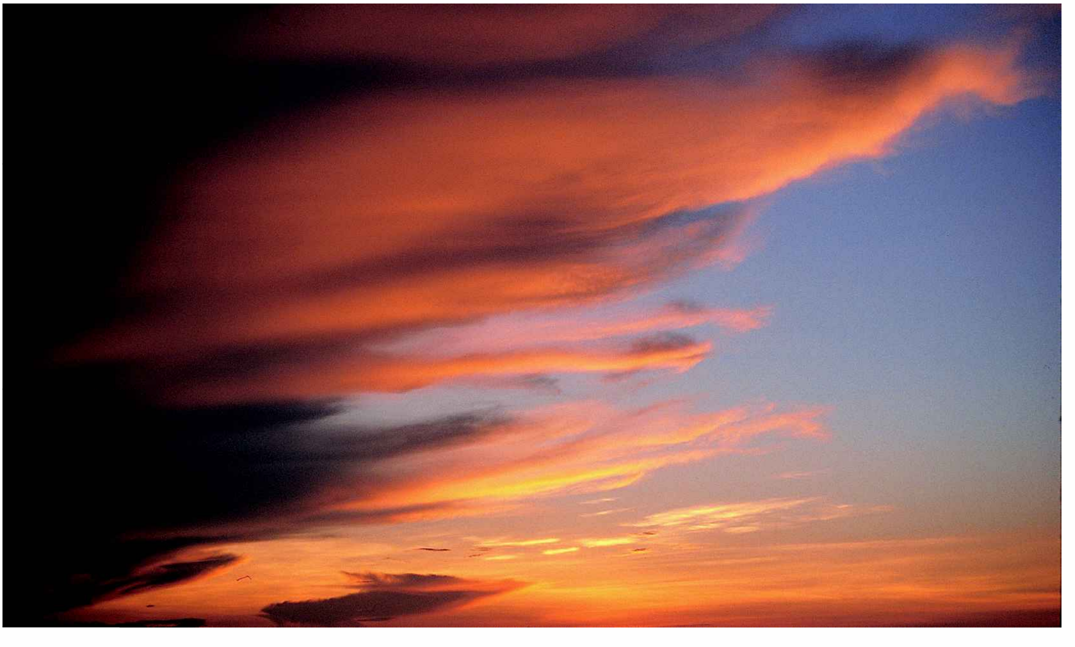
Numerous contrails lit by the morning sun over Ellel, Lancaster, at 0815 GMT on 19 December 2002



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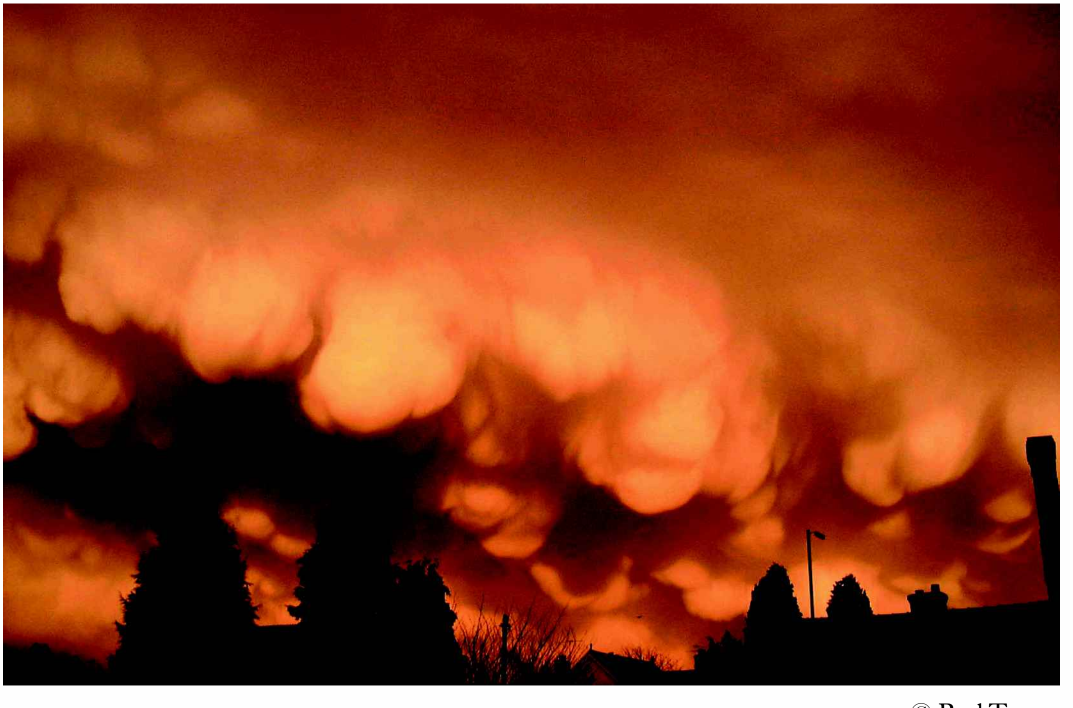
Dense cirrus cloud over Kuvula, Uganda, in June 1998

© J. F. P. Galvin

Brilliant sunset on a receding cold front over Newton Abbot, Devon, on 10 November 2002. Unusually, it is almost possible to imagine the frontal surface sloping away to the left in this picture.



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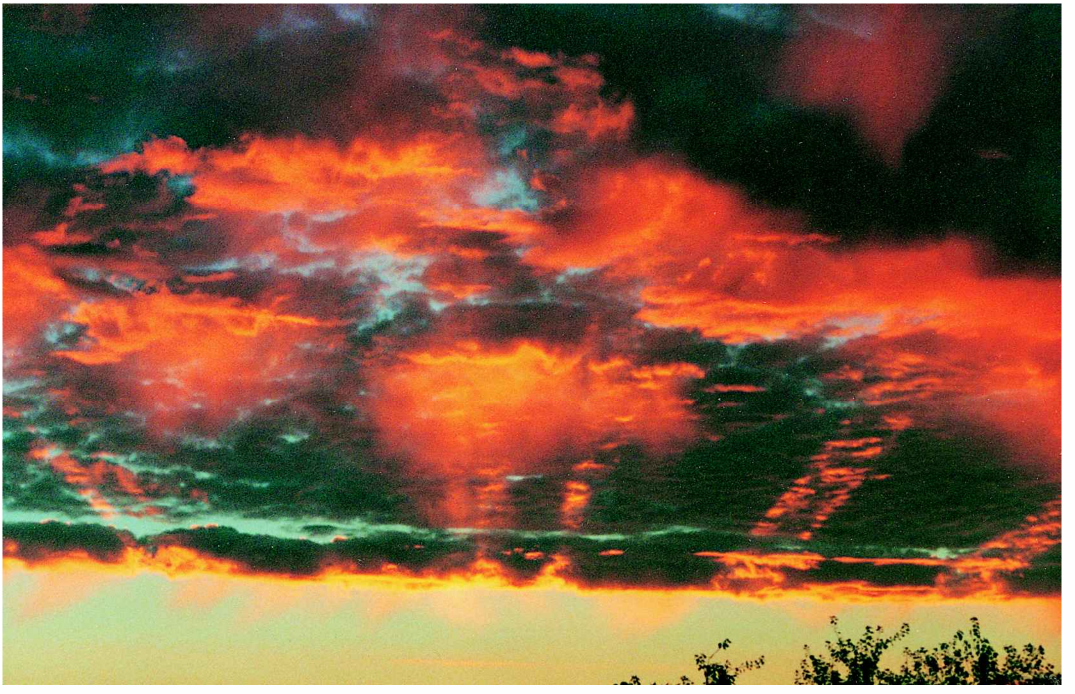
Mamma clouds following a severe storm on 1 March 2003 over Newport, Shropshire. The cloud formations were unusual and formed a talking point for many people in the area.



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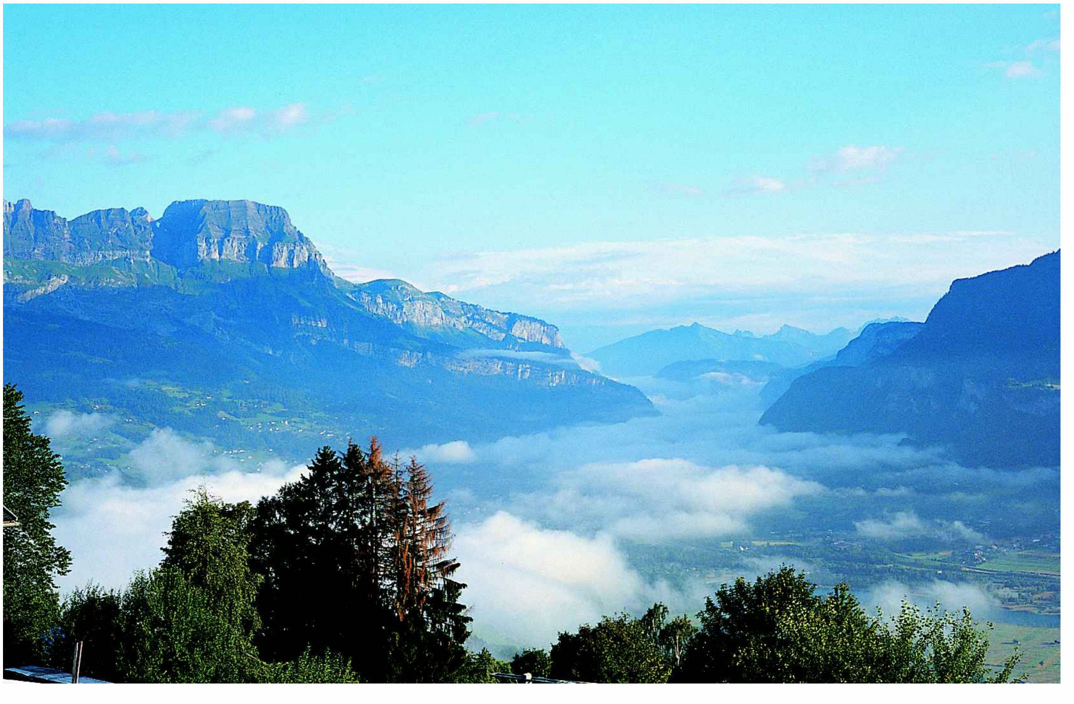
Line of high stratocumulus and an extensive area of altocumulus with virga at 2100 BST *on 26 July 2002 over Raunds, Northamptonshire. Note that the low sunlight is lighting the base of the altocumulus only where there are small gaps in the line of stratocumulus, giving the appearance of radiating lines.*



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Stratus in the Arve valley, Haut Savoi

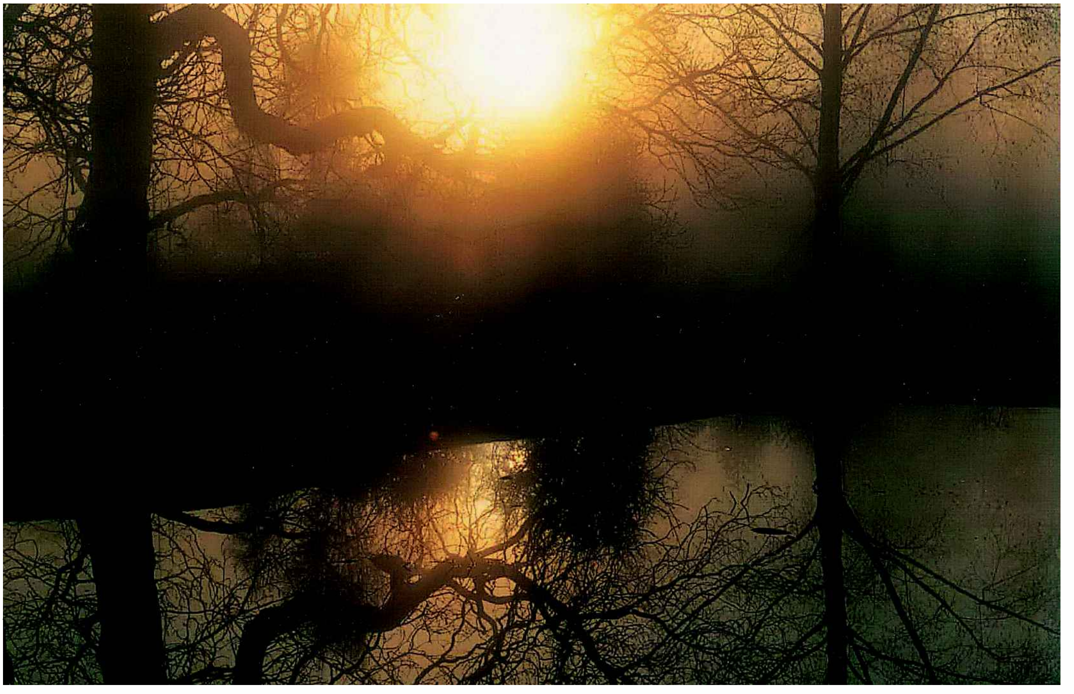

Fig. 2 "Sunsets and dawns". Early-morning mist over the lake, Whiteknights Campus, University of Reading, on 1 January 2000, following a clear still night. (© Diane Arnold.) (See letter on p. 280.)

Fig. 1 The Lynmouth storm by Geoffrey Webb. A copy of the painting hangs in the Royal Meteorological Society headquarters. (See letter on p. 281.)

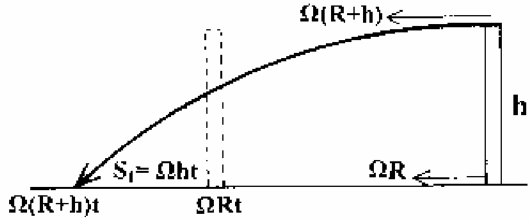

Fig. 1 A tower at the equator of height h *rotating with the earth (with radius* R *and angular velocity* Ω*) has a velocity of* ΩR *at the base and* Ω(R+h) *at the top. An object falling from the top of the tower with an acceleration* g *will have a horizontal velocity excess of* Ωh*, which, over the time of the fall,* $t = \sqrt{\frac{2h}{g}}$*, will carry the object a horizontal distance,* $S_1 = \frac{\Omega}{2}\sqrt{\frac{8h^3}{g}}$*. Away from the equator the deflection is proportional to the cosine of the latitude.*

way to prove or disprove the Copernican theory that the earth rotates and not the stars. The anti-Copernicans claimed that, if the earth was spinning around its axis, an object dropped from a tower would be 'left behind', *i.e.* deflected to the west. Galileo argued that this was wrong since the object would take part in the earth's rotation; but, he added, since the rotational velocity at the top of the tower would be slightly larger than at the surface, the falling object would actually overtake the tower and land slightly to the *east* of it (Fig.1). If we put Galileo's reasoning into mathematics, we will find that an object dropped from 100 m will, as seen from outside the earth, follow a parabolic path and be deflected 3 cm. Such small values were at that time difficult to confirm by measurements.*

The Coriolis deflection

Actually, the deflection according to Galileo's method is not quite correct and yields results which are 50% too large. We can understand this in two ways. The first is by treating the deflection as a consequence of the Coriolis effect (Fig. 2). The second way is to start from

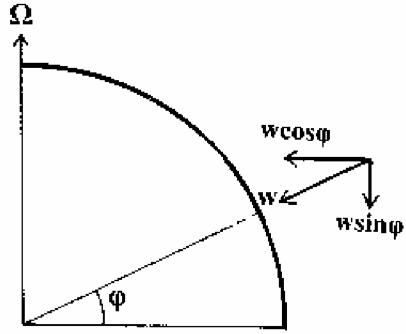

Fig. 2 The velocity, **w** = gt*, of a falling body can be split up into one component,* **w***sin*φ*, parallel to the earth's axis, and another component,* **w***cos*φ*, perpendicular to the earth's axis. The first will not be deflected since it is parallel to the rotation axis, the second will be deflected to the right (east), by a Coriolis force* $-2\Omega\mathbf{w}\cos\varphi$ *(per unit mass). Integrating this over the time of the fall from a height* h *yields a deflection* $S = \frac{\Omega\cos\varphi}{3}\sqrt{\frac{8h^3}{g}}$*.*

Galileo's approach, but to take into account that during the fall gravity will not point in the same direction. Due to the shape of the earth it will change with a component pointing increasingly back towards the starting point (Fig. 3).

The 'backward' acceleration reduces the 3 cm deflection by 1 cm to 2 cm, just as given

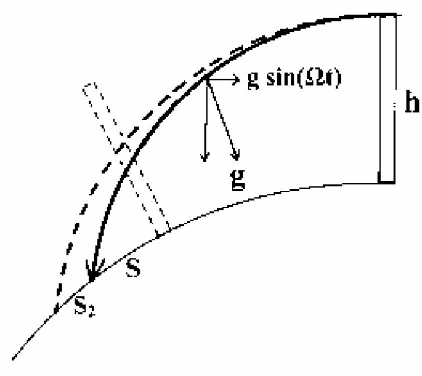

Fig. 3 The trajectory of a falling object, seen from outside the earth. Due to the curvature of the earth the object will be affected by a component of gravity, g*, pointing towards the centre of the earth. This backward acceleration,* a*, can be written* $a = -g\sin\Omega t \approx g\Omega t$*, which, integrated over* t*, the time of the fall, yields* $S_2 = -\frac{\Omega}{6}\sqrt{\frac{8h^3}{g}}$ *which, added to* S_1*, gives the correct deflection,* S*. Part of this explanation can also be used to calculate the deflection of an object shot vertically upwards with a velocity* V_0*, reaching a height* h*. Since there is no excess velocity, the only deflective mechanism is the changing direction of gravity yielding a westerly deflection twice* S_2 *(or* $4V_0^3\Omega/3g^2$*).*

* For in-depth analyses of the seventeenth-century complex, and often confused, debates on the deflection of falling objects, see Koyré (1955, pp. 358–395), Burstyn (1965, pp. 52–69) and Armitage (1947, pp. 343–346).

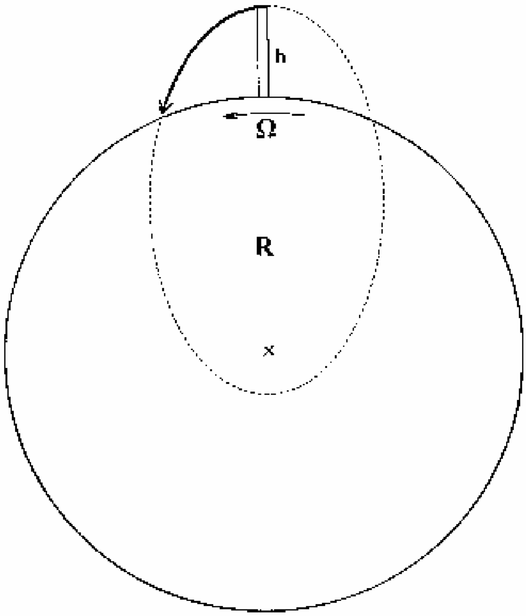

Fig. 4 Trajectory of a falling object at the equator, seen from outside the earth by an observer situated above the North Pole. The trajectory is elliptic, just like the orbits of the moon around the earth or the planets around the sun.

by the Coriolis effect.* More interestingly, retarded in its eastward motion, the object will, seen from outside the earth, follow an elliptic path (French 1971, pp. 591–592). This is, according to Kepler's laws,† the same type of curve that orbiting moons or planets follow (Fig. 4). This is no coincidence: the Coriolis effect and Kepler's laws are different ways of expressing the fundamental law of conservation of angular momentum.

"A fancy of my own"

The Italian debate fuelled the interest in the problem in England, and in 1674 Robert Hooke published a book entitled *An attempt to prove the motion of the earth*. It was in his capacity as the newly elected Secretary of the Royal Society that Hooke, in November 1679, wrote a letter to Isaac Newton. The intention was to draw Newton into a discussion on planetary motion; but Newton had something else on his mind, what he called "a fancy of my own" – the deflection of objects dropped from a high altitude as proof of the earth's rotation. Much later in his life Isaac Newton told his friends that it was watching apples fall from the tree in his family garden that made him speculate about earthly bodies and the moon being attracted by the same gravitational forces.* There are no reasons to doubt this; scientific ideas can grow out of childish inspiration. What has made scholars sceptical† is Newton's claim that the event took place in 1666, at a time we know that he was developing ideas in mathematics and optics. However, if we place the 'falling apple event' 13 years later, in 1679, it gains much more credibility. Newton had spent most of that summer and autumn at his family home in Lincolnshire. His mother had just died and he had to attend to family matters. There had been a lot of opportunities to see apples fall in the family garden.

The elliptic path

The exchange of letters with Hooke that followed during the winter of 1679/80 shows that Newton had not yet achieved a deeper understanding of celestial mechanics. His first idea was that a falling object would, in principle, approach the centre of the earth in a

* See Wild (1973) and Stirling (1983) for two short and illuminating discussions on different ways to derive the equations of vertically dropped or projected bodies.

† Kepler had never extended his planetary laws to the neighbourhood of the earth (Lohne 1960, p. 8). It was not until the late eighteenth century that it was realised that his Second Law, the "Law of areas", could also be applied to earthly objects and as such it became known as "conservation of angular momentum".

* Newton told three different persons the same story around 1726, 60 years after the alleged event, at a time when he was engaged in priority arguments with other scientists (see Gjertsen 1980, pp. 29–30; Cohen 1980, pp. 230–231; Westfall 1980, p. 154; Hall 1992, pp. 53–54).

† The scholars agree that we must settle on late 1679 as the crucial formative period in the development of the astronomical ideas which were to be synthesised in *Principia* (Cohen 1980, p. 222; Westfall 1980, p. 155, 1992, p. 64; Whiteside 1964, p. 120; Lohne 1960, p. 34).

spiral. Thanks to Hooke* he came to realise that it would follow an elliptic path as in Fig. 4. From this insight – that a falling object follows the same type of orbit as any of the planets around the sun – it is not far-fetched to infer that the motions of all these different bodies might be controlled by the same mechanism. Still, even for a genius like Newton, it took a few more years for the penny to drop.† He never discovered the Coriolis effect, but found instead the three laws of motion.

Gauss and Laplace

More than a century later there was a renewed interest in the problem of the deflection of falling objects. In 1803 an experiment, dropping iron pebbles in a 90 m deep mine shaft, was conducted in Schlebusch, Germany. The event attracted the interest of the scientific community, and the 24-year old German mathematician Carl Friedrich Gauss and the 53-year old French mathematician Pierre Simon de Laplace volunteered to calculate the theoretically expected deflection (Fig. 5).

There was a strong element of competition since Gauss had the year before managed to calculate the orbit of the newly discovered asteroid Ceres, something Laplace had deemed impossible. Both came up with the right answer by deriving the full three-dimensional equation for motions on a rotating earth. They specifically pointed out that the Coriolis terms (as we call them) were responsible for the deflection (Laplace 1803; Gauss 1804; Benzenberg 1804; Hagen 1912). Gauss and Laplace were the first scientists to contribute to the proof of the rotation of the earth some 50 years before Foucault's famous pendulum experiment, and to analyse correctly the relative motion in connection with rotation 30 years before Coriolis's mathematical paper.

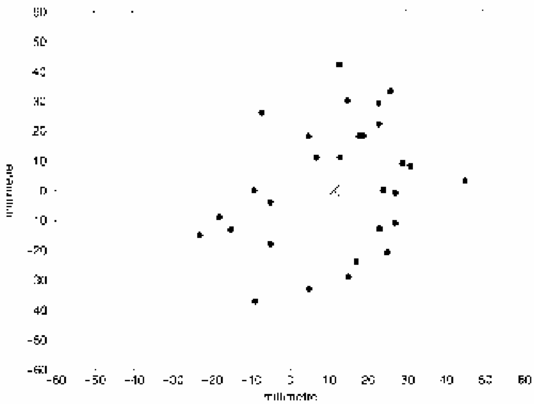

Fig. 5 Cluster of the hits from the Schlebusch experiment 1803. A cross marks the theoretically derived deflection. (From Hagen 1912.)

Erroneous treatments

The problem of the deflection of falling bodies is not trivial. Even the authorities can get it wrong. In the 1960s the problem of the deflection of a falling object was given in a nationwide Swedish school examination on the assumption that it could be solved in the incomplete way depicted in Fig. 1. The protests from the physics departments at the universities taught Swedish school administrators a healthy respect for the Coriolis force (Falk 1983). I have found two (non-Swedish!) university books on physics, which confuse the issue by considering the apparent deflection of an interstellar object, such as a meteorite, hitting the earth. Since the meteorite does not take part in the earth's rotation, approaching the earth it will, to an earthbound observer, be seen to deviate to the west.

Gyroscopic terms

We can now summarise the three-dimensional

*Nauenberg (1994) has made a strong case for Robert Hooke's "remarkable physical understanding" based on numerous experiments having a crucial importance to the development of Newton's thinking.

† For most of the 1600s Kepler's First and Second Laws were not widely understood or accepted, even by Isaac Newton. It was during his work on *Principia* in the mid-1680s that he came to realise their validity (Whiteside 1964, pp. 121 and 128–131) but also their shortcomings. It was, for example, only in 1685/86, when he questioned one of the fundamental parts of Kepler's theory, that he was able to formulate his Third Law. He then realised that the trajectory of an orbiting planet has its focus in the common centre of mass of the sun and the orbiting planet, and not just in the centre of the sun as stated by Kepler (Cohen 1992, pp. 234–236).

Coriolis deflections for different motions in an array where the mathematical terms have, for simplicity, been indicated only by their signs – see Table 1.

Table 1 Three-dimensional relation between the motion on a rotating planet and the Coriolis deflection

	Northward motion	Eastward motion	Downward motion
Northward deflection	0	−1	0
Eastward deflection	1	0	1
Downward deflection	0	−1	0

The number 0 means no deflection, 1 means deflection in the indicated direction and −1 deflection in the opposite direction. (For example −1 in the upper row represents both eastward motion deflected southwards and westward motion deflected northwards.) The deflections which involve vertical motions are proportional to the cosine of the latitude, while those which do not involve vertical motions are proportional to the sine of the latitude and hence change sign across the equator.

The three-dimensional Coriolis terms or, as Lord Kelvin called them, 'gyroscopic terms', play an important role in general laws concerning the stability of rotating systems (Sommerfeld 1952, pp. 168–169; Lyttleton 1953, p. 17). The gyroscopic terms can, if sufficiently strong, render a system stable when the usual energy conditions would indicate instability. In other words it is the three-dimensional Coriolis effect which provides a 'gyroscopic resistance' to children's spinning tops; but that is another story, perhaps to be told at some other time.

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